A Model of Money and Credit, with Application to the Credit Card Debt Puzzle

Irina A. Telyukova  Randall Wright
University of California – San Diego  University of Pennsylvania

March 22, 2007

Abstract

Many individuals simultaneously have significant credit card debt and money in the bank. The credit card debt puzzle is: given high interest rates on credit cards and low rates on bank accounts, why not pay down the debt? While economists have gone to elaborate lengths to explain this observation, we argue it is a special case of the rate of return dominance puzzle from monetary economics. We analyze the issue by extending standard monetary theory to incorporate consumer debt. This seems interesting in its own right, since developing models where money and credit coexist is a long-standing challenge, and it helps put into context recent discussions of consumer debt.

*We thank Neil Wallace, Ed Nosal, and participants of seminars at Penn, UC-Riverside, the Federal Reserve Banks of Cleveland and New York, and the 2006 SED meetings in Vancouver for feedback. We thank the National Science Foundation and the Jacob K. Javitz Graduate Fellowship Fund for research support. The usual disclaimer applies.
1 Introduction

A large number of households simultaneously have significant credit card debt and hold a significant amount of low-interest liquid assets. Although there are many ways to measure this, a simple summary statistic is that 27% of U.S. households in 2001 had credit card debt in excess of $500 and over $500 in the bank; and the median household in this group revolved around $3,800 of credit card debt and had $3,000 in the bank (see Telyukova 2006). The so-called credit card debt puzzle is this: given 14% interest rates on credit cards, and 1 or 2% on bank accounts, why not pay down the debt? According to Gross and Souleles (2001), “Such behavior is puzzling, apparently inconsistent with no-arbitrage and thus inconsistent with any conventional model.”

Economists have gone to elaborate lengths to explain such phenomena. For example, some people assume that consumers cannot control themselves (Laibson et al. 2000); others assume they cannot control their spouses (Bertaut and Haliassos 2002; Haliassos and Reiter 2003); and others hypothesize that such households are typically on the verge of bankruptcy (Lehnert and Maki 2001). While these ideas are interesting, and may contain elements of truth, we think it is useful to point out that the credit card debt puzzle is actually not a new observation: it is another manifestation of the venerable rate of return dominance puzzle from monetary economics. Hence, insights may be gained by using models and ideas from monetary theory, and in particular, taking seriously the notion of liquidity.1

Our hypothesis is simple. Households need to have money readily available – generally, they need to have liquid assets – for contingencies where it may be difficult or costly to use credit. In addition to the usual examples, like taxis and cigarettes, it is important to note that there are many big-ticket items for which this is the case. For instance, rent or mortgage payments cannot usually be made by credit card. Also, many unanticipated events such as household repairs (plumbing, heating, air conditioning, etc.) often require checks or even cash,

---

1The idea that agents may hold assets with low rates of return because they are relatively liquid – i.e. because they have an advantage as a medium of exchange – is formalized among other places in Kiyotaki and Wright (1989); as we discuss below, it goes back much further in the informal literature.
for whatever reason, and getting caught short in such situations can be very costly. Even if agents are revolving credit card debt, they need to have some cash easily accessible to meet these contingencies. The point may appear obvious; this does not mean it is unimportant.\footnote{To get some idea of the numbers involved, according to the U.S. Statistical Abstract, 77% of consumer transactions in 2001 used liquid assets defined as cash, bank deposits, and closely related instruments. According to the Consumer Expenditure Survey, the median household described above, with $3,800 of credit card debt and $3,000 in the bank, purchased goods worth $1,993 per month using liquid assets. See Telyukova (2006).}

The rate of return dominance question and the idea that some notion of liquidity ought to be part of the solution go back a long time. Hicks (1935) is well known for challenging monetary economists to “look frictions in the face” when framing “the central issue in the pure theory of money” as the need for an explanation of the fact that people hold money when interest rates are positive. The better-known version of his challenge is to explain “the decision to hold assets in the form of barren money, rather than of interest- or profit-yielding securities.” But the same issue arises in reverse: “So long as interest rates are positive, the decision to hold money rather than lend it, or use it to pay off old debts, is apparently an unprofitable one” (Hicks 1935, emphasis added). Hicks anticipated the credit card debt puzzle before there were credit cards.

We think there is something to be gained by analyzing consumer debt in the context of monetary theory. However, there does not exist an appropriate off-the-shelf model of money and credit, and it is not trivial to build one. Modern theory makes money essential by imposing some form of \textit{anonymity}, which obviously makes credit infeasible (of course, making credit difficult is precisely what makes money essential). We develop a generalized model where agents are anonymous in some situations but not others. While this seems natural, one has to specify the environment in such a way that when agents are not anonymous credit is actually useful, which is \textit{not} the case, e.g., in Lagos and Wright (2005), because of quasi-linear utility in the centralized market of that model. Moreover, if one tries to build a model where credit is useful when agents are not anonymous, without care, the analysis quickly becomes intractable, as is the case, e.g., if one abandons quasi-linear utility in the Lagos-Wright model. In our framework, money is essential and credit is useful, but the analysis is still very tractable: we can easily...
derive strong existence and characterization results, and these results help us to understand the coexistence of consumer debt and money in the bank.\footnote{To be clear from the outset, this paper is about theory; whether the approach is able to account quantitatively for salient aspects of the data is analyzed in Telyukova (2006). We summarize some of those findings in the Conclusion.}

2 The Basic Model

Here we describe the basic physical environment, focusing on a somewhat special case to make the main point; in Section 4 we discuss a variety of generalizations. To begin, there is a \([0,1]\) continuum of agents that live forever in discrete time. There is one nonstorable consumption good at each date that individuals are sometimes (stochastically) able to produce using labor. Following Lagos and Wright (2005), hereafter LW, we assume agents periodically visit both centralized markets and decentralized markets. Having some decentralized trade, with certain frictions discussed below, makes money essential. Having some centralized markets is interesting for its own sake, and makes the analysis much more tractable than models without this feature in the literature on the microfoundations of money.\footnote{See Molico (2006), Green and Zhou (1998,2002), Camera and Corbae (1999), Zhou (1999) or Zhu (2003,2005) for models where all trade is decentralized and the analysis is much more difficult. Earlier monetary models like Shi (1995) or Trejos and Wright (1995) are also simple, but only because money was assumed to be indivisible and agents were allowed to hold at most 1 unit.}

Money in this economy is a perfectly divisible and storable object that is intrinsically worthless but potentially could have value as a medium of exchange. The money supply is fixed for now at \(M\), but later we allow it to vary over time. Although we use the word money, we do not necessarily literally mean cash. It is not hard to recast the model with agents depositing their cash in bank accounts and paying for goods and services using checks or debit cards, as in He, Huang and Wright (2005, 2006). This is relevant because what we have in mind is relatively liquid assets generally: the money need not be in your pocket, it could be in the bank, but it does need to be easily accessible. Given realistic interest rates on demand deposits, except possibly for safety considerations money in the bank is about the same as may as money in your pocket, so we ignore the distinction for now and assume money is simply a perfectly liquid asset.
with 0 interest. Later we introduce an additional real asset that can always be liquidated, but at a fixed cost, and determine its return endogenously.

In LW, each period is divided into two subperiods. In one, there is a centralized Walrasian market, and in the other, there is a decentralized market where agents meet according to a random bilateral-matching process. With the additional assumption that agents are anonymous in the decentralized market, a medium of exchange becomes essential, as is well known; see Kocherlakota (1998), Wallace (2001), Corbae et al. (2003), and Aliprantis et al. (2006) for formal discussions. After each meeting of this market, agents go to a centralized market, where they engage in various activities, including working and rebalancing their money holdings. If utility is linear in hours worked, all agents take the same amount of money out of the centralized market, which is what keeps things simple.

There is no role for credit in LW, and it is important to understand why. First, credit is not possible in the decentralized market because of the assumption that agents are anonymous, which we cannot relax since this is what makes money essential. Second, credit is not necessary in the centralized market because of the assumption that all agents can work and have utility that is linear in hours, which we do not want to relax since this is what makes the analysis tractable. How to proceed? Our idea is to introduce a third subperiod – generalized later to many subperiods – where some agents want to consume but cannot produce, which makes credit useful, and where we do not assume anonymity, which makes credit feasible. We determine whether agents use cash or credit in this market endogenously, while maintaining an essential role for money plus analytic tractability due to the other two markets.5

All agents want to consume in subperiod (market) 1, and \( u_1(x_1) \) is their common utility function, which is strictly increasing and weakly concave. A random subset want to consume in \( s = 2, 3 \), and conditional on this, \( u_s(x_s) \) is their utility function, which is strictly increasing and concave. All agents are able to produce in \( s = 1 \), and the disutility of working \( h_1 \) hours is linear,

---

5Berentsen et al. (2005a) also add a third subperiod to LW, but it is another round of decentralized exchange, so there is no possibility of credit. Berentsen et al. (2005b) and Chiu and Meh (2006) introduce a third subperiod with a centralized market in order to discuss banking, but they focus on an entirely different set of issues.
\[ c_1(h_1) = h_1. \] A random subset are able to produce in \( s = 2, 3, \) and conditional on this, the disutility of working is \( c_s(h_s), \) which is strictly increasing and convex. When they can produce, agents transform labor one-for-one into goods, \( x_s = h_s. \) For simplicity, at any \( s = 2, 3 \) a random set of agents chosen in an i.i.d. manner want to consume but cannot produce, and vice-versa, while no one can do both (this is easy to relax). Let \( x_s^* \) denote the solution to \( u'_s(x_s^*) = c'_s(x_s^*). \) Let \( \beta_s \) be the discount factor between \( s \) and the next subperiod, with \( \beta_1 \beta_2 \beta_3 < 1. \)

An individual’s state is \((m_{st}, b_{st})\), denoting money and debt in subperiod \( s \) of period \( t. \) We drop the \( t \) when there is no risk of confusion, writing \( m_{st} = m_s, m_{s,t+1} = m_{s+1}, \) etc. Let \( W_s(m_s, b_s) \) be the value function. At \( s = 1, 2, \) the market value of money is \( \phi_s, \) so \( 1/\phi_s \) is the nominal price level; there is no \( \phi_3 \) since there is no centralized market at \( s = 3, \) although prices will implicitly be defined by whatever trade happens to occur. Similarly, the real interest rate in the centralized market at \( s = 1, 2 \) is \( r_s, \) but there is no \( r_3, \) where our convention for notation is as follows: if you bring debt \( b_s \) into subperiod \( s \) you owe \( (1 + r_s)b_s. \) Also, we assume away all enforcement problems with credit – repayment is simply taken for granted, as is the case in most (but not all) general equilibrium theory.\(^6\)

The plan now is to consider behavior each subperiod (market) in turn, then put the pieces together to characterize equilibrium.

### 2.1 Market 1

At \( s = 1, \) there is a centralized market where agents solve\(^7\)

\[
W_1(m_1, b_1) = \max_{x_1, h_1, m_2, b_2} \{ u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2) \} \tag{1}
\]

s.t. \( x_1 = h_1 + \phi_1(m_1 - m_2) - (1 + r_1)b_1 + b_2. \)

\(^6\)It may be interesting to make enforcement endogenous, as Berentsen et al. (2005b) do in a related model, but it would be mainly a distraction for current purposes. Still, it is important to note that our assumed exogenous enforcement mechanism does not render money inessential: we can enforce credit when the consumer is known; we can enforce nothing, and so money is needed, when the consumer is anonymous.

\(^7\)To rule out Ponzi schemes, one normally imposes a credit limit \( b_j \leq B, \) either explicitly or implicitly. We impose that agents pay off past debts at \( s = 1 \) each period, without loss in generality (given they have to pay off debt at some point, they are quite happy to pay it off in market \( s = 1 \) at any date \( t \) since then they have quasi-linear utility). Also, we always assume an interior solution for \( h; \) see LW for conditions to guarantee this is valid in these types of models.
Substituting $h_1$ from the budget constraint into the objective function and differentiating, the first-order and envelope conditions are

\begin{align}
1 & = u'_1(x_1) \quad \text{(2)} \\
\phi_1 & = \beta_1 W_{2m}(m_2, b_2) \quad \text{(3)} \\
-1 & = \beta_1 W_{2b}(m_2, b_2). \quad \text{(4)}
\end{align}

\[ W_{1m}(m_1, b_1) = \phi_1 \quad \text{(5)} \]
\[ W_{1b}(m_1, b_1) = -(1 + r_1). \quad \text{(6)} \]

Notice (2) implies $x_1 = x_1^*$ for all agents, while (3)-(4) imply $(m_2, b_2)$ is independent of $x_1$ and $(m_1, b_1)$. Also, as long as $W_2$ is strictly concave, there will be a unique solution for $(m_1, b_1)$. It is simple to check that the same conditions that guarantee strict concavity in $m$ from LW also apply here, and so $m_2 = M$ for all agents. However, we will see that $W_2$ is actually linear in $b_2$, which means we cannot pin down $b_2$ for any individual. This is no surprise: with a competitive market and quasi-linear utility, in equilibrium, agents are indifferent between the allocation they have and an alternative where they work a little more now, save the proceeds, and work a little less later. Although this is true for any individual, it cannot be true in the aggregate, since average labor input is pinned down by feasibility at $\bar{h}_1 = x_1^*$.

Given this, one can resolve the indeterminacy for individuals in two ways. First, one can focus on symmetric equilibria where all agents choose the same solution when they have the same set of solutions to a maximization problem, which is innocuous since other equilibria are payoff equivalent and observationally equivalent at the aggregate level; this pins down $b_2 = \bar{b}_2$ for all agents. Alternatively, we could impose an arbitrarily small transaction cost on revolving debt in subperiod 1, which would break agents’ indifference and refine away the other equilibria. In what follows we take the former route, and simply focus on symmetric equilibria. Then, aggregating budget equations across agents, we have

\[ \bar{x}_1 = \bar{h}_1 + \phi_1(\bar{m}_1 - \bar{m}_2) - (1 + r_1)\bar{b}_1 + \bar{b}_2. \quad \text{(7)} \]
In equilibrium, $\bar{h}_1 = \bar{x}_1 = x^*_1$, $\bar{m}_1 = \bar{m}_2 = M$, and $\bar{b}_1 = 0$ (average debt is 0 since consumers ultimately borrow only from each other). Hence, (7) implies $b_2 = \bar{b}_2 = 0$ for all agents.

### 2.2 Market 2

At $s = 2$, a measure $\pi$ of agents want to consume but cannot produce and a measure $\pi$ can produce but do not want to consume; the remaining $1 - \pi$ do neither. In equilibrium, $x^C_2 = h^P_2$, where $x^C_2$ is the consumption of consumers and $h^P_2$ the production of producers. The expected value of entering market 2 is therefore

$$W_2(m_2, b_2) = \pi W^C_2(m_2, b_2) + \pi W^P_2(m_2, b_2) + (1 - 2\pi)W^N_2(m_2, b_2),$$

(8)

where $W^C_2$, $W^P_2$ and $W^N_2$ are the value functions for a consumer, a producer and a nontrader. It is tedious but useful to study their problems one at a time.

For a nontrader,

$$W^N_2(m_2, b_2) = \max_{m_3, b_3} \beta_2 W_3(m_3, b_3)$$

s.t. $0 = \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.$

Although nontraders neither consume nor produce, they can adjust their portfolios, but we will see below that in equilibrium they choose not to. The solution $(m^N_3, b^N_3)$ satisfies

$$W_{3m}(m^N_3, b^N_3) = -\phi_2 W_{3b}(m^N_3, b^N_3),$$

(9)

plus the budget equation. The envelope conditions are

$$W^N_{2m}(m_2, b_2) = \beta_2 W_{3m}(m^N_3, b^N_3)$$

(10)

$$W^N_{2b}(m_2, b_2) = \beta_2 (1 + r_2) W_{3b}(m^N_3, b^N_3).$$

(11)

For a consumer,

$$W^C_2(m_2, b_2) = \max_{x_2, m_3, b_3} \{u_2(x_2) + \beta_2 W_3(m_3, b_3)\}$$

s.t. $x_2 = \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.$
The solution \((x^C_2, m^C_3, b^C_3)\) satisfies
\[
\phi_2 u'_2(x^C_2) = \beta_2 W_{3m}(m^C_3, b^C_3) \\
-u'_2(x^C_2) = \beta_2 W_{3b}(m^C_3, b^C_3)
\]
plus the budget equation. The envelope conditions are
\[
W^C_{2m}(m_2, b_2) = \phi_2 u'_2(x^C_2) = \beta_2 W_{3m}(m^C_3, b^C_3) \\
W^C_{2b}(m_2, b_2) = -(1 + r_2)u'_2(x^C_2) = (1 + r_2)\beta_2 W_{3b}(m^C_3, b^C_3).
\]

For a producer,
\[
W^P_2(m_2, b_2) = \max_{h_2,m_3,b_3} \{-c_2(h_2) + \beta_2 W_3(m_3, b_3)\}
\]
\[
s.t. 0 = h_2 + \phi_2(m_2 - m_3) - (1 + r_2)b_2 + b_3.
\]

The solution \((h^P_2, m^P_3, b^P_3)\) satisfies
\[
\phi_2 c'_2(h^P_2) = \beta_2 W_{3m}(m^P_3, b^P_3) \\
-c'_2(h^P_2) = \beta_2 W_{3b}(m^P_3, b^P_3)
\]
plus the budget equation. The envelope conditions are
\[
W^P_{2m}(m_2, b_2) = \phi_2 c'_2(h^P_2) = \beta_2 W_{3m}(m^P_3, b^P_3) \\
W^P_{2b}(m_2, b_2) = -(1 + r_2)c'_2(h^P_2) = (1 + r_2)\beta_2 W_{3b}(m^P_3, b^P_3).
\]

We cannot conclude that \((m_3, b_3)\) is independent of \((m_2, b_2)\), the way we could conclude that \((m_2, b_2)\) is independent of \((m_1, b_1)\) in the previous subperiod, since in this market we generally do not assume \(u_2\) and \(c_2\) are linear. But in any case,
\[
W_{2m}(m_2, b_2) = \beta_2[\pi W_{3m}(m^C_3, b^C_3) + \pi W_{3m}(m^P_3, b^P_3) + (1 - 2\pi) W_{3m}(m^N_3, b^N_3)]
\]
\[
W_{2b}(m_2, b_2) = \beta_2(1 + r_2)[\pi W_{3b}(m^C_3, b^C_3) + \pi W_{3b}(m^P_3, b^P_3) + (1 - 2\pi) W_{3b}(m^N_3, b^N_3)]
\]
2.3 Market 3

In market 3 we assume that trade occurs via anonymous bilateral meetings and bargaining, although it is important to emphasize that bargaining is not crucial: e.g. Rocheteau and Wright (2005) analyze similar models with price taking and with price posting, while Kircher and Galenianos (2006) analyze versions with auctions, and all of our results go through with these alternative pricing mechanisms. In any case, in market 3, you cannot use credit due to anonymity: I will not take your IOU because I know you could renege without fear of punishment. Still, one may ask why some institution that is not anonymous does not issue interest-bearing claims – i.e. private money – that circulate in market 3. One answer is to say that such claims can be counterfeited, and government has a monopoly on the production of non-counterfeitable notes; see Lester et al. (2007) for details. These assumptions are strong, but they are logically consistent, and provide a coherent environment with roles for both money and credit.8

Consider a meeting where one agent wants to consume and the other can produce. Call the former agent the buyer and the latter the seller. They bargain over the amount of consumption for the buyer \(x_3\) and labor by the seller \(h_3\), and also a dollar payment \(d\) from to the former to the latter. Since feasibility implies \(x_3 = h_3\), we denote their common value by \(q\). If \((m_3,b_3)\) is the state of a buyer and \((\tilde{m}_3,\tilde{b}_3)\) the state of a seller, the outcome satisfies the generalized Nash bargaining solution,

\[
(q, d) \in \arg \max S(m_3, b_3)^\theta S(\tilde{m}_3, \tilde{b}_3)^{1-\theta} \text{ s.t. } d \leq m_3,
\]

where the constraint says an agent cannot give more money than he has, \(\theta\) is the bargaining

---

8 As we said above, there are related models where agents deposit their money in banks and pay with checks or debit cards in decentralized markets. This works, even when individuals are anonymous, because these instruments are claims on the bank and not on the consumer (consider a travellers’ check as the purest example). Interest on this private (inside) money is endogenous, but it will not generally equal the market interest rate on consumer credit, for several reasons. One is the legal prohibition of interest on checking deposits, which was in place for much of the last century. Another is bank operating costs. And another is the need for banks to hold some non- or low-interest assets as reserves, either to meet legal requirements or to facilitate settlement. Since actual interest rates on demand deposits are quite low – even negative when one accounts for fees etc. – we ignore this detail and assume money bears no interest. In He et al. (2005,2006) consumers keep their money in the bank even at negative interest rates for safekeeping against theft.
power of the buyer, and the surpluses are given by

\[
S(m_3, b_3) = u_3(q) + \beta_3 W_{1,+1}(m_3 - d, b_3) - \beta_3 W_{1,+1}(m_3, b_3)
\]

\[
\tilde{S}(\tilde{m}_3, \tilde{b}_3) = -c_3(q) + \beta_3 W_{1,+1}(\tilde{m}_3 + \tilde{d}, \tilde{b}_3) - \beta_3 W_{1,+1}(\tilde{m}_3, \tilde{b}_3).
\]

Using (5) and (6), we have the neat simplification

\[
S(m_3, b_3) = u_3(q) - \beta_3 \phi_{1,+1} d
\]

\[
\tilde{S}(\tilde{m}_3, \tilde{b}_3) = -c_3(q) + \beta_3 \phi_{1,+1} \tilde{d}.
\]

This easily yields the following generalization of LW (a proof is in the Appendix).

**Lemma 1.** \(\forall(m_3, b_3)\) and \((\tilde{m}_3, \tilde{b}_3)\), the solution to the bargaining problem is

\[
q = \begin{cases} 
 g^{-1}(\beta_3 m_3 \phi_{1,+1}) & \text{if } m_3 < m_3^* \\
 q^* & \text{if } m_3 \geq m_3^* 
\end{cases}
\]

and

\[
d = \begin{cases} 
 m_3 & \text{if } m_3 < m_3^* \\
 m_3^* & \text{if } m_3 \geq m_3^* 
\end{cases}
\]

(23)

where \(q^*\) solves \(u'_3(q^*) = c'_3(q^*)\), the function \(g(\cdot)\) is given by

\[
g(q) \equiv \frac{\theta u'_3(q) c_3(q) + (1 - \theta) u_3(q) c'_3(q)}{\theta u'_3(q) + (1 - \theta) c'_3(q)},
\]

(24)

and \(m_3^* = g(q^*) / \beta_3 \phi_{1,+1}\).

Observe that the bargaining solution \((q, d)\) depends on the buyer’s money holdings \(m_3\) and not on any other element of \((m_3, b_3)\) or \((\tilde{m}_3, \tilde{b}_3)\); hence we write \(q = q(m_3)\) and \(d = d(m_3)\) from now on. We also show in the Appendix, exactly as in LW, that \(m_3 < m_3^*\) in any equilibrium. Hence, buyers always spend all their money in market 3, \(d(m_3) = m_3\), and receive \(q = g^{-1}(\beta_3 m_3 \phi_{1,+1}) < q^*\) in return, which yields \(\partial q / \partial m_3 = \beta_3 \phi_{1,+1} / g'(q) > 0\).

Let \(\sigma\) be the probability of a meeting between a buyer and seller in market 3. Then

\[
W_3(m_3, b_3) = \sigma \{ u_3[q(m_3)] + \beta_3 W_1[m_3 - d(m_3), b_3] \}
\]

\[
+ \sigma E \{ -c_3[q(\tilde{m}_3)] + \beta_3 W_1[m_3 + d(\tilde{m}_3), b_3] \} + (1 - 2\sigma) \beta_3 W_1[m_3, b_3],
\]

(25)
where $\mathbb{E}$ is the expectation of $\tilde{m}_3$ (the money holdings of a random agent one meets, which we later show is degenerate at $\tilde{m}_3 = M$). Differentiating (25) using (5) and (6),

\begin{align}
W_{3m}(m_3, b_3) &= \beta_3 \phi_{1,+1} \{ \sigma e[q(m_3)] + 1 - \sigma \} \\
W_{3b}(m_3, b_3) &= -\beta_3 (1 + r_{1,+1}),
\end{align}

where the function $e(\cdot)$ in (26) is given by

\begin{equation}
 e(q) \equiv u'_3(q)/g'(q).
\end{equation}

We assume $e'(q) < 0$.\(^9\)

## 3 Equilibrium

Our definition of equilibrium is relatively standard, except there is no market-clearing condition for market 3, since with bilateral trade it clears automatically. Also, to reduce notation, we describe every agent’s problem at $s = 1, 2$ in terms of choosing $(x_s, h_s, m_{s+1}, b_{s+1})$, which are implicitly functions of the state, and it is understood that for producers $x_2^p = 0$, for consumers $h_2^C = 0$, and for nontraders $x_2^N = h_2^N = 0$.\(^{10}\)

**Definition 1.** An equilibrium is a set of (possibly time-dependent) value functions $\{W_s\}$, $s = 1, 2, 3$, decision rules $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, $s = 1, 2$, bargaining outcomes $\{q, d\}$, and prices $\{r_s, \phi_s\}$, $s = 1, 2$, such that:

1. **Optimization:** In every period, for every agent, $\{W_s\}$, $s = 1, 2, 3$, solve the Bellman equations (1), (8) and (25); $\{x_s, h_s, m_{s+1}, b_{s+1}\}$, $s = 1, 2$, solve the relevant maximization problems; and $\{q, d\}$ solve the bargaining problem.

---

\(^9\)Sufficient conditions on preferences to guarantee $e'(q) < 0$ for all $q$ can be found in LW. A simple assumption that works for any preferences is $\theta \approx 1$, since $\theta = 1$ implies $g(q) = c(q)$. But even if we do not have $e'(q) < 0$ for all $q$, it is easy to check that $e'(q) < 0$ for any $q$ that satisfies the second order conditions to the maximization problem in market 2, so this assumption is really not much of a restriction.

\(^{10}\)We do not include the distribution of the state variable in the definition of equilibrium, but it is implicit: given an initial distribution $F_1(m, b)$ at the start of subperiod 1, the decision rules generate $F_2(m, b)$; then the decision rules at $s = 2$ generate $F_3(m, b)$; and the bargaining outcome at $s = 3$ generates $F_{1,+1}(m, b)$. Also, as we said above, we only consider equilibria where we have an interior solution for $h$.\[^{12}\]
2. Market clearing: In every period,

\[ \bar{x}_s = \bar{h}_s, \bar{m}_{s+1} = M, \bar{b}_{s+1} = 0, s = 1, 2 \]

where for any variable \( y \), \( \bar{y} = \int y \, di \) denotes the aggregate.

**Definition 2.** A steady state equilibrium is an equilibrium where the endogenous variables are constant across periods, although generally not across subperiods within a period.

We are mainly interested in equilibria where money is valued, which means it must be valued in all subperiods in every period.

**Definition 3.** A monetary equilibrium is an equilibrium where, in every period, \( \phi_s > 0, s = 1, 2, \) and \( q > 0. \)

**Theorem 1.** There exists a steady state monetary equilibrium, and it is characterized by:

1. At \( s = 1 \), all agents choose the same consumption \( x_1 = x_1^* \), portfolio \((m_2, b_2) = (M, 0)\), and as a function of their individual states \((m_1, b_1)\) hours

\[ h_1 = h_1(m_1, b_1) = x_1^* - \phi_1(m_1 - M) + (1 + r_1)b_1, \]

which implies that on average \( h_1 \) is \( \bar{h}_1 = x_1^* \).

2. At \( s = 2 \),

- consumers choose \( x_2 = x_2^* \), \( m_3 = M \) and \( b_3 = x_2^* \);  
- producers choose \( h_2 = x_2^* \), \( m_3 = M \) and \( b_3 = -x_2^* \);  
- nontraders choose \( m_3 = M \) and \( b_3 = 0 \).

3. At \( s = 3 \), in every trade \( d = M \) and \( q \) solves

\[ 1 + \frac{\rho}{\sigma} = e(q), \tag{29} \]

where \( e(q) \) is given by (28) and \( \rho \) is defined by \( \frac{1}{1 + \rho} = \beta_1 \beta_2 \beta_3 \), which implies \( q < q^* \).
4. Prices are given by:

\[ r_1 = \frac{u'(x^*_2) - \beta_2 \beta_3}{\beta_2 \beta_3}, \quad r_2 = \frac{\beta - r_1}{1 + r_1}. \]

\[ \phi_1 = \frac{q(q)}{\beta_3 M}, \quad \text{and} \quad \phi_2 = \frac{\phi_1 [\sigma e(q) + 1 - \sigma]}{1 + r_1}. \]

**Proof:** To begin, insert the envelope condition for \( W_{3b} \) from (27) into (13) and (17) to get

\[ u'_2(x^C_2) = \beta_2 \beta_3 (1 + r_{1,+1}) \]

(30)

\[ c'_2(h^P_2) = \beta_2 \beta_3 (1 + r_{1,+1}). \]

(31)

This implies \( u'_2(x^C_2) = c'_2(h^P_2) \), and hence \( x^C_2 = h^P_2 = x^*_2 \). Similarly, insert the envelope condition for \( W_{3m} \) from (26) into the first order conditions (12) and (16) to get

\[ \phi_2 u'_2(x^C_2) = \beta_2 \beta_3 \phi_{1,+1} \{ \sigma e[q(m^C_3)] + 1 - \sigma \} \]

(32)

\[ \phi_2 c'_2(h^P_2) = \beta_2 \beta_3 \phi_{1,+1} \{ \sigma e[q(m^P_3)] + 1 - \sigma \}. \]

(33)

Given \( e'(q) < 0 \) and \( q'(m) > 0 \) for all \( m < m^*_3 \), plus \( x^C_2 = h^P_2 = x^*_2 \), we conclude \( m^C_3 = m^P_3 \).

Similarly, inserting (27) and (26) into the first order condition for a nontrader,

\[ \phi_{1,+1} \{ \sigma e[q(m^N_3)] + 1 - \sigma \} = \phi_2(1 + r_{1,+1}). \]

(34)

Exactly the same condition results from combining (30) and (32) for a consumer, or (31) and (33) for a producer. Hence, we conclude \( m^N_3 = m^C_3 = m^P_3 = M \).

>From the budget equations,

\[ b^C_3 = x^*_2 + (1 + r_2)b_2 \]

\[ b^P_3 = -x^*_2 + (1 + r_2)b_2 \]

\[ b^N_3 = (1 + r_2)b_2. \]

This completes the description of market 2. Moving back to market 1, clearly (2) implies \( x_1 = x^*_1 \). Inserting the envelope conditions for \( W_2 \) and \( W_3 \) into (3) and (4), we have

\[ \phi_1 = \beta_1 \beta_2 \beta_3 \phi_{1,+1} \{ \sigma e[q(M)] + 1 - \sigma \} \]

(35)

\[ 1 = \beta_1 \beta_2 \beta_3 (1 + r_2)(1 + r_{1,+1}). \]

(36)
where we use in the first case the result that $W_{3m}$ depends on $m_3$ but not $b_3$, and $m_3 = M$. Notice (36) is an arbitrage condition between $r_2$ and $r_{1,+1}$: if it does not hold there is no solution to the agents’ problem at $s = 1$; and if it does hold then any choice of $b_2$ is consistent with optimization. Hence we can set $b_2 = 0$. On the other hand, (35) implies

$$
(1 + \rho) \frac{\phi_1}{\phi_{1,+1}} = \sigma e[q(M)] + 1 - \sigma.
$$

(37)

In steady state this implies (29). It is now standard (see LW) to show $q < q^*$. This pins down the allocation. Now consider prices. We get $r_1$ from (30) with $x_2 = x_2^*$, and then set $r_2$ in terms of $r_1$ to satisfy the arbitrage condition (36). Given $q$, Lemma 1 tells us $\phi_1$, and (34) tells us $\phi_2$, as described in the statement of the Theorem. This is all we need for the definition of equilibrium (plus the value functions, but these are obvious). By construction, this constitutes a steady state monetary equilibrium. ■

The central result of Theorem 1 is that, at $s = 2$, consumers buy on credit ($b_3 = x_2^*$), even though they have cash on hand ($m_2 = m_3 > 0$). The reason, of course, is they know that at $s = 3$ they may need the money. Notice the theorem not only characterizes equilibrium, it also establishes existence. We can actually say more: the steady state equilibrium is unique subject to the caveat mentioned above; so it is not only that consumers with cash may buy on credit – this must be true in equilibrium.\(^{11}\)

We now discuss rates of return. Condition (34) equates the value of a dollar’s worth of cash and a dollar’s worth of credit coming out of market 2. The left side is a weighted average of the marginal gain if the dollar is spent in market 3, $u'(q)q'(m_3) = \beta_3 \phi_{1,+1} e(q)$, and the return if it is not spent but carried forward to the next period, $\beta_3 \phi_{1,+1}$. The right side of (34) is the real return (the interest saved) from using the same dollar to pay down debt, $\beta_3 \phi_2 (1 + r_{1,+1})$. Notice the return to money includes a liquidity premium: since $e(q) > 1$ in equilibrium (see below) the

\(^{11}\)The caveat concerning uniqueness is that we impose in market 1 that all agents choose the same solution for $b_2$ when they have multiple solutions. As we said, other equilibria are payoff equivalent and observationally equivalent in the aggregate, so this is not much of a restriction. Moreover, in any other equilibria, prices and consumption are exactly as stated in Theorem 1, but individuals may choose to roll over debt between periods, which affects the timing of their labor supply but not the results concerning money and credit at $s = 2$. 

15
value of spending a dollar at \( s = 3 \) is higher than the value of carrying it to next period. If one ignores the liquidity premium and simply considers the return on carrying money across periods, then it looks like – indeed, it is – rate of return dominance.

**Theorem 2.** (Rate of Return Dominance) In any steady state monetary equilibrium,

\[
\frac{\phi_{1,+1}}{\phi_2} < 1 + r_{1,+1}.
\]

**Proof:** By (34),

\[
\frac{\phi_{1,+1}}{\phi_2} = \frac{1 + r_{1,+1}}{1 - \sigma + \sigma e(q)}.
\]

The result follows if \( e(q) > 1 \). By (29), in steady state \( e(q) = 1 + \rho/\sigma > 1 \). \( \blacksquare \)

4 Discussion

Here we make several remarks about the above results and discuss some extensions. First, we can always price nominal bonds traded in \( s = 2 \) at \( t \) and redeemed in \( s = 1 \) at \( t + 1 \) using the no-arbitrage condition known as the Fisher equation to get the nominal interest rate

\[
1 + i_{1,+1} = (1 + r_{1,+1}) \frac{\phi_2}{\phi_{1,+1}}. \tag{12}
\]

Then Theorem 2 can be equivalently stated as \( i_{1,+1} > 0 \). The nominal rate is the opportunity cost of carrying cash, which agents are willing to pay, for the benefit of liquidity. Also, the results above are framed in terms of rates of return between \( s = 2 \) at \( t \) and \( s = 1 \) at \( t + 1 \), because it is at \( s = 2 \) that the key decision is made to pay with cash or credit, but we can alternatively measure returns over the entire period. From \( s = 1 \) at \( t \) to \( s = 1 \) at \( t + 1 \), the gross return on money in steady state is 1, while the return on credit (paying down debt) is \((1 + r_2)(1 + r_{1,+1})\). We readily get \((1 + r_2)(1 + r_{1,+1}) > 1 \) from (36), so obviously rate of return dominance holds across the entire period, too.

Another point is that in *any* equilibrium, and not just in steady state, essentially everything in Theorems 1 and 2 holds, except that (37) does not reduce to (29). However, we can insert

---

12 For the sake of this discussion, assume that bonds are illiquid – i.e. they cannot be used as a medium of exchange in market 3, either because they are not tangible objects but merely book-keeping entries, say, or because they are not recogniziable (i.e. they are counterfeitable) in market 3, even though they are recogniziable (can be verified) in market 1.
\( g(q) = \beta_3 m_3 \phi_{1,+1} \) from Lemma 1 to get

\[
(1 + \rho) \frac{g(q)}{g(q+1)} = \sigma e(q+1) + 1 - \sigma. \tag{38}
\]

A monetary equilibrium is characterized by a (positive) bounded path \( \{q_t\} \) solving the difference equation (38), along with values for the other objects satisfying the conditions above. As in most monetary models, there will be multiple equilibrium paths \( \{q_t\} \), including exotic dynamic and sunspot equilibria (Lagos and Wright 2003). But in all of these equilibria \( x_s, b_s, \) and \( r_s \) are exactly as in Theorem 1, and although \( \phi_s \) will change over time, Theorem 2 holds exactly as stated.

Another generalization is to allow the money supply to vary over time. Suppose e.g. \( M_{t+1} = (1 + \gamma)M \), with money being injected or withdrawn in market 1 via lump sum transfers or taxes. Consider equilibria where all real variables, including \( q \) and \( \phi M \), are stationary, which means \( \phi_1 / \phi_{1,+1} = 1 + \gamma \) and (37) becomes

\[
(1 + \rho)(1 + \gamma) = \sigma e[q(M)] + 1 - \sigma.
\]

From the Fisher equation, the left side is \( 1 + i \), so we get

\[
1 + \frac{i}{\sigma} = e(q). \tag{39}
\]

Thus, \( q \) is decreasing in \( i \), but this does not affect the real allocation in markets 1 and 2. As is standard, the Friedman rule \( i = 0 \) is achieved by a policy setting \( \gamma = \beta_1 \beta_2 \beta_3 - 1 \), and provides a lower bound on \( \gamma \). At the Friedman rule, the returns on money and credit are the same; for any other feasible policy, we still get rate of return dominance.

Next, note that although our baseline 3-subperiod model gets some agents to carry debt and money simultaneously, they never need to roll over this debt for more than a period (they could, but we refined away such behavior because, as long as we rule out Ponzi schemes, they have to pay it off sometime, and given quasi-linear utility at \( s = 1 \), this is as good a time as any). Of course this does not mean that our model speaks only to high-frequency observations,
since we can make a period as long as we like, but if one wants to make debt rollover necessary, consider a version with \( n \) subperiods. For this exercise, we can generally let the centralized and decentralized markets be open \textit{simultaneously} each subperiod, and let agents transit between them as follows: those in the centralized market at \( s \) move to the decentralized market at \( s + 1 \) with probability \( \delta_s \); and those in the decentralized market at \( s \) move to the centralized market at \( s + 1 \) with probability \( 1 \).\(^{13}\)

As long as \( \delta_s > 0 \), agents are willing to pay an opportunity cost of carrying money at \( s \), since they might need it at \( s + 1 \). For convenience, set \( \delta_n = 0 \), so everyone is in the centralized market at \( s = 1 \), and let them all produce with quasi-utility linear, so they all settle their debts at \( s = 1 \), as in the benchmark model.\(^{14}\) In each \( s \in \{2, ..., n\} \), the centralized markets are like market 2 in the benchmark, except now we more generally let productivity \( \omega_s \) differ across agents and subperiods in any i.i.d. manner (\( \omega_s = 0 \) is the case where you cannot produce at all). Also, agents in the centralized markets at \( s > 1 \) now have general utility functions \( U_s(x_s, h_s) \). In the decentralized markets, agents trade bilaterally, as in market 3 in the benchmark, and sellers produce output one-for-one with labor.

In the working paper Telyukova and Wright (2006) we establish the following generalizations of Theorem 1 (existence and characterization) and Theorem 2 (rate of return dominance).

\textbf{Theorem 3.} In the model with \( n \) subperiods, there exists a steady state monetary equilibrium and it implies:

1. For all \( s \), every agent leaves the centralized market with the same \( m_{s+1} = M \).

2. For all \( s \), \( U_{sx}(x_s, h_s) = k_s \) and \( U_{sh}(x_s, h_s) = -k_s \omega_s \) where \( k_s \) is constant across agents in the centralized market.

\(^{13}\)This specification was adapted from Williamson (2005), and implies you cannot be in a decentralized market two periods in a row.

\(^{14}\)This means the nominal rate is 0 between \( s = n \) at \( t \) and \( s = 1 \) at \( t + 1 \) since there is no demand for liquidity at this one instant in time. To get around this, and make things more realistic, in general, one simply has to stagger the subperiods when different agents are in the centralized market with quasi-linear utility.
3. If two agents have different \((m_s, b_s)\) and the same productivity \(\omega_s\), their \(h_s, x_s\) and \(m_{s+1}\) are the same, so they have different \(b_{s+1}\); if two agents have the same \((m_s, b_s)\) and different \(\omega_s\), their \(h_s\) will differ and they typically have different \(b_{s+1}\).

4. Agents may roll over or run up debt between \(s = 2\) and \(s = n\) while maintaining their holdings of \(m_s\).

**Theorem 4.** In the model with \(n\) subperiods, in monetary equilibrium for all \(s \neq n\),

\[
\frac{\phi_{1,+1}}{\phi_s} < (1 + r_{s+1})(1 + r_{s+2})...(1 + r_n)(1 + r_{1,+1}).
\]

(40)

The framework thus accommodates many trading rounds between times when agents settle accounts, consistent with the observation that they may roll over debt for a long time before paying it off while having money in the bank, where this money is dominated in rate of return. Given this is understood, we return for simplicity to the baseline model with 3 markets.

A referee suggested the next generalization: in market 3, in addition to having a probability \(\sigma\) of an anonymous meeting where you cannot use credit, there is a probability \(\tilde{\sigma}\) of a nonanonymous meeting where you can. One can interpret this in terms of preference shocks: sometimes you want a good produced by someone who knows you, so you do not need cash; other times you want a good produced by someone who does not, so you need cash. It is easy to prove that in nonanonymous meetings, where credit is feasible, agents trade the efficient quantity \(q = q^*\), and are indifferent between payments in any combination of cash and credit with the same value, although they must use some credit since \(m_3\) is never enough to afford \(q^*\). The value of \(q\) in anonymous meetings satisfies (29), as before, which depends on \(\sigma\) but not on \(\tilde{\sigma}\). As \(\sigma \to 0\) (which must happen when \(\tilde{\sigma} \to 1\)) money becomes worthless.

Comments by another referee suggested the final extension. He was concerned that although the baseline model may account for the observation that consumers choose not to pay

---

15 The following model is related to recent work by Lagos and Rocheteau (2006), Lagos (2006), and Geromichalos, Licari and Suárez-Lledó (2006) on search-based models with multiple assets, although they do not have a fixed cost, which is an important component of our setup.
down debt when they have money in the bank, it could not account for the observation that they also hold other assets with rates of return that are positive but lower that those on debt. Suppose there is, in addition to \(m\) and \(b\), another asset \(a\). For concreteness, suppose it is a standard “Lucas tree” in fixed supply \(A\) that pays a real dividend \(\delta\) each period at \(s = 1\), and can be traded at price \(\psi_s\) at \(s = 1, 2\). It can also be used as a means of payment at \(s = 3\), but only if one pays a fixed cost \(p\). This is meant to capture the “penalty for early withdrawal” on some time deposits, the fixed cost involved in taking out a second mortgage or home equity loan, and so on. Thus, \(a\) is less liquid than \(m\), but it can be accessed if necessary.

The market 1 problem is now

\[
W_1(m_1, b_1, a_1) = \max_{x_1, h_1, m_2, b_2, a_2} \{u_1(x_1) - h_1 + \beta_1 W_2(m_2, b_2, a_2)\}
\]

s.t. \(x_1 = h_1 + \phi_1(m_1 - m_2) - b_1(1 + r_1) + b_2 + a_1(\psi_1 + \delta) - \psi_1 a_2\)

We get the same first-order and envelope conditions (2)-(6) as in the base model, plus obvious new conditions concerning \(a_3\). The main results go through about everyone choosing the same portfolio \((m_2, b_2, a_2) = (M, 0, A)\), and about \(W\) being linear in all arguments. At \(s = 2\), similarly, we have the first-order and envelope conditions from the base model plus the obvious new conditions concerning \(a_3\). At \(s = 3\), agents meet and bargain over \(q\) and payments \(d_m\) and \(d_a\) in terms of \(m\) and \(a\), while the buyer has to pay liquidation cost \(p(d_a)\), with \(p(d_a) = p > 0\) if \(d_a > 0\) and \(p(0) = 0\).

For simplicity, at \(s = 3\), let us assume for this presentation \(\theta = 1\) (a take-it-or-leave-it offer by the buyer) and \(c(q) = q\). The key assumption is that the buyer’s preferences are random: with probability \(\sigma\) he has a regular meeting with utility \(u_3(q)\); with probability \(\hat{\sigma}\) he has an emergency meeting with \(\hat{u}_3(q)\), where \(\hat{u}_3(q) > u_3(q)\) for all \(q\). With \(\theta = 1\), in a regular meeting a buyer chooses \((q, d_m, d_a)\) to solve

\[
\max_{q, d_m, d_a} \{u_3(q) - \beta_3[\phi_{1,+1} d_m + (\psi_{1,+1} + \delta) d_a] - p(d_a)\}
\]

s.t. \(q = \beta_3[\phi_{1,+1} d_m + (\psi_{1,+1} + \delta) d_a],\ d_m \leq m_3,\ d_a \leq a_3\).
In an emergency meeting he chooses \((\hat{q}, \hat{d}_m, \hat{d}_a)\) to solve the analogous problem. Thus,

\[
W_3(m_3, b_3, a_3) = \sigma \{ u_3(q) - p(d_a) + \beta_3 W_1(m_3 - d_m, b_3, a_3 - d_a) \} \\
+ \hat{\sigma} \{ \hat{u}_3(\hat{q}) - p(\hat{d}_a) + \beta_3 W_1(m_3 - \hat{d}_m, b_3, a_3 - \hat{d}_a) \} \\
+ (1 - \sigma - \hat{\sigma}) \beta_3 W_1(m_3, b_3, a_3).
\]

We look for an equilibrium where \(d_m = \hat{d}_m = m_3\), while \(d_a = 0\) and \(\hat{d}_a > 0\) — i.e., regularly buyers spend all of \(m\), and they liquidate \(a\) just in cases of emergency. It should be clear that we can always pick the liquidation cost \(p\) so that we get \(d_a = 0\) and \(\hat{d}_a > 0\). As always, we have \(q < q^*\). One can also show now that we can either have \(\hat{q} < \hat{q}^*\) or \(\hat{q} = \hat{q}^*\) depending on whether the asset supply \(A\) is below or above some cutoff value (see e.g. Geromichalos et al. 2006). Let us assume \(A\) is low, so that \(\hat{d}_a = a_3\) and \(\hat{q} < \hat{q}^*\) — i.e. when you liquidate you spend all your \(a\) holdings, but even this is not enough to get you the first-best quantity \(\hat{q}^*\). Then, after some algebra, we have

\[
W_{3b}(m_3, b_3, a_3) = -\beta_3(1 + r_{1,+1}) \\
W_{3a}(m_3, b_3, a_3) = \beta_3(\psi_{1,+1} + \delta)[\hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \hat{\sigma}] \\
W_{3m}(m_3, b_3, a_3) = \beta_3\phi_{1,+1}[\sigma u_3'(q) + \hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \sigma - \hat{\sigma}].
\]

Using these results, we can compute the rates of return on the three assets as follows:

\[
1 + i_b = (1 + r_1)(1 + r_2) = 1 + \rho \\
1 + i_a = \frac{\psi_{1,+1} + \delta}{\psi_1} = \frac{1 + \rho}{\hat{\sigma}u_3'(\hat{q}) + 1 - \hat{\sigma}} \\
1 + i_m = \frac{\phi_{1,+1}}{\phi_1} = \frac{1 + \rho}{\sigma u_3'(q) + \hat{\sigma}\hat{u}_3'(\hat{q}) + 1 - \sigma - \hat{\sigma}}
\]

We have generalized rate of return dominance: \(i_b > i_a > i_m\). The reason is of course that \(b\) has no liquidity premium, \(m\) has a high liquidity premium, and \(a\) is somewhere in between.

Consumers at \(s = 2\) do not pay down debt even though they hold both \(m\) and \(a\). As in the baseline model, they do not pay down debt with \(m\) at \(s = 2\) because they know that with some
probability they will find themselves in a situation at $s = 3$ where they want to consume and credit is not available. If they are not sufficiently desperate to consume at $s = 3$, it is not worth liquidating $a$, so money is useful. Now, additionally, they also choose to not pay down debt with $a$ at $s = 2$, even though $i_a < i_b$, because they value the fact that they can liquidate $a$ in an emergency as long as they pay the fixed cost $p$.

5 Conclusion

The coexistence of assets with different returns has been dubbed the credit card debt puzzle, although in some sense it is a special case of the rate of return dominance puzzle. We build on recent developments in monetary theory to construct a model where agents sometimes have the option to trade using credit and sometimes do not, by assuming in some situations they are anonymous and in some situations they are not. The framework is tractable: we have a constructive existence proof, and for all intents and purposes we have uniqueness (obviously the quasi-linear structure is key for this). It also yields strong predictions. A key prediction is that agents may not pay down debt even when it is costly in terms of interest and they have liquid assets at hand. The explanation is that they may need the liquidity later, when credit is not available. One might say this is reasonable, but perhaps obvious; it has not previously been considered, however, in the context of the credit card debt puzzle.\(^\text{16}\)

How does the approach do quantitatively? After relaxing some assumptions made here for theoretical purity, in the interest of realism, Telyukova (2006) calibrates the model. The analysis is too involved to describe in detail, but we can summarize the findings as follows. First, compared to the data, the calibrated model generates almost but not quite as many households

\(^{16}\)Of course, one can sometimes get cash advances on credit cards, so even if a purchase cannot be made on credit, one need not actually have money at hand. However, cash advances have strict limits, and, more importantly, entail very high interest charges (higher than interest on purchases). It should not be hard to model this very much along the lines of the extension in the previous section, where one could liquidate $a$ but only at cost $p$. In any case, as one referee mentioned, a good question is: why are interest rate charges so high for cash advances? We do not know for sure, but offer a conjecture. Credit card companies want agents to hold liquid assets rather than use them to pay down debt, since, after all, revolving debt is mainly how they make profit. By making cash advances costly, credit card companies increase the demand for liquidity, keep people from paying down debt, and increase profit.
simultaneously holding credit card debt and liquid assets. Second, for these households, the model easily accounts for at least half of their liquidity. So the theory is quantitatively very relevant, if not the whole story. Studying other pieces of the puzzle and discriminating between them is of course worthwhile. A referee thought we might discriminate between theories as follows. Our baseline model predicts agents may have \( m > 0 \) and \( b > 0 \), but not another asset \( a \) with \( i_b > i_a > 0 \), as some agents in the data clearly do. The extension of the model in the previous section, with a fixed cost of liquidating \( a \), generates exactly \( m, a, b > 0 \) with \( i_m < i_a < i_b \), however.

Therefore, it is not as easy as the referee thought to discriminate between our liquidity-based approach and other (e.g. behavioral) theories, at least not using his suggestion. There are other ways of discriminating. As mentioned in the Introduction, one alternative idea is that people roll over credit card debt to control spending by spouses – but the problem with this is not only that it is a fairly expensive way to proceed, it turns out that single people also have significant debt and liquidity (Telyukova 2006). Another alternative mentioned in the Introduction is that people in this position are on the verge of bankruptcy – but the problem with this is that they do not actually go bankrupt with particularly high probability, and they often have sizeable wealth in other assets (Telyukova 2006). So, as we said, although these alternatives may be part of the puzzle, it seems crucial to consider liquidity as well.
Appendix

First we derive the bargaining solution in Lemma 1. The necessary and sufficient conditions for (22) are

\[ \theta \left[ \beta_3 \phi_{1,+1} d - c_3(q) \right] u'_3(q) = (1 - \theta) \left[ u_3(q) - \beta_3 \phi_{1,+1} d \right] c'_3(q) \]  

(41)

\[ \theta \left[ \beta_3 \phi_{1,+1} d - c_3(q) \right] \beta_3 \phi_{1,+1} = (1 - \theta) \left[ u_3(q) - \beta_3 \phi_{1,+1} d \right] \beta_3 \phi_{1,+1} - \lambda \left[ u_3(q) - \beta_3 \phi_{1,+1} d \right]^{1-\theta} \left[ \beta_3 \phi_{1,+1} d - c_3(q) \right]^\theta \]  

(42)

where \( \lambda \) is the Lagrange multiplier on \( d \leq m_3 \). There are two possible cases: If the constraint does not bind, then \( \lambda = 0, \ q = q^* \) and \( d = m^* \). If the constraint binds then \( q \) is given by (41) with \( d = m_3 \), as claimed.

We now argue that \( m_3 < m^*_3 \). First, as is standard, in any equilibrium \( \phi_{1,+1} \leq (1 + \rho) \phi_1 \) (the nominal interest rate \( i \) is nonnegative). In fact, again as is standard, although we allow \( i \to 0 \), we assume \( i > 0 \), so that \( \phi_{1,+1} < (1 + \rho) \phi_1 \). Now suppose \( m_3 > m^*_3 \) at some date for some agent. Since the bargaining solution tells us he never spends more than \( m^*_3 \), he could reduce \( m_3 \) by reducing \( h_1 \) at \( t \), then increase \( h_1 \) at \( t + 1 \) and not change anything else. It is easy to check that this increases utility, so \( m_3 > m^*_3 \) cannot occur in equilibrium. Hence \( m_3 \leq m^*_3 \). To show strict inequality, suppose \( m_3 = m^*_3 \) for same agent. Again he can reduce \( h_1 \) at \( t \) and carry less money. If he is a buyer in subperiod 3, he gets a smaller \( q \), but the continuation value is the same since by the bargaining solution he still spends all his money. If he does not buy then he can increase \( h_1 \) at \( t + 1 \) so that he need not change anything else. It is easy to check that the net gain from carrying less money is positive, as in LW.
References


