Credit, Vacancies and Unemployment Fluctuations*

Nicolas Petrosky-Nadeau†

September 24, 2008

Abstract

The propagation properties of the standard model of equilibrium unemployment are significantly altered when vacancy costs require some external financing on frictional credit markets. Agency problems lead to higher costs of vacancies that, when counter-cyclical, greatly increase the elasticity of vacancies to productivity through two distinct channels: (i) a cost channel - lowered unit costs during an upturn as credit constraints are relaxed increase the incentive to post vacancies; (ii) a wage channel - the improved bargaining position of firms afforded by the lowered cost of vacancies neutralizes part of the upward pressure of market tightness on wages. As a result, the model can match the observed volatility of unemployment, vacancies and labor market tension. Moreover, the progressive reaction of financing constraints to innovations generates persistence in the response of market tightness and vacancies, a robust feature of the data and shortcoming of the standard model. Extending the model to allow for endogenous job separation improves its ability to match gross labor flows statistics while preserving a Beveridge curve between unemployment and vacancies.

*This research has benefited from discussions with Alain Delacroix and Étienne Wasmer. Financial support from FQRSC is gratefully acknowledged.

†Contact: Nicolas Petrosky-Nadeau, Université du Québec à Montréal, Département de Sciences Économiques, B.P. 8888, Station Centre-Ville, Montréal (QC) H3C 3P8, Canada. E-mail: petrosky.nadeau.nicolas@gmail.com. Web: http://www.er.uqam.ca/nobel/d347771/
1 Introduction

The standard Mortensen and Pissarides (1994) search and matching model of equilibrium unemployment has been argued in many places to be inconsistent with key business cycle facts (e.g. Shimer, 2005, Fujita and Ramey, 2007). In particular it cannot explain the high volatilities of unemployment, vacancies and market tightness, the strong negative correlation between vacancies and unemployment, nor the persistence in the adjustment of these variables to exogenous shocks. Subsequent research has focused on whether the lack internal propagation stems from the structure of the model itself (e.g., Shimer 2004, Fujita and Ramey, 2007) or whether it is a question of setting an appropriate calibration (e.g., Hagedorn and Manovskii, 2007).

This paper extends the baseline equilibrium unemployment framework by assuming that external finance must be called upon to fund part of a firm’s vacancies, and that agency costs cause credit markets to be frictional. While there exists large body of evidence suggesting that credit market frictions play an important role for firm behavior, both empirical and theoretical work focusing on their implications for firm growth and investment decisions, recent work has developed on linking credit market imperfections to job creation.\(^1\) Both Acemoglu (2001) and Wasmer and Weil (2004), for example, show how credit market imperfections can lead to higher equilibrium unemployment.\(^2\)

Firms in the model write standard debt contracts, in the spirit of Gale and Hellwig (1985), to fund vacancies over net worth. Agency problems lead to higher, time varying, costs of vacancies which are persistently relaxed during an economic upturn. This greatly increases the elasticity of vacancies to productivity through two distinct channels: (i) a cost channel, driving a time-varying wedge in the job creation condition in which lowered unit costs during an upturn as constraints are relaxed increase the incentive to post vacancies. Amplification arises by inducing a change in costs for a given expected profit from a filled vacancy; (ii) a wage channel - under Nash bargaining as a wage mechanism, the lowered cost of vacancies neutralizes part of the upward pressure of market tightness on wages by improving the bargaining position of firms. This provides amplification by increasing the elasticity of expected profits from new hires to shifts in productivity, and hence the incentive to post vacancies.

---

\(^1\)Empirically, panel data studies find that small firms with more difficult access to credit pay fewer dividends, take on more debt, and have investment rates that are more sensitive to cash flows even after controlling for future profitability. See Hubbard (1998) and Stein (2000) for surveys.

\(^2\)Rendon (2001) provides evidence that job creation at Spanish firms is restricted by limited access to credit. Linking current costs to financial markets is also a features of bank loan models as in Chirstiano et al (2005) or debt model as in Carlstrom and Fuerst (). ***
This financial accelerator is distinct from previous mechanisms to address the issue of propagation in two fundamental ways. First, amplification is a result of both a vacancy cost and wage channel. The former, which plays a dominant role, is a novel feature in which the key is a time varying cost of recruiting new workers due to the necessity to raise external funds on frictional credit markets. The latter is distinct from previous work in that source of wage rigidity is a consequence of frictional credit markets and not an inherent feature of the wage rule or a particular calibration of the model. The second fundamental distinction relates to the persistence of the adjustments to productivity shocks. In a standard search model, or models with increased wage rigidity for that matter, the largest response of market tightness is contemporaneous to the exogenous shock. In this setting, financing constraints relax progressively during a cyclical upturn as net worth is accumulated. This persistent relaxation of credit constraints is such that the height of the response of labor market variables is reached several quarters after the innovation. Amplification and persistence here are inextricably linked.

The model’s quantitative results, detailed in section 3, are set against a comparable framework without credit frictions. The propagation potential is significant, generating a highly pro-cyclical labor market tightness that matches the volatility relative to output observed in the data (13.35 against 15.41 in the data and 3.78 in the standard model). As a result, the relative volatility of unemployment, which is 6.82 in the data, rises to 4.50 in the presence of credit frictions compared to 1.46 in the standard model. Importantly, the model remains consistent with the empirical observation of a strong negative correlation between vacancies and the unemployment rate, or the Beveridge curve (-0.68 against -0.89 in the data). The second significant implication is a sluggish response of vacancies and market tightness to a technological innovation. U.S. quarterly data display a high degree of persistence measured as positive autocorrelations in the growth rate of market tightness of 0.67, 0.48 and 0.33 at the first, second and third lags respectively. The benchmark calibration leads to autocorrelations of 0.73, 0.38 and 0.19 at the first, second and third lags in the growth rate of market tightness, whereas a standard search model generates virtually no autocorrelation. The main results are robust to

---

3Backward looking social norms (Hall, 2003), staggered wage contracting (Trigari and Gertler, 2006) or information asymmetries over productivity (Menzie, 2006) have all been proposed as sources of wage rigidities to address the issue of amplification in the standard labor search framework. Hagerdorn and Manovskii (2007) follow an alternative route by calibrating the model on observed cyclical properties of real wages with parameter value for the value of non-market activities and relative Nash bargaining weights that result in rigid wages.

4The model is set in a DSGE framework as in Merz (1996) or Andolfatto (1997), extended to frictional credit markets in a manner similar to Carlstrom and Fuerst’s (1997) work with the canonical real business cycle model.

5Second moments correspond to Hodrick-Prescott filtered data.
various parametrizations of the credit and labor market frictions.

The benchmark model allows only for exogenous separation of workers out of employment, a feature that not only fits poorly with the data but results in an inability of the model to be consistent with observations on gross labor flows. Section 4 extends the model to allow for endogenous labor separation by introducing a job specific productivity shock observed at the beginning of each period. Jobs drawing a productivity below a certain threshold are terminated. However, contrary to Mortensen and Pissarides (1994), some of the separations are inefficient owing to restrictions on current losses that push the cut off productivity above that for which the surplus of the job match is null.

The impact of counter-cyclical separation is twofold. On the one hand, the response of output is amplified as the separation rate directly impacts employment. On the other hand, the drop in separations during an upturn, by rapidly increasing employment, puts downward pressure on the value of an additional vacancy, resulting in less volatile job vacancies. Overall, the main results regarding propagation remain, and the model is largely consistent with the cyclical properties of gross labor flows generating counter-cyclical gross hires and job losses. In addition, the extensions maintains the Beveridge relationship whereas the standard model with endogenous separation generates a positive correlation between unemployment and vacancies of 0.53.

While the macroeconomic consequences of credit market imperfections have generally focused on their consequences for capital investment, e.g. models of financial intermediation and agency costs by Bernanke and Gertler (1989) or Kiyotaki and Moore (1997), their implications for labor markets have been largely overlooked. Moreover, this paper contributes to the growing literature on the quantitative ability of the job matching framework to explain labor market business cycle facts. A first branch has focused on the dynamics of wages as a means to generate amplification of exogenous innovations (see Hagedorn and Manovskii, 2007, and the references therein). A second branch has focused on the persistence of market adjustments. This paper concurs with the conclusion drawn in Fujita and Ramey (2007) that costs to creating new vacancies play a role in accounting for the observed pattern in employment adjustment. While they offer a sunk costs to vacancy creation story to impose the observed persistent adjustment,

---

6Two notable exceptions are Acemoglu (2001) and Wasmer and Weil (2004) cited earlier. This paper is closest in spirit to the latter which first identifies the financial accelerator at play when hiring is conditional on the availability of external funds. Both papers, however, are mainly concerned with steady state implications, not the dynamic propagation of shocks.
costs here evolve endogenously as a function of credit market conditions.7

2 Model

The model is populated by two types of agents: firms that produce using labor and households who decide on optimal consumption and purchases of risk free bonds. The allocation of labor from households to firms involves a costly and time consuming matching process, following the now common approach of Mortensen and Pissarides (1994), adapted to a representative household framework as in Merz (1996) or Andolfatto (1997).8 The additional assumption is that firms must seek external funds over their net worth to finance current vacancies, and that the lending relationship is subject to a credit market friction of the costly state verification type. The resulting debt contract is characterized by an optimal monitoring threshold and vacancy postings.

2.1 Labor markets and households

Firms post job vacancies $V_t$ to attract unemployed workers $U_t$ at a unit cost of $\gamma$. Jobs are filled via a constant returns to scale matching function taking vacancies and unemployed workers $M(U_t,V_t)$. Define $\theta_t = \frac{V_t}{U_t}$ as labor market tightness from the point of view of the firm, or the v-u ratio. The matching probabilities are $M(U_t,V_t) = p(\theta_t) \frac{V_t}{U_t}$ and $M(U_t,V_t) = q(\theta_t)$ for firms and workers respectively, with $\frac{\partial p(\theta_t)}{\partial \theta_t} < 0$ and $\frac{\partial q(\theta_t)}{\partial \theta_t} > 0$. Note that $q(\theta_t) = \theta_t p(\theta_t)$.

Once matched, jobs are destroyed at the exogenous rate $\delta$ per period. Thus employment $N_t$ and unemployment $U_t$ evolve according to

$$N_{t+1} = (1 - \delta)N_t + p(\theta_t)V_t \quad (1)$$
$$U_{t+1} = (1 - q(\theta_t))U_t + \delta N_t \quad (2)$$

The representative household, given existing employment and unemployment, chooses optimal consumption and purchases of risk free bonds, which pay a rate $r$ the following period, in

---

7More recently, Shao and Silo (2008) consider a model of endogenous entry which, in effect, leads to a counter-cyclical net present value of vacancies.

8For a formal treatment of the set-up, see Mortensen and Pissarides (1994). The introduction of labor search to quantitative business cycle research is owed to the contributions of Merz (1996) and Andolfatto (1997). Labor force participation choices are not considered here, individuals are either employed or unemployed. See Wasmer and Garibaldi (2005) or Haefke and Reite (2006) for models of labor market participation.
order to maximize the value function:

\[ V(N_t, U_t) = \max_{C_t, B_t} \left[ U(C_t) + \beta E_t V(N_{t+1}, U_{t+1}) \right] , \]

subject to the budget constraint \( W_t N_t + b U_t + (1 + r_{t-1}) B_{t-1} + \Pi_t = C_t + B_t + T_t \), and the laws of motion for matched labor (1) and unemployment (2). The government raises \( T_t \) in taxes to fund unemployment benefits \( U_t b \), while employed workers earn the wage \( W_t \). \( \Pi_t \) are firm profits rebated lump sum at the end of the period. Denoting the multiplier on the budget constraint by \( \lambda_t \), the first order conditions are

\[
(C_t) : \quad U_C(C_t) = \lambda_t \tag{3} \\
(B_t) : \quad \lambda_t = \beta E_t \lambda_{t+1}(1 + r_t) \tag{4}
\]

### 2.2 Financial contract and vacancy decisions

The informational assumptions are chosen to generate standard debt contracts, in the tradition of Gale and Hellwig (1985) and Williamson (1987), set in a quantitative macroeconomic framework as in Carlstrom and Fuerst (1997). The contracts are written in a competitive capital market (in the sense that there is a large number of insignificant lenders and firms) and lenders are assumed to hold sufficiently large and diversified portfolios to ensure perfect risk pooling, with the result that investors behave as if they were risk neutral. Repayment of the debt is assumed to occur within the period such that there is a unit opportunity cost to funds.\(^{10}\) The competitive pressure ensures that each lender-firm pair will write a contract which maximizes the expected utility of the firm subject to the constraint that the expected return to the lender cover the opportunity cost of funds.\(^{11}\)

Define period net revenues as \( x(X - W) N \), where \( X \) is the aggregate level of technology, \( W \) is the wage rate, and \( x \) is a random variable with positive support, cdf \( F(x) \), pdf \( f(x) \) and \( E(x) = 1 \).\(^{12}\) The crucial assumption is that agents have asymmetric information over the

---

\(^{9}\)As in Andolfatto (1997), each worker is a member of a household that offers perfect insurance against labor market outcomes and is involved in a passive search process.

\(^{10}\)The present contract is written for intra-period loans while Bernanke et al (1999) consider inter-period contracts which take into account aggregate uncertainty.

\(^{11}\)If the expected utility of the firm is not maximized subject to this constraint, some other investor can offer a contract which is more attractive to the firm and still make a profit, see Gale and Hellwig (1985).

\(^{12}\)Alternatively the firm’s period net revenue could be expressed as \( xX - W \) with \( x \) drawn from a positive
realization of the random variable \( x \). This state can only be observed by lenders at some cost proportional to realized net revenues, \( 0 < \mu < 1 \).

The timing of events in each period is as follows. Assume that vacancy costs \( \gamma V \) must be paid before production occurs. All agents observe the aggregate state \( X \) and, given initial assets \( A \), firms borrow \((\gamma V - A)\) from financial markets to pay for period vacancy postings.\(^{13}\) Lenders and borrowers agree on a contract that specifies a cutoff productivity \( \overline{x} \) such that if \( x > \overline{x} \), the borrower pays \( \overline{x} (X - W) N \) and keeps the equity \((x - \overline{x}) (X - W) N\). If \( x < \overline{x} \), the borrower receives nothing and the lender claims the residual net of monitoring costs.

Define the expected gross share of returns going to the lender as

\[
\Gamma(\overline{x}) = \int_{0}^{\overline{x}} xdF(x) + \int_{\overline{x}}^{\infty} \overline{x}dF(x)
\]

noting that \( \Gamma'(\overline{x}) = 1 - F(\overline{x}) > 0 \) and \( \Gamma''(\overline{x}) = -f(\overline{x}) < 0 \), and expected monitoring costs as

\[
\mu G(\overline{x}) = \mu \int_{0}^{\overline{x}} xdF(x)
\]

with \( \mu G'(\overline{x}) = \mu \int_{0}^{\overline{x}} x F(x) \).\(^ {14}\) It is easy to see that the expected gross share to the lender will always be positive.\(^ {15}\) Given this set of definitions we can conveniently express the lender’s participation constraint as \( [\Gamma(\overline{x}) - \mu G(\overline{x})] (X - W) N = (\gamma V - A) \), which states that the returns net of monitoring costs must equal the value of the loan.

Given the assumptions on the functional forms, notably constant returns to scale in production and a linear monitoring technology, the loan amount will be proportional to net worth and each firm will face the same interest rate (see Carlstrom and Fuerst, 1997). The result is a straightforward aggregation in which only the evolution of aggregate net worth is needed to know the cost faced by firms on credit markets. These evolve according to \( A_{t+1} = \varsigma [1 - \Gamma(\overline{x}_t)] (X_t - W_t) N_t \), where the parameter \( 0 < \varsigma < 1 \) ensures self-financing does not oc-

\(^{13}\)Bank loan models, as in Christiano, Eichenbaum and Evans (2005) for example, assume that all current costs, in there case the wage bill, must be financed by bank loans. The assumption of a fraction of vacancy cost needing external financing is less extreme.

\(^{14}\)The expected share of returns going to the borrower under the contract is \( \Upsilon(\overline{x}) = \int_{0}^{\infty} \overline{x}dF(x) \). Note that \( \Gamma(\overline{x}) + \Upsilon(\overline{x}) = 1 \).

\(^{15}\)To do so, take the limits \( \lim_{\overline{x} \to 0} \Gamma(\overline{x}) = \int_{0}^{\infty} \overline{x}dF(x) = 0 \), \( \lim_{\overline{x} \to \infty} \Gamma(\overline{x}) = \int_{0}^{\infty} xdF(x) = 1 > 0 \) and recall that \( \Gamma(\overline{x}) \) is strictly increasing and concave in \( \overline{x} \).
Rearranging as

$$A_{t+1} = \varsigma \left[ (X_t - W_t)N_t - \left( 1 + \frac{\mu G(\bar{T}_t)(X_t - W_t)}{N_t} \right) (\gamma V_t - A_t) \right]$$

(5)

focuses on the premium associated with external funds, \( \frac{\mu G(\bar{T}_t)(X_t - W_t)}{N_t} \), which for any \( \mu > 0 \) is strictly positive.

We can now write the optimal incentive compatible contracting problem with non-stochastic monitoring and repayment within the period. Vacancy postings and the threshold \( \bar{T} \) are chosen to maximize the expected gross return to the firm subject to the lender’s participation constraint

$$J(N_t, A_t) = \max_{V_t, \bar{T}_t} \left[ 1 - \Gamma(\bar{T}_t) \right] (X_t - W_t)N_t + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(N_{t+1}, A_{t+1})$$

subject to \( \left[ \Gamma(\bar{T}_t) - \mu G(\bar{T}_t) \right] (X_t - W_t)N_t = (\gamma V_t - A_t) \),

and the laws of motion for employment (1) and aggregate assets (5). Note that firms use the stochastic discount factor \( \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \).

The optimality condition for vacancy postings, denoting the multiplier on the lender’s participation constraint by \( \phi \),

$$\frac{\gamma \phi_t}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1})$$

equates the average cost of a vacancy, \( \frac{\gamma \phi_t}{p(\theta_t)} \), to the expected marginal value of an additional employed worker \( \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1}) \). This describes a job creation condition guiding firms’ decision to create job vacancies****

In order to derive the marginal value of a worker to the firm, \( J_n(N_t, A_t) \), differentiate the firm’s value function with respect to \( N \),

$$J_n(N_t, A_t) = \left[ 1 - \Gamma(\bar{T}_t) \right] (X_t - W_t) + \phi_t \left[ \Gamma(\bar{T}_t) - \mu G(\bar{T}_t) \right] (X_t - W_t) + (1 - \delta) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(N_{t+1}, A_{t+1})$$

The first term corresponds to the net return on an employee accruing to the firm under the debt contract. The second term captures the value an additional worker brings to the firm by relaxing the financing constraint in terms of an increased ability to reimburse the loan. The final

---

16 The assumption of some depletion in the stock of assets is needed to rule out eventual self-financing. Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1998) assume that a fraction of the entrepreneurial population exits every period consuming their assets on the way out.
term captures the value of the continued relationship. For the sake of simplifying the notation, call \( \Omega(\pi_t) = \phi_t [\Gamma(\pi_t) - \mu G(\pi_t)] - \Gamma(\pi_t) \). Combining the value of a marginal worker with the optimality condition for vacancies, and making use of the household bond Euler equation (4), yields the intertemporal condition for vacancy postings

\[
\gamma \frac{\phi_t}{p(\theta_t)} = \frac{1}{1 + r_t} E_t \left[ \left( 1 + \Omega(\pi_{t+1}) \right) (X_{t+1} - W_{t+1}) + (1 - \delta) \frac{\gamma \phi_{t+1}}{p(\theta_{t+1})} \right] \tag{6}
\]

At this stage it is useful to show how this setting with credit frictions compares with a standard labor search model. Consider first the credit constraint multiplier \( \phi_t \) on the cost side of the job creation condition. From the first order condition for the cutoff productivity, the multiplier may be expressed as

\[
\phi_t = \frac{\Gamma'(\pi_t)}{\left[ \Gamma'(\pi_t) - \mu G'(\pi_t) \right]} \tag{7}
\]

In the absence of monitoring costs the threshold \( \pi \) tends to the lower bound of its support. It is straightforward to show that \( \partial \phi_t / \partial \pi_t > 0 \), and that in the limit \( \lim_{\pi_t \to 0} \phi_t = 1 \).\(^{17}\) That is, for any positive monitoring cost, the presence of credit frictions drives up the average cost of vacancy postings to \( \gamma \frac{\phi_t}{p(\theta_t)} \) as opposed to \( \gamma \frac{1}{p(\theta_t)} \).

Second, one can show that \( \lim_{\pi_t \to 0} \Omega(\pi_t) = 0 \), such that in the absence of monitoring costs the first order condition (6) collapses to the standard job creation condition in a stochastic discrete time setting:

\[
\gamma \frac{p(\theta_t)}{p(\theta_{t+1})} = \frac{1}{1 + r_t} E_t \left[ X_{t+1} - W_{t+1} + (1 - \delta) \gamma \frac{p(\theta_{t+1})}{p(\theta_t)} \right] \tag{8}
\]

The received argument for the lack of amplification of productivity shocks is easily understood by this job creation condition equating the average cost of a vacancy to the expected benefit of a new job. A sudden rise in productivity, increasing the profits to the firm of a job, increases the incentive to post vacancies. The same rise in productivity, however, leads to a rise in the wage reducing the profits to the firms. For most applications of the Nash bargaining solution, the wage is highly elastic to productivity such that the profits from a job for the firm are relatively inelastic to productivity shocks and, as a consequence, so are vacancy postings (Shimer, 2005, Hall 200X).\(^{***}\)

The natural response to this issue has been to induce greater wage rigidity by either changing

\(^{17}\)The reader is referred to the technical appendix for details.
the structure of the model, i.e. settling on different wage determination mechanisms (Hall, 2003, Trigari and Gertler, 2006, Menzio, 2006), or choosing a calibration resulting in a wage less elastic to productivity (Hagedorn and Manovskii, 2007). Here, credit frictions have the potential to amplify productivity shocks in manner that is fundamentally different, operating through the cost side of the job creation condition. Recall that in the presence of credit frictions the average cost to filling a vacancy is \( \gamma \phi_{t} \theta_{t} \), whereas in the standard model it is \( \gamma \theta_{t} \). The multiplier on the lender’s participation constraint, \( \phi_{t} \), which indicates how binding the credit constraints are, in effect drives a time varying wedge on the cost side relative to the frictionless model. If these constraints are countercyclical, or \( \phi_{t} \) decreases during an economic upturn, there is a downward push on the average cost of vacancies that increases the incentive for firms to post vacancies.

2.3 Workers and wages

The model is fully described once the rule for wages is determined. In order to define the values of a job \( (V_{n}) \) and unemployment \( (V_{u}) \) to a worker, differentiate the household’s value function with respect to \( N \) and \( U \):

\[
V_{n}(N_{t}, U_{t}) = \lambda_{t} W_{t} + \beta E_{t} [(1 - \delta) V_{n}(N_{t+1}, U_{t+1}) + \delta V_{u}(N_{t+1}, U_{t+1})]
\]

\[
V_{u}(N_{t}, U_{t}) = \lambda_{t} b + \beta E_{t} [(1 - q(\theta_{t})) V_{u}(N_{t+1}, U_{t+1}) + q(\theta_{t}) V_{n}(N_{t+1}, U_{t+1})]
\]

The current value of a job corresponds to the wage measured in utils and the discounted expected values of next period’s state, which with probability \( (1 - \delta) \) remains employment. The value of unemployment is derived from the value of non-market activities, \( \lambda_{t} b \), and the discounted expected value of next period’s state, which with probability \( q(\theta_{t}) \) is employment.

Splitting the surplus of a worker-firm match, defined as \( S(t) = J_{n}(t) + \frac{V_{n}(t) - V_{u}(t)}{\lambda_{t}} \), under Nash bargaining yields the wage rule\(^{18}\)

\[
W_{t} = \eta \omega_{t} [(1 + \Omega(\pi_{t})) X_{t} + \gamma \phi_{t} \theta_{t}] + (1 - \eta) \omega_{t} b
\]

where \( \omega_{t} = 1/ [1 + \eta \Omega(\pi_{t})] \). As with the job creation condition, when monitoring costs tend to

---

\(^{18}\)Wages are negotiated at the beginning of the period once the aggregate state is observed but before the firm draws an idiosyncratic productivity. The wage is not a function of the idiosyncratic productivity but will reflect the terms faced by the firm on credit markets. It is assumed that wages cannot be renegotiated ex-post.
the wage rule (9) collapses to

\[ W_t = \eta [X_t + \gamma \theta_t] + (1 - \eta) b \]  

This is simply the usual the wage rule without credit frictions and leads to the following proposition

Proposition 1 - The canonical model of equilibrium unemployment is a special case of the model with frictional credit markets when the cost of monitoring tends to zero.

While we will discuss in the next section the steady state and quantitative implications for the dynamics labor markets, one important aspect of the modified wage rule is worth stressing here. A principal force in the cyclical properties of the wage rule is the term \( \gamma \phi_t \theta_t \) which, along with the value of non-market activities, captures the relative bargaining positions of workers and firms. During an upturn, market tightness rises making it more costly for firms to pull out of the wage negotiations to search for another worker (the rise in \( \theta \) implies a drop in the probability of meeting a worker \( p(\theta) \)). In the presence of credit market frictions, the cost of an empty vacancy \( \gamma \phi_t \) actually decreases during good times as conditions on credit markets improve. The improved bargaining position of firms limits somewhat the upward pressure on wages stemming from the rise in market tightness. The end result is to induce some degree of wage rigidity which will contribute to amplifying productivity shock in the manner outlined above.\(^{19}\)

2.4 Closing the model

From the household’s budget constraint it is straightforward to derive an aggregate resource constraint

\[ Y_t [1 - \mu G(\bar{x}_t)] = C_t + \gamma V_t. \]

where \( Y_t = X_t N_t, \mu G(\bar{x}_t) \) are resources consumed in monitoring and \( \gamma V_t \) are vacancy costs.

The equilibrium of the model is then defined by equations (3) and (4) from household optimization, a job creation condition (6), optimality condition for the threshold \( \bar{x}_t \) in (7), the definition of market tightness, the lender’s participation constraint, a wage rule (9), the

\(^{19}\) As a note, both \( \omega_t \) and \( \Omega(\bar{x}_t) \) are relatively inelastic to productivity and will contribute only marginally to fluctuations in wages. \(***\)
aggregate resource constraint and laws of motion for asset accumulation, aggregate employment and unemployment.

3 Propagation properties of financial and labor market frictions

Before discussing some of the steady state labor market implications of credit market frictions in this setting, the assumptions on functional forms and calibration are presented in detail. The model is then solved by computing the unique rational expectations solution for a linearization around the deterministic steady state, and the dynamics effects are evaluated with a series of unconditional second moments and impulse response functions. The performance of the model is assessed by simulating a standard labor search model as a basis for comparison and performing a series a sensitivity analysis to key parameters and aspects of the calibration.

3.1 Functional forms and calibration

Following much of the real business cycle literature, aggregate technology is assumed stationary and to evolve according to

$$\log X_t = \rho_X \log X_{t-1} + \epsilon_t^X,$$

with $\epsilon_t^X \sim (0, \sigma^2_X)$ and $0 < \rho_X < 1$. For household preferences, period utility is defined as $u(C) = \log C$. The idiosyncratic shock $x$ is assumed to follow a log-normal distribution with mean $E(x) = 1$; i.e. $\log(x) \sim N(-\frac{\sigma^2_{\log(x)}}{2}, \sigma^2_{\log(x)})$. Finally, following much of the labor search literature, the matching technology is a Cobb-Douglas $M(U, V) = \chi U^\epsilon V^{1-\epsilon}$, with $0 < \epsilon < 1$ and $\chi > 0$.

The model is calibrated to quarterly data. The discount factor $\beta = 0.992$ is set so as to match an average annual real yield on a risk less 3-month treasury bill of 3.3%. The parameters of the exogenous process are $\rho_X = 0.975$ and $\sigma_X = 0.0072$. The quarterly default rate is set to 2.5%, larger than the value of 1% used in Carlstrom and Fuerst (1997) and slightly smaller than the value of 3% used in Bernanke et al. (1999). The resource cost of monitoring is set to $\mu = 0.075$. This value is much lower than Carlstrom and Fuerst (1997) who use a value of 0.25 and the value of 0.12 in Bernanke et al. (1999). Along with the value of $\varsigma$, this ensures a steady state premium on external finance of 134 basis points, and a ratio of liquidity to vacancy costs

---

20Find more recent numbers from Dunn and Bradstreet.***
of one third. The extent to which firms require external funds, and at what cost, is a central feature of the model. While the results will be examined below along these dimensions, evidence in XX suggests that firms fund around X% of their current expenses with internal funds. Other investigations, such as Christiano et al (2005) assume that the entire wage bill must be financed through bank loans. The present calibration implies a standard deviation of the idiosyncratic productivity $\sigma_x$ of 0.30.

In deciding on the rate of unemployment to target, several authors have argued that it should include more than the rate of workers counted as unemployed as the model does not account for non-participation. Krause and Lubik (2005), for example, choose an unemployment rate of 12%, above the average rate observed for the United States, while Gertler and Trigari (2006) set the rate at 7%. The benchmark calibration here will aim for a steady state unemployment rate of 10%. The cost of job vacancies is set to $\gamma = 0.125$. The elasticity in the labor matching function, $\epsilon$, is set to 0.6, which lies below the value of 0.72 used in Shimer (2005) but well within the range of values identified by Petrongolo and Pissarides (2001) in their survey of the matching function. The bargaining weight of the household in the wage negotiation, $\eta$, is set to 0.5. This mid-point is chosen to strike a balance between the extremes advocated in Hagedorn and Manovskii (2007) and Shimer (2005). Finally, the quarterly rate of job separation is set to 6%, corresponding to the evidence presented in Davis, Faberman and Haltiwanger (2006), and the value of $\chi$ is chosen to obtain a job filling rate of 0.6.

The benchmark calibration results in an average job finding rate of 0.54 and a replacement rate $b/w$ of 0.81. It is well known that the properties of labor search models change dramatically as this ratio tends to unity, and setting a high value as advocated by Hagedorn and Manovskii (2007) has the unappealing implication that workers gain little utility from accepting a job (see Mortensen and Nagypal, 2007). While there is no definitive value for the replacement rate, the present calibration tries to stay clear of such issues by straying closer to the value used in Elsby and Michaels (2007).

---

22 Reasons put forward by each author. What of Hosios and efficiency xxxxx.
23 The strategy employed here is to pin down the value of non-market activity such as to match an observed unemployment rate. This approach avoids some of the controversy surrounding the value of this parameter. Hagedorn and Manovskii (2007) reconcile the standard search model with key labor market statistics by employing an elevated value of the replacement rate of 0.96. Rotemberg (2006) chooses a value of 0.9, while Elsby and Michaels (2007) set the rate at a lower 0.86.
3.2 Steady state implications

Proposition 2 - There exists a unique steady state equilibrium in which the rate of unemployment in strictly increasing in the degree of agency costs, \( \mu \).

Proof. The job creation condition in the presence of credit constraints can express the wage as a decreasing function of market tightness

\[
w = 1 - \left( \frac{1}{\beta} - (1 - \delta) \right) \frac{\phi \gamma}{(1 + \Omega(x)) p(\theta)}
\]

where aggregate technology has been normalized to 1. Relative the to case with perfect credit markets, the additional cost induced by the necessity of external funds implies a steeper curve by the factor \( \frac{\phi}{1 + \Omega(x)} > 1 \).\(^{24}\) Figure 1 plots in \((\theta, w)\) the job creation curve for the model with (solid line) and without (dashed line) credit frictions. The wage rule in the presence of credit frictions

\[
w = \eta \omega (1 + \Omega(x) + \gamma \phi \theta) + (1 - \eta) \omega b
\]

has a slope \( \frac{\eta \phi}{\omega (1 + \Omega(x))} \) greater than in the absence of credit market friction given since \( \frac{\phi}{\omega (1 + \Omega(x))} > 1 \).\(^{25}\) The wage is increasing in market tightness and, conditional on \((\eta \omega (1 + \Omega(x)) + (1 - \eta) \omega b) < 1\), the intersection of the wage rule and job creation condition is unique.

Combined, the two labor market equilibrium conditions, the job creation and wage rule, pin down equilibrium market tightness \( \theta \) as

\[
\gamma \left( \frac{\phi}{\omega (1 + \Omega(x))} \frac{(r + \delta)}{\chi} \theta^e + \eta \phi \theta \right) = (1 - \eta) [1 - b]
\]

which in the absence of credit friction is given by

\[
\gamma \left( \frac{r + \delta}{\chi} \theta^e + \eta \theta^e \right) = (1 - \eta) [1 - b]
\]

where \( \theta^e \) denotes equilibrium market tightness in the frictionless case. \( \theta < \theta^e \) follows from the fact that \( \phi > 1 \) and \( \frac{\phi}{\omega (1 + \Omega(x))} > 1 \) for any strictly positive value of monitoring cost \( \mu \). To see the

\(^{24}\) The \( \phi \) is strictly increasing in \( \pi \) and \( \lim_{\omega \to 0} \frac{\phi}{\omega (1 + \Omega(x))} = 1 \).

\(^{25}\) This is evident since \( \omega = \frac{1}{\phi (1 + \Omega(x))} < 1 \) and we have shown that \( \frac{\phi}{1 + \Omega(x)} > 1 \).
Figure 1: Steady state labor market equilibrium

effect of an increase in \( \mu \) on market tightness, consider first that \( \frac{\partial \phi}{\partial \mu} > 0 \), or that the measure of credit constraint is increasing in monitoring costs. Since it is also the case that \( \frac{\partial \phi}{\partial \mu} \omega (1 + \Omega (x)) > 0 \), an increase in monitoring costs leads to a decrease in equilibrium labor market tightness.\(^{26}\)

### 3.3 Dynamic results

Several authors have noted the failure of the Mortensen-Pissarides framework to generate sufficient internal propagation of exogenous shocks to match key labor market statistics. Table 1 reports the Hodrick-Prescott filtered standard deviation relative to aggregate output of variables central to the labor market, along with their contemporaneous correlation with the cyclical component of aggregate output. The first columns set the performance of the standard labor search model against moments from U.S. data and highlight its shortcomings. The relative volatility of vacancies generated by the standard model is only 40% of that in the data. The performance in terms of unemployment or employment is hardly any better: the model generates a relative standard deviation for unemployment of 1.46 against a relative standard deviation of 6.82 in the data, or just 20% of the relative volatility observed in the data. The dismal performance of the model extends to the measure of labor market tightness, which has a relative volatility of 15.41 in the data and 4.61 for the standard model.

The second significant shortcoming regards the persistence in the adjustment to exogenous shocks. Evidence uncovered from reduced form VARs show that market tightness (and vacan-

\(^{26}\)The effect on the equilibrium wage is ambiguous as higher recruiting costs both lowers job offers and affects the threat in wage bargaining to the advantage of workers.
cies) have a sluggish response to productivity shocks, peaking several quarters after the innovation (see Fujita and Ramey 2007). Another measure of this persistence, the auto-correlation in the growth rate of market tightness, is reported in the last three rows of Table 1. The data are characterized by a high degree of positive auto-correlation at the first three lags while the standard search model generates virtually no persistence.\footnote{This criticism resembles that addressed to RBC models regarding the persistence in the response of output to productivity shocks (see Cogley and Nason, 1995). The standard search model does generate some persistence in output growth, essentially because of the predetermined nature of employment, but still falls short of being consistent with the data.}

\begin{center}
Table 1: Unconditional 2nd moments
\end{center}

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable:</td>
<td>a</td>
<td>b</td>
<td>a</td>
</tr>
<tr>
<td>$V$</td>
<td>8.83</td>
<td>0.89</td>
<td>3.78</td>
</tr>
<tr>
<td>$\theta$</td>
<td>15.41</td>
<td>0.90</td>
<td>4.61</td>
</tr>
<tr>
<td>$U$</td>
<td>6.83</td>
<td>-0.88</td>
<td>1.46</td>
</tr>
<tr>
<td>$N$</td>
<td>0.48</td>
<td>0.82</td>
<td>0.16</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>1.40</td>
<td>1.05</td>
<td>1.38</td>
</tr>
<tr>
<td>$corr(\Delta \theta, \Delta \theta_{-1})$</td>
<td>0.67</td>
<td>0.02</td>
<td>0.73</td>
</tr>
<tr>
<td>$corr(\Delta \theta, \Delta \theta_{-2})$</td>
<td>0.48</td>
<td>0.001</td>
<td>0.38</td>
</tr>
<tr>
<td>$corr(\Delta \theta, \Delta \theta_{-3})$</td>
<td>0.33</td>
<td>-0.007</td>
<td>0.19</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments, but market tightness growth, are Hodrick-Prescott filtered; Data sources: BLS, BEA.

### 3.3.1 Vacancies and labor market tension

We begin by examining, in Figure 2, the response of vacancies and market tightness to a persistent expansionary productivity shock in the standard (dashed line) and proposed (solid line) models. The introduction of credit frictions yields two improvements: first, the response is largely amplified; second, the response is persistent, or the adjustment to the exogenous innovation is "sluggish". The unconditional second moments for the proposed model, presented in the last columns of Table 1, show that relative volatility of vacancies is large, at 9.89, and close to
the value of 8.82 found in the data. The labor market tightness generated is also remarkably close to its empirical counterpart with a relative volatility of 13.35, compared to 15.41 in the data. In term of persistence, vacancies, and market tightness, peak several quarters after the shock. More precisely, the model generates elevated positive auto-correlation in the growth rate of market tightness that are very close to the data at the first two lags, and only slightly too low at the third (see the last three rows of Table 1).

![Figure 2: IRFs to an expansionary productivity shock, vacancies and job finding hazard](image)

Understanding the present results lies in the dynamics of the cost and wage channels of propagation outlined earlier. As both depend on the evolution $\phi$, the first panel of Figure 3 plots the response of the measure of credit constraints following the same persistent shock to productivity. While the constraint is relaxed on impact, the slow accumulation of net worth pushes the constraint to its lowest several periods after the shock. The effect on the job creation condition through the cost of vacancies is strongest therefore several periods after the shock. The wage channel is illustrated in the second panel of Figure 3. Following an innovation to productivity, wages do not respond initially as strongly as in the standard model, and increases progressively for several quarters. This rigidity contributes to the elasticity of the initial response of vacancies to a productivity shock which is greater in the model with credit frictions (see the first panel of Figure 2). The ensuing rise in the wage, as market tightness continues to rise faster than $\phi$ decreases, counters some of the relaxing of the financing constraint for job creation.

---

28 The value is in fact somewhat large, a point we will return to in the extension below.

29 The relative standard deviation of wages in the standard model is 0.82 whereas with credit friction it is lowered to 0.49. Both are highly pro-cyclical. In a single shock economy this is approximately the elasticity of wage with respect to productivity. HM 2007 use the low elasticity to productivity to pin down the relative bargaining power of workers in their calibration strategy. ***/
However, the continued rise in vacancies is testimony to the fact that the cost channel is largely dominant. The joint effect of these channels explains why the peak in market tightness is reached 6 quarters after the initial shock.

The large propagation potential of financial frictions results in a standard deviation of aggregate unemployment of 4.5, whereas the volatility of unemployment in the standard model is only 1.46 against 6.83 in the data. The standard deviation of aggregate output in the model with credit frictions, at 1.38, is close to the value of 1.40 in the data. A standard labor search model generates a volatility of aggregate output barely beyond the impulse provided by the exogenous process with a standard deviation of 1.05. Figure 4, which plots the responses of output and unemployment to a persistent productivity shock, illustrates the full impact of this financial accelerator on aggregate activity. Output continues to expand several quarters after the standard model has reached its peak, and the strong flows of hiring lead to a deep drop in the unemployment rate.

3.3.2 Beveridge curve and cross-correlations

One concern for extensions to the standard framework is the violation of a robust empirical observation of a strong negative correlation between unemployment and vacancies, or the Beveridge curve.\textsuperscript{30} Table 2 presents the contemporaneous cross-correlations of key labor market variables in the data and as generated by the model. In this respect the proposed model is

\textsuperscript{30}Examples***
Figure 4: IRFs to an expansionary productivity shock, output and unemployment

again an improvement on the standard model with a correlation of -0.68, half way in between the correlation in the data, -0.89, and the correlation generated by the standard model, -0.45.

The data are also characterized by a very strong negative correlation between the unemployment rate and the measure of labor market tightness with a contemporaneous correlation of -0.97. The standard model generates a somewhat weak correlation -0.68. The presence of credit friction brings the correlation closer to the data at -0.84. By extension, the proposed model also improves on the correlation between the unemployment and job finding rates.

Another significant improvement of the proposed model is to reduce the correlation between unemployment and labor productivity, almost perfectly replicating a correlation of -0.42 in the data. This correlation is too strong in the standard labor search model, which generates a correlation of -0.65. Both models fall short, however, of being consistent with the correlations between labor productivity and vacancies or the market tightness. These have a mild positive correlation in the data, around 0.4, whereas both models generate very high positive correlations.
Table 2: Labor market cross-correlations

<table>
<thead>
<tr>
<th>U</th>
<th>V</th>
<th>θ</th>
<th>q(θ)</th>
<th>Y/N</th>
<th>U</th>
<th>V</th>
<th>θ</th>
<th>q(θ)</th>
<th>Y/N</th>
<th>U</th>
<th>V</th>
<th>θ</th>
<th>q(θ)</th>
<th>Y/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1.00</td>
<td>-0.89</td>
<td>-0.97</td>
<td>-0.95</td>
<td>-0.97</td>
<td>1.00</td>
<td>-0.45</td>
<td>-0.68</td>
<td>-0.65</td>
<td>1.00</td>
<td>-0.68</td>
<td>-0.84</td>
<td>-0.42</td>
<td>-0.53</td>
</tr>
<tr>
<td>V</td>
<td>-1</td>
<td>0.98</td>
<td>0.90</td>
<td>0.36</td>
<td>-1</td>
<td>0.96</td>
<td>0.96</td>
<td>0.97</td>
<td>-1</td>
<td>0.97</td>
<td>0.97</td>
<td>0.91</td>
<td>-1</td>
<td>0.97</td>
</tr>
<tr>
<td>θ</td>
<td>-1</td>
<td>1</td>
<td>0.95</td>
<td>0.40</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.99</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.82</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>q(θ)</td>
<td>-1</td>
<td>1</td>
<td>0.40</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.99</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.82</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>Y/N</td>
<td>-1</td>
<td>1</td>
<td>0.95</td>
<td>0.40</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.99</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.82</td>
<td>-1</td>
<td>1</td>
</tr>
</tbody>
</table>

All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA.

Table 3: Robustness - baseline model with credit friction and exogenous labor separations

<table>
<thead>
<tr>
<th>Premium</th>
<th>A</th>
<th>γ</th>
<th>ε</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5%</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>2%</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>3/4</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.2</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.1</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.25</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.5</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.75</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.07</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>0.12</td>
<td>a</td>
<td>b</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>U</td>
<td>3.17</td>
<td>-0.70</td>
<td>5.04</td>
<td>-0.83</td>
</tr>
<tr>
<td>V</td>
<td>6.95</td>
<td>0.99</td>
<td>11.21</td>
<td>0.93</td>
</tr>
<tr>
<td>θ</td>
<td>9.39</td>
<td>0.97</td>
<td>4.62</td>
<td>0.99</td>
</tr>
<tr>
<td>σ(y)</td>
<td>1.18</td>
<td>1.51</td>
<td>1.14</td>
<td>1.61</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA.
3.4 Sensitivity analysis

This section examines the sensitivity of the main results along the dimensions of credit and labor market imperfections. With results presented in Table 3, we look at the effects of calibrating on a different external finance premium and liquidity to vacancy costs ratio first, then look at the impact of variations in the unit cost of vacancies, the elasticity of the matching function with respect to unemployment, and to different steady state rates of unemployment.

Calibrating to a lower premium on external finance, as the first column of Table 3 shows, markedly reduces the volatility of the v-u ratio, and thus the volatility of unemployment and aggregate output. The inverse is observed when the premium on external finance is raised. Changing the liquid assets to vacancy cost ratio has a similar impact, although it operates mainly by changing the elasticity of liquidity to aggregate shocks, thereby playing on the external financing constraint over time.

The dynamic properties of all labor search models are very sensitive to the value of unit search costs $\gamma$. Increasing its value from 0.125 to 0.25 reduces the standard deviation of output from 1.38 to 1.22, while reducing the value of $\gamma$ to 0.1 increases the standard deviation of output to 1.81. With regards to the moments of concern here, changes in the unit cost also significantly affect relative volatilities of labor market variables. For example, the relative volatility of vacancies ranges from 7.80 to 13.18 with the choices for $\gamma$.

The elasticity of the matching function with respect to unemployment plays a large role in translating movements in the labor market tightness into new jobs. Increasing the elasticity with respect to vacancies, i.e. reducing the elasticity with respect to unemployment, greatly increases the response of the model to exogenous innovations. For example, decreasing $\epsilon$ from 0.6 to 0.5 increases the standard deviation of output to 1.55 and the relative volatility of unemployment to 5.36. On the other hand, the relative volatilities of vacancies and $\theta$ decrease.

Finally, raising the target steady state rate of unemployment, while increasing the volatility of aggregate output, has only a minor downward impact on the relative volatilities of vacancies, unemployment and the v-u ratio. The converse is true for a reduction in the target steady state rate of unemployment.
4 Extension to endogenous job separation

The previous section assumed that all labor separations occurred at a constant, exogenous rate. Endogenous job separations have been argued an important feature of equilibrium employment models both because time varying separations are a salient empirical observations and, by directly affecting the stock of matched job, offer an important mechanism for the propagation of exogenous shocks.\textsuperscript{31} This section extends the basic set up to allow for an endogenous labor separation margin.

4.1 An endogenous job separation margin

Assume that each job within a firm draws an i.i.d. productivity \( z \), where \( z \in [0, \infty) \) with cdf \( H(z) \), pdf \( h(z) \) and \( E(z) = 1 \), and that this job productivity is observed by both the worker and the firm before the idiosyncratic productivity \( x \) is known. Firms and individual workers negotiate a wage conditional on the productivity of the job, \( W(z) \), and a job drawing productivity \( z < \overline{z} \) is not profitable and terminated.\textsuperscript{32} Given frictional credit markets, this threshold is defined such that current net revenues are non negative, or \( \overline{z} \) is such that \( \overline{z}X - W(\overline{z}) = 0 \). This job destruction margin differs for the efficient separation rule in Mortensen and Pissarides (1994) in which the value of the cut off corresponds to that for which the job yields no surplus to either the worker or the firm. A separation rule that is efficient from the point of view of both parties involves a cut off for which the losses in current revenue are equal to the expected value of the job in the future

\[
z^*_t X_t - W_t(z^*_t) = -\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_N(N_{t+1})
\]

where \( z^*_t \) is the job productivity threshold in the absence of credit market frictions. The restriction that firms cannot run current period losses implies that \( \overline{z}_t > z^*_t \). In other words the cut-off productivity is higher in the presence of credit market frictions resulting in a higher rate of endogenous separations, and part of these separations will be inefficient.\textsuperscript{33}

In fact, there is evidence that more financially constrained firms engage in stronger employment contractions during periods of financial distress. Using bankruptcy filings as an indicator

---

\textsuperscript{31} Summarize literature here. ###

\textsuperscript{32} Wages are not modeled as a function of the firm’s idiosyncratic productivity as separations could be used by creditors as a source of information on the firm’s productivity draw for the current period.

\textsuperscript{33} The efficient threshold in the model with credit frictions would be \( \overline{z}_t X_t - W_t(\overline{z}_t) = -\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_N(N_{t+1}) \), and all job destructions in between the job productivities \( z \) and \( \overline{z} \) are inefficient. Adopting this threshold would, however, be inconsistent with the assumption of frictional credit markets.
of financial distress and size for as a proxy for access to credit markets, as in Gertler and Gilchrist (1994), the slowdown and contraction in employment growth at small firms is more pronounced than at larger, presumably less credit constrained, firms during the run-up to a bankruptcy filing (see the first panel of Figure 5). A more direct measure of financial constraint is a firm’s credit rating which dictates the terms of external financing. The contraction in employment at firms with a lower credit rating, as seen in the second panel of Figure 5, is much more severe than at high rating firms. Unfortunately this information cannot distinguish changes in employment due to a hiring freeze or a rise in job separations, due to layoffs or quits, as only information on the number of employees is provided.

Nonetheless, the work of Davis, Faberman and Haltiwanger (2006) indicates that an overwhelming majority of the change in employment at contracting firms is due to separations. This evidence provides support for an extension in which the rate of job separations as influenced by the degree of financial constraints.

With non-profitable jobs terminated before vacancies and debt contracts are determined, workers available for production are given by \( \bar{N}_t = \int_{\bar{z}_t}^{\infty} N_t dH(z) \), where \( N_t \) is beginning of period employment. It is also assumed that some separations occur exogenously at the end of the period at the rate \( \delta^x \) such that total job separation is \( \delta_t = \delta^x + \int_0^{\bar{z}_t} dH(z) \).

The timing assumption affords the following two benefits. First, ex-ante all firm face identical problems and make identical choices such that the analysis continues in a representative firm framework. Second, expected net revenues \( \int_{\bar{z}_t}^{\infty} (zX - W(z)) dH(z) \bar{N} \) are always positive,

\[ \text{Figure 5: Employment growth during financial distress - by size and credit rating} \]

---

34 Firm level observations on employment are obtained from the Compustat data base and bankruptcy filing dates from the Bankruptcy Research Database.

35 Indeed, it may be that current employees, observing private information concerning the productivity of the firm before outsiders, begin to search for other jobs on the current job. On the importance of job-to-job transitions see, for example, Fallick and Fleischmann (2004), Nagypal (2004) or Faberman and Nagypal (2007).

36 Had the job productivity shock been drawn after the firm’s idiosyncratic shock, the heterogeneity would lead to a multiplicity of separation conditions. It would then be necessary to follow a distribution of employment at each firm according to its history.
which leaves the expected gross shares of net revenue under the debt contract unchanged, and the optimal contracting problem is naturally expressed in a similar fashion with the appropriately modified participation constraint and laws of motion for employment and aggregate assets. This results in optimality conditions for vacancy postings and the monitoring threshold that retain the same form as earlier. Closing the extension to endogenous job separation, the job creation condition and wage rule are now

\[
\frac{\gamma \phi_t}{p(\theta_t)} = \frac{1}{1 + r_t} E_t \left[ [1 + \Omega(x_{t+1})] \int_{x_{t+1}}^{\infty} (zX_{t+1} - W_{t+1}(z)) dH(z) + (1 - \delta_{t+1}) \frac{\gamma \phi_{t+1}}{p(\theta_{t+1})} \right]
\]

(11)

\[
W_t(z) = \eta \omega_t \left[ (1 + \Omega(x_t)) zX_t + \gamma \theta_t \phi_t \right] + (1 - \eta) \omega_t b
\]

(12)

4.2 Quantitative results

The idiosyncratic shock \(z\) is assumed to follow a log-normal distribution with mean \(E(z) = 1\); i.e. \(\log(z) \sim N\left(-\frac{\sigma^{2}_{\log(z)}}{2}, \sigma^{2}_{\log(z)}\right)\). The steady state endogenous job separation rate set to 0.03, or 1/2 of total separations, which is in-line with evidence on the rate of layoffs in Davis, Faberman and Haltiwanger (2006). This results in a standard deviation of the job specific shocks of \(\sigma_{\log(z)} = 0.17\).

Endogenous job separation generate two opposing forces on the amplification of productivity shocks, with the second often overlooked in the literature. On the one hand, counter-cyclical movements in the rate of separation contribute to rapidly increasing employment and production during an economic upturn. In this sense endogenous separations provide a degree of amplification. On the other hand, this same mechanism puts a downward pressure on the expected benefit of new jobs or hires for firms, dampening their incentive to post vacancies. The results in Table 4 clearly show the impact of the two mechanisms outlined above. First, the volatility of aggregate output is greatly amplified, rising from a standard deviation of 1.38 to 1.66. Second, the relative volatility of vacancies in the model with endogenous separation is lower than is the model with exogenous separation, dropping to 7.12. The same holds for the relative volatility of market tightness. Nonetheless the results are still a large improvement on the standard labor search model with either exogenous or endogenous separation. Results for the latter are reported

\[37\]The contracting problem to this extension is fully set up in the appendix.
in the last columns of Table 4. While output and unemployment are clearly more volatile, the relative volatility of market tightness is only 25% of that in the data.

Table 4: Unconditional 2nd moments

<table>
<thead>
<tr>
<th>Variable:</th>
<th>Exogenous δ</th>
<th>Endogenous δ</th>
<th>Exogenous δ</th>
<th>Endogenous δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>8.83</td>
<td>9.89</td>
<td>7.12</td>
<td>3.78</td>
</tr>
<tr>
<td>θ</td>
<td>15.41</td>
<td>13.35</td>
<td>9.77</td>
<td>4.61</td>
</tr>
<tr>
<td>U</td>
<td>6.83</td>
<td>4.50</td>
<td>3.41</td>
<td>1.46</td>
</tr>
<tr>
<td>N</td>
<td>0.48</td>
<td>0.50</td>
<td>0.38</td>
<td>0.16</td>
</tr>
</tbody>
</table>

σ(y)  1.40  1.38  1.66  1.05  1.59

a: Standard deviation relative to output; b: contemporaneous correlation with output.

An important concern when introducing endogenous separation is the preservation of the Beveridge relationship between unemployment and vacancies (see Ramey, 2008). As Table 2 reports, the model extension retains a strong negative correlation that characterizes the data, whereas the standard model with endogenous separation generates a positive correlation of 0.53. Broadly speaking, extending the model to endogenous job separations brings little changes to the contemporaneous cross-correlation of other labor market variables.

4.2.1 Endogenous separation and gross labor flows

Table 5 presents unconditional second moments for transition hazards and gross flows, in particular their HP filtered standard deviation relative to output, and their contemporaneous correlation with output. While there is broad consensus concerning the strong procyclicality of the job finding hazard, which all models can replicate, the models with credit friction alone generate relative volatilities close to the data. While models of exogenous separation cannot match the dynamics of the separation rate in the data by construct, both extended models do a good job of matching the relative volatility and counter-cyclicality in the data.38

38I thank Shigeru Fujita and Garey Ramey for kindly sharing their data. The raw monthly series were first adjusted by a 12 month backward-looking moving average, as in Fujita and Ramey (2006). Quarterly series
The following decomposition is useful in understanding the predictions in terms of worker flows. The law of motion for aggregate employment separates all changes into flows of hires and job losses:

\[ N_{t+1} - N_t = q(\theta_t)U_t - \delta_t N_t \]

\[ \Delta N_{t+1} = \text{Hires}_t - \text{Losses}_t \]

where gross hires are given by \( \text{Hires}_t = q(\theta_t)U_t \) and gross job losses by \( \text{Losses}_t = \delta_t N_t \).

Gross hirings are the product of movements in the rate of unemployment and the job finding hazard. Since both move in opposite direction over the cycle, the model with endogenous separation is able to generate a negative correlation of gross hiring because the pool of the unemployed is shrinking sufficiently to counter the effect of the rise in the job finding hazard.

The assumption of constant job separation rates, on the other hand, leads to counter-factual dynamics for gross labor flows as the gross hires generated are pro-cyclical (see Table 5). This stands in contradiction of the strong negative correlation with the business cycle uncovered in the work of Blanchard and Diamond (1990) and Fujita and Ramey (2006).

It is no surprise that both models of constant separation rates are far outdone at matching the counter-cyclical gross job losses over the business cycle. The statistics on job losses for these models are simply those of aggregate employment as this is the only time-varying component of gross job losses, and these models generate pro-cyclical gross job losses. For the current calibration gross job losses are almost as volatile as in the data in the extended model and are strongly counter-cyclical.

---

were then computed by averaging over monthly observations.
Table 5: Unconditional 2nd moments - labor market variables

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>endogenous $\delta$</td>
<td>exogenous $\delta$</td>
</tr>
<tr>
<td>Hazard rates:</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>$q(\theta)$</td>
<td>4.83</td>
<td>0.6</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>3.34</td>
<td>-0.86</td>
</tr>
<tr>
<td>Worker flows:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross hires</td>
<td>2.02</td>
<td>-0.53</td>
</tr>
<tr>
<td>Gross losses</td>
<td>2.98</td>
<td>-0.86</td>
</tr>
</tbody>
</table>

a: Standard deviation relative to output; b: contemporaneous correlation with output. All moments are Hodrick-Prescott filtered; Data sources: BLS, BEA, Fujita and Ramey (2006).

5 Conclusion

TO BE COMPLETED

This paper introduced a financial friction in the funding of current job vacancy costs in an otherwise standard labor search model to investigate the potential for propagating exogenous shocks. Counter-cyclical constraints on external finance are found to greatly increase the volatility of vacancies and unemployment through a powerful financial accelerator. An extension to endogenous labor separation furthers the propagation properties of credit market imperfections while remaining consistent with labor market statistics of interest such as the Beveridge curve, or the negative correlation between unemployment and vacancies.
References


