The Game of Love and Hazard: A Structural Model of Consumption and Saving

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Abstract
In the present paper, we consider a two-person household and study the risk that results from the stochastic fluctuations in the bargaining power of the household members. The model is inspired from the bargaining theory developed by Rubinstein and Binmore. We obtain the following results. i) The level of household investments in assets tends to increase in presence of this specific form of risk. ii) Spouses choose to invest funds in assets that have the largest negative effect on the variance of bargaining power. iii) Consumption inequality among spouses is lower in the richest couples.

JEL codes: D11, D14, D31, J12
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1 Introduction
One of the main economic justifications for the formation of a couple is the possibility of risk sharing among spouses; see Pollak (1985) for a discussion. However, this ignores the fact that, in general, a couple is not able to credibly commit to a fair division of future consumption since any current agreement can be renegotiated. If the outside opportunities are very favorable to the husband (say), he can be inclined to take advantage of the situation and renege on the “contract” made with his wife. Peoples living in multi-person households have thus to face another form of risk — what we call “conjugal
risk” in this paper — which is related to the uncertainty of the future distribution of bargaining power. Even if conjugal risk is probably less important than income risk, recent investigations (e.g., Mazzocco, 2004a) indicate that its magnitude should not be underestimated.

The present paper focuses exclusively on this specific form of risk and investigates its impact on saving and consumption decisions. To do that, we build a “fully structural” framework to model the behavior of multi-person households in a stochastic context. The model is inspired from the bargaining theory developed by Rubinstein and Binmore. The marriage surplus stems from a relationship-specific asset that vanishes in case of divorce. The variations in the balance of power are thus caused by a change in the outside opportunities of spouses, which are simply represented here by divorce. More precisely, the personal incomes are supposed to be stochastic and, consequently, influence the utility obtained in case of a separation of the couple. Finally, the income at the household level is assumed to be deterministic so that the effects of the conjugal risk are isolated from those of other risks.

These assumptions, although more restrictive than in many other investigations, allow us to have a better understanding of consumption and saving behavior. The first — but not very original — message of the present paper is that limited commitment in households generally leads to inefficiencies because of the aversion of agents to fluctuations in utilities. The basic idea underlying the model is that a high level of assets can be used to reduce fluctuations in bargaining power — this is what we call the “insurance motive” of saving.¹ This results from our conjecture that the spouses’ outside opportunities are less attractive if the household has largely invested in assets. We then draw two main conclusions. Everything else being equal,

1. The level of investments in assets will be higher in presence of conjugal risk. Moreover, spouses will choose to invest funds in assets that have the largest insurance effect, namely the relationship-specific assets.

2. Conversely, the households in which the level of assets is high will be more equal. In particular, the distribution of consumption among spouses will be more equitable in the richest households.

¹This motive has not been accorded much attention until now. See, for example, the introduction of the survey by Browning and Lusardi (1996), in which more than ten saving motives are listed (including precaution and consumption smoothing).
The study of the intertemporal decision process of multi-person households has attracted considerable attention from economists in recent years. Several papers are connected to the present contribution. Basu (1999), Konrad and Lonnerud (2000), Lundberg (2002), Lundberg, Startz and Stillman (2001), Rainer (2003) and Wells and Maher (1998) adopt a fully structural framework, like ours, where the decision process is completely specified, and then suppose that spouses are able to influence their respective bargaining power in the household by specific actions (such as the number of worked hours). The objective of these authors is then to characterize the optimal strategy of each spouse. Our investigation differs from these because the variation in bargaining power is exogenously determined and our main concern is the modelling of saving and consumption behavior. Aura (2003) supposes that the variations in bargaining power are exogenously determined, instead of being the result of spouses’ actions. He then considers the intertemporal allocation of consumption in a certainty context and investigate the impact of these variations on the saving patterns. Lich-Tyler (2004) derives Euler equations in a certainty context for various assumptions on the decision process. These authors, however, do not incorporate uncertainty in their specification. Mazzocco (2004a, 2004b) follows a similar line of research in a model with uncertainty. In that case, the optimal consumption path will also be influenced by the degree of prudence of household members. In another paper which is closely related to ours, Mazzocco (2004c) investigates saving and consumption behavior in a two period model. He demonstrates the counterintuitive result that risk sharing can entails an increase in saving. These various models, however, do not specify the cause of the uncertainty and, consequently, the conclusions that can be drawn are quite limited. Finally, the results of this paper can be related to several contributions on risk-sharing in a limited-commitment context. See Gobert and Poitevin (1998) and Ligon, Thomas and Worrall (2002) for example.2

2To be complete, we have also to cite the precursory studies by Browning (1996, 2000) on life-cycle consumption.

2 The Model

Hypotheses A two-person household, consisting of a husband (A) and a wife (B), makes decisions about consumption and saving over two periods in a stochastic environment. Spouses consumes a private good, denoted
by $x_I$, and benefit from a relationship-specific asset, denoted by $X$, which constitutes the marriage surplus. The latter results from household public goods (including children) and the various emotional attachments between spouses. There is neither labor supply nor domestic production. The spouses are characterized by identical, intertemporally additive utility functions with VNM felicity functions at each period of the following form:\[^3\]

$$U_I = u(x_I) + X, \quad I = A, B,$$

where $u$ is three times differentiable and satisfies $u' > 0$, $u'' < 0$, $u''' \geq 0$ (i.e., agents are risk-averse and non-imprudent) and $u'(0) = \infty$. The relationship-specific asset is assumed “exogenous” in the present section. The “endogenous” case is considered at the end of Section 3.

The household as a whole receives a non-stochastic income, denoted by $Y_1$ for the first period, and $Y_2$ for the second period. This information is common knowledge at the beginning of the life-cycle. During the first period, the household members choose the levels of consumption and saving. The saving is then invested in a riskless asset with a return equal to zero. The total resources of the household available at the second period are thus equal to $Y_2 + S$ where $S$ denotes the (positive or negative) saving of the first period. The important point is that, even if the household income of the second period $Y_2$ is deterministic, the personal incomes, that make up the household income at the second period, are stochastic and such that

$$y_A = \frac{Y_2}{2} - \Sigma \varepsilon, \quad y_B = \frac{Y_2}{2} + \Sigma \varepsilon$$

where $\varepsilon$ is a random term which follows an uniform distribution with support $[-\frac{1}{2}, +\frac{1}{2}]$ and $\Sigma \leq Y_2$ is a parameter of dispersion. Each household member is informed of his or her personal income $y_I$ at the beginning of the second period.\[^4\] Then, the household members have to choose: either they stay

\[^3\]The fact that the wife and the husband have the same preferences is undoubtedly a very strong assumption. For instance, there are reasons to believe that the discount rate of future is larger for husbands than for wives; see Browning (1996, 2000). Supposing that household members are different, however, complicates our arguments and does not significantly alter the most important results of this paper.

\[^4\]The distribution of incomes between household members at the second period is thus the sole source of uncertainty (since the total of personal incomes is deterministic). In other words, the form of risk that we study is purely idiosyncratic and could be eliminated by an efficient system of insurance. Our idea is to be concentrated on this specific form of risk that has never been investigated in depth.
together or they divorce. If they decide to stay together, knowing \( y_A, y_B \) and \( S \), they have to divide consumption among them. If, on the contrary, they decide to divorce, they give up the marriage surplus and obtain the level of felicity equal to:

\[
U_I = u(y_I + \theta IS)
\]

The underlying assumption behind Eq. (3) is that, in case of divorce, each member receives a proportion of total saving equal to \( \theta_I \), with \( \theta_A + \theta_B = 1 \). Suppose that \( |y_J - y_I| \leq S \), i.e., saving is at least as great as the difference in personal incomes in terms of absolute values. A simple and general specification for \( \theta_I \) is then the following:

\[
\theta_I = \theta \frac{1}{2} + (1 - \theta) \frac{y_J - y_I}{2S},
\]

where \( \theta \) is a constant comprised between 0 and 1. The share of saving of each spouse is thus a weighted mean of the equal-sharing and the full-compensating situations. If \( \theta = 0 \) (full-compensating), household members receive after divorce the following endowment:

\[
y_I + y_J - y_I S = \frac{Y_2}{2}.
\]

If \( \theta = 1 \) (equal-sharing), they receive the following endowment:

\[
y_I + \frac{S}{2}.
\]

For the sake of simplicity, we consider the sole case of equal-sharing\(^5\) in what follows. However, the main conclusions remain valid in the more general specification described by Eq. (4) as long as the full-compensating case is excluded.

**Distribution of Consumption**  The main idea of our approach is inspired by the bargaining models à la Rubinstein–Binmore where outside opportunities are given here by the level of felicity obtained in case of divorce.\(^6\) In

\(^5\)Thus, saving is purely public. Spouses are not able, in case of divorce, to take the personal saving they have amassed.

\(^6\)The application of this bargaining model to the household context has been initially proposed by Bergstrom (1997). See also Muthoo (1999). Our specification is also consistent with the bargaining model of Ligon (2002).
other words, the distribution of consumption is determined by the maximization of the product of the felicity functions subject to the constraint that the household members obtain at least the level of felicity of divorce. Since the environment that we consider is symmetrical, for an interior solution, each spouse receives the same endowment. At the first period, the level of felicity obtained by each spouse is given by

\[ U_I = u\left(\frac{Y_1 - S}{2}\right) + X, \] (5)

if the sustainability constraint is not binding, i.e., if the level of utility given by Eq. (5) is inferior or equal to the level given by Eq. (3). This is plausible if, at the moment of the marriage, the partners are not too different. At the second period, the level of felicity is equal to

\[ U_I = u\left(\frac{Y_2 + S}{2}\right) + X, \] (6)

if the sustainability constraint is not binding. Otherwise, the level of felicity is given by

\[ U_I = u\left(y_I + \frac{S}{2}\right), \]

i.e., the level of the reservation utility obtained from divorce.

The sustainability constraint is binding if, at the second period, the distribution of personal incomes is very unequal. The threshold of \( y_I \) at which it happens is denoted by \( y_I^* \), where \( y_I^* \) is implicitly defined by

\[ u\left(\frac{Y_2 + S}{2}\right) + X = u\left(y_I^* + \frac{S}{2}\right). \] (7)

Because of the symmetry of the model, this threshold is the same for each household member. Hence the solution to Eq. (7) can be denoted as \( y_I^* = y^* (Y_2, X, S) \). The so-called threshold function has very simple properties. In particular, Lemma 5 in the appendix demonstrates that it is everywhere greater than or equal to \( Y_2/2 \) and satisfies:

\[ \frac{\partial y_I^*}{\partial Y_2} \geq \frac{1}{2}, \quad \frac{\partial y_I^*}{\partial X} > 0, \quad \frac{\partial y_I^*}{\partial S} \geq 0. \]
That means, in particular, that an increase in the second period income, the marriage surplus or saving discourages divorce. This result is quite intuitive and does not deserve a discussion; see the appendix for more properties.

Theoretically, the threat of divorce should never be carried out even if this threat is credible. A separation is indeed inefficient for spouses because of the loss of the marriage surplus. They have to strike an agreement. If the personal income of one spouse is greater than the threshold, he or she will be able to demand a greater share of consumption. The function $\phi_I$ is such that member $I$ is indifferent between divorce and marriage and is formally defined by

$$u\left(y_I + \frac{S}{2}\right) = u\left(\phi_I + \frac{S}{2}\right) + X,$$

for any $y_I \geq y^*$. The solution to this equation is the sharing function, denoted by $\phi_I = \phi(y_I, X, S)$. Lemma 6 in the appendix demonstrates that this function is everywhere comprised between $Y_2/2$ and $y_I$ and satisfies:

$$\frac{\partial \phi_I}{\partial y_I} > 0, \quad \frac{\partial \phi_I}{\partial X} < 0, \quad \frac{\partial \phi_I}{\partial S} \leq 0.$$

Hence, an increase in the marriage surplus or in savings has a negative impact on what can be demanded by the spouse with a credible opportunity of leaving. An increase in the personal income has a positive impact. Moreover, we have the following identity:

$$\phi(y^*, X, S) = \frac{Y_2}{2}.$$  \hspace{1cm} (9)

Finally, if $y_I \geq y^*$, the partner of member $I$ obtains:

$$Y_2 + \frac{S}{2} - \phi(y_I, X, S).$$

That is, the total resources from which the share of consumption of member $I$ is substracted.

**Saving Motives** The level of felicity of household members at the first period, since the sustainability constraint is assumed to be non-binding, is perfectly determined and given by Eq. (5) as a function of $S$. Consider now
the level of felicity at the second period and, to simplify, suppose that there exists at least one value of $\varepsilon$ such that the threat of divorce is credible, i.e.,

$$2y^* - Y_2 \leq \Sigma \leq Y_2,$$

from Eq. (2). Under the assumption that the distribution of $\varepsilon$ is uniform, the probability that the sustainability constraint be not binding (conditionally on $S$) is given by

$$\Pr \left( \frac{Y_2 - 2y^*}{2\Sigma} < \varepsilon < \frac{2y^* - Y_2}{2\Sigma} \right) = \frac{2y^* - Y_2}{\Sigma},$$

and the sustainability constraint is binding only if

$$\frac{Y_2}{2} \pm \Sigma \varepsilon > y^*,$$

with $\varepsilon \in \left[ -\frac{1}{2}, \frac{1}{2} \right]$. The (conditional) expected felicity function at the second period is then given by

$$E(u|Y_2, S, X) = \frac{2y^* - Y_2}{\Sigma} \times u \left( \frac{Y_2 + S}{2} \right)$$

$$+ \int_{-\frac{1}{2}}^{\frac{1}{2}} u \left( Y_2 + S - \frac{\phi \left( \frac{Y_2}{2} - \Sigma \varepsilon \right)}{2} \right) d\varepsilon$$

$$+ \int_{\frac{1}{2}}^{\frac{1}{2}} u \left( \frac{S}{2} + \frac{\phi \left( \frac{Y_2}{2} + \Sigma \varepsilon \right)}{2} \right) d\varepsilon,$$

where, to keep notation as simple as possible, only the first argument of the sharing function $\phi$ is made explicit. If this expression is differentiated with respect to $S$, the expected marginal felicity function is obtained. Before deriving this expression, we have to introduce the definition of the variance of the second period consumption. Straightforward (but tremendous) computations give

$$\text{var}(x_t) = \frac{2}{\Sigma} \int_{y^*}^{y^*+\Sigma} \left( \phi(t) - \frac{Y_2}{2} \right)^2 dt.$$  

Thus the dispersion of the second period consumption is directly related to the deviation between the sharing function and the average consumption.
This expression represents the conjugal risk that results from the sole fluctuations in bargaining power.\(^7\) Differentiating this expression with respect to \(S\), using the Leibniz Rule and Eq. (9), yields:

\[
\frac{\partial \text{var}(x_I)}{\partial S} = 4 \sum_{y^*}^{Y_2+\Sigma} \left( \phi(t) - \frac{Y_2}{2} \right) \frac{\partial \phi(t)}{\partial S} \, dt
\]

This expression is obviously negative since, as seen above, \(\phi \geq \frac{Y_2}{2}\) and \(\frac{\partial \phi}{\partial S} \leq 0\). The expected marginal felicity function and its interpretation can now be given in the next proposition.

**Proposition 1** The expected marginal felicity function is equal to the sum of three non-negative terms, i.e.,

\[
\frac{\partial E(u|Y_2, S, X)}{\partial S} = \text{Cons} + \text{Prec} + \text{Insu}
\]  \hspace{1cm} (11)

where

\[
\text{Cons} = \frac{1}{2} u'(\frac{Y_2 + S}{2})
\]  \hspace{1cm} (12)

is the consumption motive of saving,

\[
\text{Prec} \simeq \frac{1}{4} u'''(\frac{Y_2 + S}{2}) \times \text{var}(x_I)
\]  \hspace{1cm} (13)

is the precautionary motive of saving,

\[
\text{Insu} \simeq \frac{1}{2} u''(\frac{Y_2 + S}{2}) \times \frac{\partial \text{var}(x_I)}{\partial S}
\]  \hspace{1cm} (14)

is the insurance motive of saving, and the approximations are valid if the sharing function \(\phi\left(\frac{Y_2+\Sigma}{2}\right)\) is close to \(\frac{Y_2}{2}\).

This proposition deserves some comments. We first must note that the terms (13) and (14) result from Taylor expansions and thus approximate

\(^7\)Note also that \(\text{var}(x_I) < \text{var}(y_I)\) since the sharing function \(\phi\) is concave with respect to \(y_I\) as is demonstrated in Lemma 6. That is to say, the conjugal risk is smaller than the income risk from which it results.
the exact terms which enter the expected marginal felicity function. These approximations are valid if the risk is relatively small.\textsuperscript{8}

Consider now the motives of savings. The \textbf{consumption motive} is clear and has not to be explained. The \textbf{precautionary motive} is also well-known in the economic literature on uncertainty (e.g., Gollier, 2001). It implies that, if the bargaining power at the second period stochastically fluctuates, an increase in saving gives a supplement of welfare because of the prudence of household members. This effect is proportionate to the convexity of the marginal felicity function and to the variance in the second period consumption. Hence, more convex the marginal felicity function is and more dispersed the distribution of consumption is, larger the precautionary motive is.\textsuperscript{9} The \textbf{insurance motive} is more specific to our model and, as such, more interesting. It translates the fact that, everything else being equal, if the level of saving increases, the share of consumption that the spouse with a credible threat of divorce can demand will be reduced. This effect is proportionate to the concavity of the felicity function and the effect of saving on the dispersion of consumption. The intuition is that, if spouses are averse to fluctuations of consumption, and if an increase in saving has a negative impact on the variance of consumption, the expected marginal felicity of saving will be relatively large.\textsuperscript{10}

\section{The Main Results}

\textbf{Intertemporal Optimization} Consider first the intertemporal optimization problem when spouses are able to commit to an allocation of resources for the future and, therefore, eliminate idiosyncratic risks. In other words, whatever the distribution of personal incomes, a level of consumption equal to \((Y_2 + S)/2\) is assured to spouses. If the discount factor of the second

\textsuperscript{8}The exact terms can be found in the proof of Proposition 1 in the appendix. Our interpretation of the saving motives uses these approximations but it is still valid with the exact formulation (at the cost of additional complexities). These approximations are exact only if the felicity function belongs to the quadratic family.

\textsuperscript{9}The positivity of the precautionary term is not assured in more general specifications as is demonstrated by Mazzocco (2004). Some counterintuitive results can be obtained as soon as spouses are assumed to have different felicity functions or discount rates. Be that as it may, the case that is studied here is certainly the most common in empirics.

\textsuperscript{10}If the return rate of the riskless asset was different from zero, there would be a fourth motive of saving, namely, speculation. But this generalization brings nothing.
period is equal to one (i.e., there is no time impatience), the choice of the intertemporal allocation is simply determined by the following program:

$$\max_S u \left( \frac{Y_1 - S}{2} \right) + u \left( \frac{Y_2 + S}{2} \right).$$  (15)

The marriage surplus can be disregarded here since it is additively separable from the other variables. This program yields “full-commitment” or “first best” solutions. The first order conditions are:

$$\frac{1}{2} u' \left( \frac{Y_1 - S}{2} \right) = \frac{1}{2} u' \left( \frac{Y_2 + S}{2} \right).$$  (16)

From this condition, it is obvious that the optimal level of saving is equal to:

$$S = \frac{Y_1 - Y_2}{2}$$

In that case, the role of saving is simply to smooth consumption over time; there is neither precaution, nor insurance.

In the general case, where the spouses are not able to commit to a sharing of future consumption, the intertemporal allocation is given by:

$$\max_S u \left( \frac{Y_1 - S}{2} \right) + E (u|Y_2, S, X)$$

Note that, in this case, the marriage surplus enters the objective function. This program yields the “limited-commitment” or “second best” solutions. Using Proposition 1, the first order condition is:

$$\frac{1}{2} u' \left( \frac{Y_1 - S}{2} \right) = \frac{1}{2} u' \left( \frac{Y_2 + S}{2} \right) + \text{Prec} + \text{Insu}. $$  (17)

The second order condition is globally satisfied because of the convexity of the marginal felicity function. The proof of this point is given in the appendix. The main conclusion, at this stage, is that the investment in the riskless asset is greater in the limited-commitment case than in the full-commitment one. The idea behind this result is that the expected marginal felicity of saving is more elevated in a stochastic context, since

$$\text{Prec} + \text{Insu} > 0.$$
Mazzocco (2004c) considers a more general model in which the outside opportunities of spouses are not specified, and obtains some slightly different results. In particular, he shows that, in general, intrahousehold risk sharing can entail an increase in saving. This counterintuitive result is excluded by our assumptions about the decision process and, incidentally, the fact that spouses have the same preferences.\textsuperscript{11}

**Comparative statics** To begin with, we consider the impact on saving of a variation in the marriage surplus or incomes. The marriage surplus influences behavior indirectly through the threshold and sharing functions. Incomes have also a direct effect through consumption. The next proposition presents these natural results.

**Proposition 2** The level of investments in the riskless asset $S$ decreases if

\begin{itemize}
  \item The marriage surplus $X$ increases;
  \item The first period income $Y_1$ decreases or the second period income $Y_2$ increases; the marginal propensity to save in terms of absolute values is not necessarily equal to 0.5.
\end{itemize}

The intuition of the first statement in this proposition is simple. If the marriage surplus is important, divorce is costly. Consequently, the insurance and precautionary motives of saving are reduced since the probability that the divorce threat be credible decreases. As simple as it may be, this result has several interesting, and sometimes curious applications depending on the source of the marriage surplus. For example, it implies that the level of saving of a couple with children or with a passionate relationship will be relatively small, all other things being the same. This result can also be compared to a statement by Cubeddu and Ríos-Rull (1997). These authors say that changes in the likelihood of divorce may induce a desire to save more because of standard precautionary motives. The insurance motives are, however, ignored in their framework. Moreover, our result indicates that a simple threat of divorce, even if never carried on, is sufficient to entail an increase in saving.

\textsuperscript{11}The full-commitment model that is studied here can also be compared to the unitary framework. One difference with what is made by Mazzocco (2004a, 2004b) is that our set-up is completely structural and thus introduce the insurance motive that is generally ignored in alternative approaches.
The second statement in Proposition 2 is obvious and does not deserve a discussion. Finally, it can be shown that the sign of the impact of an increase in the second period income dispersion (i.e., the parameter $\Sigma$) on savings is undetermined.

**Intrahousehold Inequality**  When the spouses are able to commit to an allocation of resources for the future, there is no intrahousehold inequality: the spouses can insure against idiosyncratic risks and they obtain the same endowment of consumption. However, in the limited-commitment context, the outside opportunities influence the intrahousehold distribution of consumption. The following propositions indicate how the distribution of resources in the household is affected by a variation in the exogenous variables.

**Proposition 3**  The variance $\text{var}(x_I)$ of the second period consumption decreases if

- The marriage surplus $X$ increases;
- The first period income $Y_1$ or the second period income $Y_2$ increases.

The first statement can be seen as follows: the distribution of consumption in households where marriage surplus are important (e.g., in couples with children or with a passionate relationship) are more equal than in other households. The second statement in this proposition is worth emphasizing. It implies, all other things being the same, that the distribution of consumption in richer households is less unequal than in poorer households. The intuition is that divorce is less profitable when household resources are important. The policy relevance of this result is clear: in less-developed countries, it suffices to treat the question of low income households and the intrahousehold inequality problem will be solved in the same time. A similar conclusion — based on a completely different model yet — is drawn by Haddad and Kanbur (1992) and Kanbur and Haddad (1994). These authors support the idea that the economic development of households should finally lead to a decrease in intrahousehold inequality (even if this conclusion may be inverted in the first step of development). Note also that empirical evidence (e.g., Lise and Seitz, 2004) seems to indicate that intrahousehold inequality has decreased over the twenty last years.
Endogenous Determination of the Marriage Surplus  Until now, the marriage surplus was supposed to be exogenously determined. The modification of this assumption can significantly altered our main conclusions. Suppose now that the marriage surplus is chosen by the household members, with a price set to one. In the first period, the household members choose the levels of saving and consumption and the marriage surplus. To begin with, consider the full-commitment case. The first order conditions become:

\[
\frac{1}{2} u' \left( \frac{Y_1 - S - X}{2} \right) = \frac{1}{2} u' \left( \frac{Y_2 + S}{2} \right),
\]

\[
\frac{1}{2} u' \left( \frac{Y_1 - S - X}{2} \right) = 2.
\]

The first equation corresponds to the allocation of savings and the second equation to the allocation of the marriage surplus. The resolution of this system gives:

\[
S = k - Y_2
\]

\[
X = Y_1 + Y_2 - 2k
\]

where \(k\) is a constant which depends on the form of felicity functions. The role of saving is thus different in the endogeneity case since, quite surprisingly, the level of saving is not a function of the first period income. However, these results have to be compared with the limited-commitment case. The first order conditions in this case are given by:

\[
\frac{1}{2} u' \left( \frac{Y_1 - S - X}{2} \right) = \frac{\partial E(u|Y_2, S, X)}{\partial S},
\]

\[
\frac{1}{2} u' \left( \frac{Y_1 - S - X}{2} \right) = 2 + \frac{\partial E(u|Y_2, S, X)}{\partial X}.
\]

Since the marriage surplus is determined at the first period, and remains constant at the second period, the expected marginal felicity function has the same decomposition as in Section 2. Hence, we note that the level of total investments, i.e., \(S + X\), is higher in the limited-commitment case than in the full-commitment one. This results as previously from the following inequality:

\[
\text{Prec} + \text{Insu} > 0.
\]
The question is whether the investments in the riskless asset or the relationship-specific asset are higher in the limited-commitment case. To examine that and simplify notations, we suppose that the felicity function belongs to the quadratic family, i.e.,

\[ u = x_I - \beta x_I^2. \]

The quadratic felicity function is generally considered as not being realistic, because it exhibits increasing absolute risk aversion. However, the advantage of this specification is that the precautionary motive of saving disappears and the insurance motive is then isolated. The next proposition summarizes the main results.

**Proposition 4** Assume that the level of the relationship-specific asset is endogenous and that the felicity function belongs to the quadratic family. Then,

- The level of investments in the riskless assets \( S \) is lower in the limited-commitment case than in the full-commitment case;
- The level of investments in the relationship-specific asset \( X \) is higher in the limited-commitment case than in the full commitment case.

This result states, in a sense, that the investments in the relationship-specific asset are more adapted for the motive of insurance because these investments are sunk once they have been made. Hence, the household reduces its investment in the riskless asset and dedicates these funds to the relationship-specific asset. In particular, this implies that the household will overinvest in children to reduce the conjugal risk. This is in strike contrast with what is generally shown in the literature. For example, Konrad and Lommerud (2000) demonstrates, using a Nash-bargained model of household, that the spouses are incited to underinvest in the relationship-specific asset. This conclusion is also drawn by Lundberg (2002) or Rainer (2003). The difference between these investigations and ours comes mainly from the role played by uncertainty.

4 Conclusion

In this paper, we propose a theoretical framework that can be used to investigate the impact of variations in bargaining power on saving and consumption
choices. We show, in particular, that saving can be motivated by a desire of insurance, which is different from prudence. In that case, we observe that spouses overinvest in assets to reduce conjugal risk. However, our investigation is above all exploratory. Several extensions are worth underlining.

- **Liquidity Constraints:** If there are liquidity constraints, the level of saving should increase. Since only the poorer households are affected by liquidity constraints, this may counterbalance the traditional effect of income on intrahousehold inequality.

- **Private Savings and Moral Hazard:** If saving is assumed private, and personal incomes are imperfectly observed by the partner, saving decisions will be influenced by moral hazard. The intuition suggests that it should increase saving.

- **Other Sources for Conjugal Risk:** Conjugal risk does not come necessarily from the variations in personal incomes. For example, the valuation of the marriage surplus by spouses can stochastically fluctuate. In this case, investments in assets are probably less appropriate as an insurance instrument.

One final remark is that the results of this paper, instead of explicitly referring to saving, can be interpreted in more general terms. The message is then the following. Suppose that some goods consumed by spouses — or some instruments at spouses’ disposal — have a direct impact on the variance of the future distribution of the intrahousehold consumption. The formula that describes the insurance motive of buying these goods is given by Eq. (14) and the prediction is that, if they are risk-averse, spouses will buy a relatively large quantity of these goods. Conversely, if spouses hold a large quantity of these goods, the distribution of consumption within the household will be relatively less unequal.

## A Appendix

### A.1 Some Useful Results about $y^*$ and $\phi$

We present below two Lemmas that are extensively used in the proofs of the various propositions of this paper.
Lemma 5  The threshold function $y^* (Y_2, S, X)$ is everywhere greater than $Y_2/2$ and satisfies the following:

1. $\frac{\partial y^*_I}{\partial Y_2} \geq \frac{1}{2}$;
2. $\frac{\partial y^*_I}{\partial X} > 0$;
3. $\frac{\partial y^*_I}{\partial S} = \frac{\partial y^*_I}{\partial Y_2} - \frac{1}{2} \geq 0$.

Proof  Applying the Implicit Function Theorem to Eq. (7) yields:

$$\frac{\partial y^*_I}{\partial Y_2} = \frac{1}{2} \frac{u'(Y_2 + S)}{u'(y^*_I + S)}$$

(this expression is at least as great as 1/2 because of the concavity of the felicity function);

$$\frac{\partial y^*_I}{\partial X} = \frac{1}{u'(y^*_I + S)}$$

(this expression is positive because of the positivity of the marginal felicity function);

$$\frac{\partial y^*_I}{\partial S} = \frac{1}{2} \frac{u'(Y_2 + S)}{u'(y^*_I + S)} - \frac{1}{2} = \frac{\partial y^*_I}{\partial Y_2} - \frac{1}{2}$$

(this expression is non-negative because of the first statement in Lemma 5).

Lemma 6  The sharing function $\phi (y_I, X, S)$ is defined for any $y_I \geq y^*$. It is everywhere comprised between $Y_2/2$ and $y_I$ and satisfies the following:

1. $0 < \frac{\partial \phi_I}{\partial y_I} < 1$;
2. \( \frac{\partial \phi_I}{\partial X} < 0; \)

3. \[-\frac{1}{2} < \frac{\partial \phi_I}{\partial S} = \frac{1}{2} \frac{\partial \phi_I}{\partial y_I} - \frac{1}{2} \leq 0; \]

4. \( \frac{\partial^2 \phi_I}{\partial S^2} > 0; \)

5. \( \phi (y^*, X, S) = \frac{Y_2}{2}. \)

**Proof**  Applying the Implicit Function Theorem to Eq. (8) yields:

\[
\frac{\partial \phi_I}{\partial y_I} = \frac{u'(y_I + \frac{S}{2})}{u'(\phi_I + \frac{S}{2})}
\]

(this expression is positive and inferior to 1 because of the concavity of the felicity function);

\[
\frac{\partial \phi_I}{\partial X} = -\frac{1}{u'(\phi_I + \frac{S}{2})}
\]

(this expression is negative because the felicity function is increasing in \( x_I \));

\[
\frac{\partial \phi_I}{\partial S} = \frac{1}{2} \frac{u'(y_I + \frac{S}{2})}{u'(\phi_I + \frac{S}{2})} - \frac{1}{2}
\]

(this expression is comprised between \(-1/2\) and 0 because of the first statement in Lemma 6); after rearrangements,

\[
\frac{\partial^2 \phi_I}{\partial S^2} = \frac{u''(y_I + \frac{S}{2}) \left( u' \left( \phi_I + \frac{S}{2} \right) \right)^2 - u'' \left( \phi_I + \frac{S}{2} \right) \left( u' \left( y_I + \frac{S}{2} \right) \right)^2}{4 \left( u' \left( \phi_I + \frac{S}{2} \right) \right)^3}
\]
(this expression is negative because of the convexity of the marginal felicity function). The last statement is proved by the comparison of Eq. (7) and Eq. (8), evaluated at \( y_f = y^* \). It gives:

\[
u \left( \frac{y^* + S}{2} \right) = u \left( \frac{\phi Y + S}{2} \right) + X = u \left( \frac{Y_2 + S}{2} \right) + X.
\]

From the second equality, we obtain: \( \phi(y^*, X, S) = Y_2/2 \).

### A.2 Proofs of Propositions

#### A.2.1 Proof of Proposition 1

If the expression for the marginal utility is differentiated with respect to \( S \), using the Leibniz rule, and rearrange, the expected marginal felicity function is obtained under the form of a sum of various terms:

\[
\frac{\partial E(u|Y_2, S, X)}{\partial S} = \text{Cons} + \text{Prec} + \text{Insu}. \tag{18}
\]

The interpretation and the sign of each term in Eq. (11) can be precisely determined. The first term is of the following form:

\[
\text{Cons} = \frac{1}{2} u \left( \frac{Y_2 + S}{2} \right).
\]

This expression is positive and represents the consumption motive of saving. The development of the other terms is more complicated. The proof uses Lemma 5 and Lemma 6 and follows in stages.

**Precautionary Motive** The second term of Eq. (11) is:

\[
\text{Prec} = \frac{1}{2} \int_{-1}^{\frac{1}{2}} u' \left( Y_2 + \frac{S}{2} - \phi \left( \frac{Y_2}{2} - \Sigma \epsilon \right) \right)\,d\epsilon
\]

\[
+ \frac{1}{2} \int_{2y^*-Y_2}^{\frac{1}{2}} u' \left( \frac{S}{2} + \phi \left( \frac{Y_2}{2} + \Sigma \epsilon \right) \right)\,d\epsilon
\]

\[
- \frac{1}{2} \left( 1 - \frac{2y^* - Y_2}{\Sigma} \right) u' \left( \frac{Y_2 + S}{2} \right)\]

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Using a convenient change of variable yields:

\[
\text{Prec} = \frac{1}{\Sigma \gamma^*} \left( \frac{1}{2} u' \left( \frac{S}{2} + \phi(t) \right) + \frac{1}{2} u' \left( Y_2 + \frac{S}{2} - \phi(t) \right) - u' \left( \frac{Y_2 + S}{2} \right) \right) dt.
\]

Define \( \psi = \frac{Y_2 + S}{2} \) and \( h(t) = \phi(t) - \frac{Y_2}{2} \). Then,

\[
\text{Prec} = \frac{1}{\Sigma} \int_{y^*}^{\frac{Y_2 + S}{2}} \left( \frac{1}{2} u' (\psi + h(t)) + \frac{1}{2} u' (\psi - h(t)) - u' (\psi) \right) dt.
\]

Note that this expression is positive from the Jensen Inequality and the convexity of the marginal felicity function. Consider the second order Taylor approximation of \( u' \) around point \( \psi \). That gives:

\[
u' (\psi \pm h) = u' (\psi) \pm u'' (\psi) h + \frac{1}{2} u''' (\psi) h^2 \pm r(h)
\]

where

\[
r(h) = \frac{1}{6} u'''(\bar{\psi}) h^3
\]

for some point \( \bar{\psi} \) interior to the interval \( \psi \) and \( \psi \pm h \), is the Lagrange remainder. Now, if we introduce this approximation in \( \text{Prec} \) and simplify, we obtain:

\[
\text{Prec} = \frac{1}{2} u'''(\psi) \int_{y^*}^{\frac{Y_2 + S}{2}} \frac{h^2(t)}{\Sigma} dt + \frac{1}{12} (u'''(\bar{\psi}_1) - u'''(\bar{\psi}_2)) \int_{y^*}^{\frac{Y_2 + S}{2}} \frac{h^3(t)}{\Sigma} dt
\]

or

\[
\text{Prec} = \frac{1}{4} u'''(\psi) \times \var(x_I) + \text{remainder}
\]

where

\[
\var(x_I) = 2 \int_{y^*}^{\frac{Y_2 + S}{2}} \frac{h^2(t)}{\Sigma} dt,
\]

remainder \( = \frac{1}{12} (u'''(\bar{\psi}_1) - u'''(\bar{\psi}_2)) \int_{y^*}^{\frac{Y_2 + S}{2}} \frac{h^3(t)}{\Sigma} dt.\)

The remainder is likely very small since \( u'''(\bar{\psi}_1) \approx u'''(\bar{\psi}_2).\)
Insurance Motive  The third term of Eq. (11) is:

\[
\text{Insu} = \int_{\frac{y_2}{2}}^{\frac{y_2}{2} - y_2} u' \left( \frac{S}{2} + \phi \left( \frac{Y_2}{2} + \Sigma \varepsilon \right) \right) \frac{\partial \phi \left( \frac{Y_2}{2} + \Sigma \varepsilon \right)}{\partial S} d\varepsilon \\
- \int_{-\frac{1}{2}}^{\frac{y_2}{2} - 2y_2} u' \left( Y_2 + \frac{S}{2} - \phi \left( \frac{-Y_2}{2} - \Sigma \varepsilon \right) \right) \frac{\partial \phi \left( \frac{-Y_2}{2} - \Sigma \varepsilon \right)}{\partial S} d\varepsilon.
\]

Using a convenient change of variable yields:

\[
\text{Insu} = \frac{1}{\Sigma} \int_{y^*}^{y^* + \Sigma} \left( u' \left( \frac{S}{2} + \phi (t) \right) - u' \left( Y_2 + \frac{S}{2} - \phi (t) \right) \right) \frac{\partial \phi (t)}{\partial S} dt,
\]

or

\[
\text{Insu} = \frac{1}{\Sigma} \int_{y^*}^{y^* + \Sigma} (u' (\psi + h(t)) - u' (\psi - h(t))) \frac{\partial \phi (t)}{\partial S} dt.
\]

This expression is clearly positive since \( \frac{\partial \phi}{\partial S} \) is negative. Consider now the second order Taylor approximation of \( u \) around point \( \psi \). That gives:

\[ u' (\psi \pm h) = u' (\psi) \pm u'' (\psi) h + \frac{1}{2} u''' (\psi) h^2 \pm r (h) \]

where

\[ r (h) = \frac{1}{6} u''' (\overline{\psi}) h^3. \]

If we introduce this expression in \( \text{Insu} \), we obtain:

\[
\text{Insu} = \frac{2u'' (\psi)}{\Sigma} \int_{y^*}^{y^* + \Sigma} h(t) \frac{\partial \phi (t)}{\partial S} dt + \frac{u''' (\overline{\psi}_1) + u''' (\overline{\psi}_2)}{6\Sigma} \int_{y^*}^{y^* + \Sigma} \frac{\partial \phi (t)}{\partial S} h^3 (t) dt.
\]

or

\[
\text{Insu} = \frac{1}{2} u'' \left( \frac{Y_2 + S}{2} \right) \frac{\partial \text{var}(x_I)}{\partial S} dt + \text{remainder},
\]

where

\[
\frac{\partial \text{var}(x_I)}{\partial S} = \frac{4}{\Sigma} \int_{y^*}^{y^* + \Sigma} h(t) \frac{\partial \phi (t)}{\partial S} dt
\]

remainder

\[
= \frac{u''' (\overline{\psi}_1) + u''' (\overline{\psi}_2)}{6\Sigma} \int_{y^*}^{y^* + \Sigma} \frac{\partial \phi (t)}{\partial S} h^3 (t) dt.
\]

||
A.2.2 Proof of Proposition 2

Consider the proof of the first statement. Applying the Implicit Function Theorem to Eq. (17) gives:

$$\frac{\partial S}{\partial X} = -\frac{\frac{\partial^2 E(u|Y_2, S, X)}{\partial S \partial X}}{\frac{\partial^2 E(u|Y_2, S, X)}{\partial S^2} + \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right)}$$

where the denominator of this expression is negative because of the second order condition. Hence, the sign of this expression is the same as the sign of $\frac{\partial^2 E(u|S)}{\partial S \partial X}$. This can be written as follows:

$$\frac{\partial^2 E(u|Y_2, S, X)}{\partial S \partial X} = \frac{\partial \text{Cons}}{\partial X} + \frac{\partial \text{Prec}}{\partial X} + \frac{\partial \text{Insu}}{\partial X}$$

It is obvious that the first term in right-hand-side of this expression is equal to zero, i.e.,

$$\frac{\partial \text{Cons}}{\partial X} = 0.$$  

The second term deserves more attention. We actually have:

$$\frac{\partial \text{Prec}}{\partial X} = \frac{1}{\Sigma} \int_{t^*}^{y*} A \frac{\partial \phi(t)}{\partial X} dt,$$

where

$$A = u'' \left( \frac{S}{2} + \phi(t) \right) - u'' \left( \frac{Y_2}{2} - \phi(t) \right).$$

This expression is negative because $A$ is positive. The third term is equal to

$$\frac{\partial \text{Insu}}{\partial X} = \frac{1}{\Sigma} \int_{t^*}^{y*} A \frac{\partial \phi(t)}{\partial S} \frac{\partial \phi(t)}{\partial X} dt,$$

where $A$ is positive as above. Using Lemma 6, this expression is negative. This proves the first statement of Proposition 2.
Consider now the proof of the second statement in Proposition 2. Suppose that the first period income increases and consider the impact on saving. Applying the Implicit Function Theorem to Eq. (17) gives:

\[
\frac{\partial S}{\partial Y_1} = \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right) + \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right) \frac{\partial^2 E(u|Y_2, S, X)}{\partial S^2} + \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right).
\]

This expression is comprised between 0 and 1. To evaluate the impact of a variation in the second period income, we must note that a variation in income is analogue to a variation in saving. Thus:

\[
\frac{\partial^2 E(u|Y_2, S, X)}{\partial S \partial Y_2} = \frac{\partial^2 E(u|Y_2, S, X)}{\partial S^2},
\]

and from the Implicit Function Theorem, we have:

\[
\frac{\partial S}{\partial Y_2} = - \frac{\partial^2 E(u|Y_2, S, X)}{\partial S^2} + \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right).
\]

This expression is comprised between \(-1\) and 0. Hence, an increase in the second period income has a negative impact on savings.

**A.2.3 Proof of Proposition 3**

The proof of this proposition is based on the relation between \( S \) and \( \text{var}(x_I) \), i.e.,

\[
\frac{\partial \text{var}(x_I)}{\partial S} = 4 \sum \int_{y^*}^{Y_2 + \Sigma} \left( \phi(t) - \frac{Y_2}{2} \right) \frac{\partial \phi(t)}{\partial S} dt.
\]

This expression is clearly negative, i.e., an increase in savings has a negative impact on the dispersion of consumption. This proves that an increase in marriage surplus or the first period income has a negative impact on the distribution of consumption. Note also that

\[
\frac{\partial \text{var}(x_I)}{\partial Y_2} = \frac{\partial \text{var}(x_I)}{\partial S}.
\]

Hence using Proposition 2 proves that an increase in the second period income has also a negative effect on the distribution of income.
A.2.4 Proof of Proposition 4

From the first order conditions, we obtain a relation between the marginal expected felicity functions:

\[ \frac{\partial E(u|Y_2, S, X)}{\partial S} - \frac{\partial E(u|Y_2, S, X)}{\partial X} = 2. \tag{20} \]

Recall that the form of marginal expected felicity function with respect to \( S \) is equal to

\[ \frac{\partial E(u|Y_2, S, X)}{\partial S} = \frac{1}{2} u'(\frac{Y_2 + S}{2}) + \frac{2}{\Sigma} u''(\frac{Y_2 + S}{2}) \int_{y^*}^{\frac{Y_2 + \Sigma}{2}} (\phi(t) - \frac{Y_2}{2}) \frac{\partial \phi(t)}{\partial S} dt. \]

Moreover, it is straightforward to show that the marginal expected utility with respect to \( X \) is equal to

\[ \frac{\partial E(u|Y_2, S, X)}{\partial X} = \frac{2}{\Sigma} u''(\frac{Y_2 + S}{2}) \int_{y^*}^{\frac{Y_2 + \Sigma}{2}} (\phi(t) - \frac{Y_2}{2}) \frac{\partial \phi(t)}{\partial X} dt. \]

Thus, from Eq. (20), we have:

\[ \frac{1}{2} u'(\frac{Y_2 + S}{2}) = 2 - \frac{2}{\Sigma} u''(\frac{Y_2 + S}{2}) \int_{y^*}^{\frac{Y_2 + \Sigma}{2}} (\phi(t) - \frac{Y_2}{2}) \left( \frac{\partial \phi(t)}{\partial X} - \frac{\partial \phi(t)}{\partial S} \right) dt \]

From Eq. (8), and the assumption made on preferences, we have:

\[ \frac{\partial \phi}{\partial X} - \frac{\partial \phi}{\partial S} = -\frac{2 - \beta y_I + \beta \phi_I}{2 - 2\beta (\phi_I + \frac{S}{2})} \]

It is easy to show that this expression is negative since

\[ 1 - \beta \left( \frac{y_I - \phi_I}{2} \right) > 0. \]

Eq. (19) then becomes

\[ \frac{1}{2} u'(\frac{Y_2 + S}{2}) > 2. \]

This following condition determines the distribution of investment between \( X \) and \( S \). If we compare this inequality with the first order conditions in the full-commitment case, we obtain that the investment in the riskless asset is lower than in the full-commitment case and, consequently, the investment in the relationship-specific capital is higher. ||
A.3 Second Order Condition

The second order condition can be written as follows:

\[
\frac{1}{4} u'' \left( \frac{Y_2 + S}{2} \right) + \frac{1}{4} u'' \left( \frac{Y_1 - S}{2} \right) + \frac{\partial \text{Prec}}{\partial S} + \frac{\partial \text{Insu}}{\partial S} < 0
\]  

Using the expressions for Prec and Insu and the identity \( \phi (y^*) = Y_2 / 2 \), the third term of Eq. (21) writes down as follows:

\[
\frac{\partial \text{Prec}}{\partial S} = \frac{1}{2 \Sigma} \int_{y^*}^{\frac{Y_2 + \Sigma}{2}} A_1(t) dt - \frac{1}{4} \left( 1 - \frac{2y^* - Y_2}{\Sigma} \right) u'' \left( \frac{Y_2 + S}{2} \right)
\]

with

\[
A(t) = u'' \left( \frac{S}{2} + \phi(t) \right) \left( \frac{1}{2} + \frac{\partial \phi(t)}{\partial S} \right) + u'' \left( \frac{Y_2 + S}{2} - \phi(t) \right) \left( \frac{1}{2} - \frac{\partial \phi(t)}{\partial S} \right).
\]

The term \( A(t) \) is negative for any \( t \) because of the concavity of \( u \) and the fact that \( |\partial \phi(t)/\partial S| < 1/2 \). Moreover, since \( (2y^* - Y_2) / \Sigma \geq 0 \), we have:

\[
\frac{1}{4} u'' \left( \frac{Y_2 + S}{2} \right) + \frac{\partial \text{Prec}}{\partial S} < 0
\]

The last term of Eq. (21) is equal to

\[
\frac{\partial \text{Insu}}{\partial S} = \frac{1}{\Sigma} \int_{y^*}^{\frac{Y_2 + \Sigma}{2}} \frac{\partial \phi(t)}{\partial S} B_1(t) dt + \frac{1}{\Sigma} \int_{y^*}^{Y_2} \frac{\partial^2 \phi(t)}{\partial S^2} B_2(t) dt
\]

where

\[
B_1(t) = u'' \left( \frac{S}{2} + \phi(t) \right) \left( \frac{1}{2} + \frac{\partial \phi(t)}{\partial S} \right) - u'' \left( \frac{Y_2 + S}{2} - \phi(t) \right) \left( \frac{1}{2} - \frac{\partial \phi(t)}{\partial S} \right)
\]

\[
B_2(t) = u' \left( \frac{S}{2} + \phi(t) \right) - u' \left( \frac{Y_2 + S}{2} - \phi(t) \right)
\]

The term \( B_1 \) is positive for any \( t \) because of the convexity of \( u' \), the term \( B_2 \) is non-positive for any \( t \) because of the concavity of \( u \). Thus, the term \( \partial \text{Insu} / \partial S \) is negative and, consequently, the second order condition is satisfied.
References


