THE LAFFER CURVE IN AN INCOMPLETE-MARKETS ECONOMY

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ABSTRACT. This paper examines quantitative issues related to the Laffer curve in a neoclassical growth model with endogenous labor supply and complete or incomplete financial markets where distortionary taxes on labor, capital and consumption are used to finance government consumption, lump-sum transfers and debt repayments. We show that the shape of the Laffer curve related to each type of taxation differs a lot for the two model versions, especially when public debt is adjusted to fulfill the government budget constraint. In the incomplete markets setup, a given level of the fiscal revenues can be associated to three different levels of labor or capital income taxes. This finding occurs because the tax rates change non monotonically with public debt when markets are incomplete.


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1. Introduction

In this paper, we inspect how allowing for liquidity-constrained agents and incomplete financial markets impacts on the shape of the Laffer curve. This object, i.e. the relation between fiscal revenues and taxes, and more specifically its slippery slope have been often used as a key ingredients for fiscal policy prescriptions in macroeconomic models (see, e.g., Flodén and Lindé [2001], Ireland [1994], Schmitt-Grohé and Uribe [1997]). In particular, if a permanent taxation level is such that fiscal revenues lie on the slippery slope of the Laffer curve, then a tax cut can be self-financed.

Recently, Trabandt and Uhlig (2009) analyzed thoroughly Laffer curves in the standard, representative-agent, complete-markets (CM), neoclassical growth model when distortionary taxes on labor, capital, and consumption are used to finance government consumption, lump-sum transfers, and debt repayments. As is well-known, in such a Ricardian framework, it does not matter whether public debt or transfers are adjusted to make the government budget constraint hold. However, in an Incomplete-Markets (IM) economy à la Aiyagari (1994), Ricardian equivalence fails so that opting to adjust debt rather than transfers can potentially make a big difference. Given that this class of models has now become a standard framework for analyzing fiscal policy (see, e.g., Domeij and Heathcote [2004], Heathcote, 2005), assessing how the shape of the Laffer curve is modified in such an environment is a priori worthwhile. This is the main goal of the present paper.

There are at least two reasons why allowing for an IM setup can affect the shape of the Laffer curve. First, because of market incompleteness and individual risk, agents accumulate more assets (Aiyagari, 1994) and supply more labor (Pijoan-Mas, 2006) compared to a CM setup. Indeed, these are the only ways agents can self-insure to smooth consumption. This tends to lower the elasticity of aggregate labor and capital supplies and, thus, can affect the shape of the Laffer curve, especially when it comes to labor and capital taxes. Second, in a CM economy calibrated along the lines of (Trabandt and Uhlig, 2009), depending on the variable adjusted to make the government budget constraint hold, one can easily get debt-output ratios exceeding 1000% or transfer-output ratios exceeding 40%, for sufficiently high tax rates. While such wild variations are innocuous in a CM setup, it is worth exploring their consequences in an IM setup. As suggested above, Ricardian equivalence fails in such an economy. As a consequence, the real interest rate is no longer invariant to the particular mode used to balance the government budget, i.e. either by adjusting lump-sum transfers or public debt. In particular, the effects of changing public debt on the shape of the Laffer curve are no longer trivial since the real interest will vary positively with the level of debt. Indeed, in such a model, as Aiyagari and McGrattan (1998) showed, public debt crowds out physical capital in households portfolio, thus exerting an upward pressure on the equilibrium interest rate. Thus, in addition to the usual general equilibrium effects emphasized by Trabandt and Uhlig (2009), fiscal revenues will now depend on whether the government saves and supplies capital to the production sector or issues new bonds. To see this more precisely, let us assume that the tax rates on capital and consumption are zero, so that
all fiscal revenues derive from labor income taxation. These revenues are used to finance government expenditures (assumed to be constant), lump-sum transfers and debt repayments. Debt repayments are defined as the product of the real interest rate (adjusted for growth) and the level of public debt. If transfers are adjusted to make the government budget constraint hold, these adjustments will monotonically affect tax revenues for any non-zero level of debt through their (positive) aggregate effect on real interest rate. Indeed, everything else constant, an increase in transfers reduces the need to self-insure. Instead, if debt is adjusted, tax revenues will not change monotonically with debt. This is because any change in tax revenues is a combination of changes in the real interest rate, that increase with debt (due to a crowding-out effect of debt on private capital accumulation) and adjustments in the debt itself. When debt is positive, the change in tax revenues is an increasing function of debt, but for negative values of debt this change is negative. This effect is completely absent in the CM version of the model since neutrality implies that the stationary real interest rate is invariant to the level of debt or lump-sum transfers. Knowing how these two channels affect the shape of the Laffer curve is actually a quantitative issue.

To address this question, we formulate a neoclassical growth model with liquidity-constrained agents and incomplete financial markets along the lines of Aiyagari and McGrattan (1998) (see also Flodén 2001, Röhrs and Winter 2010). In our economy, households are subject to persistent idiosyncratic productivity shocks and face a borrowing constraint. As in Trabandt and Uhlig (2009), the model includes distortionary taxes on labor, capital, and consumption. These taxes are used to finance government consumption, lump-sum transfers, and interest repayments on previous debt. A nice feature of our setup is that it nests the standard neoclassical model. Setting the variance of idiosyncratic labor productivity shocks to zero and eliminating the borrowing constraint, our model replicates exactly the steady-state results of the standard neoclassical model with distortionary taxation, as retained by Trabandt and Uhlig (2009). This makes easier a quantitative comparison of the two model versions. The model is calibrated to the US economy to mimic great ratios as well as moments related to the wealth distribution. We then investigate how the Laffer curve changes shape in our IM setup. For each of the three tax rates considered, we compare our findings to those deriving from the CM version. For each tax rate, we consider alternatively the case where lump-sum transfers are adjusted to make the government budget constraint hold and the case where public debt is adjusted instead.

Our main findings are the following. When it comes to labor income taxes, the CM and IM models deliver similar Laffer curves when transfers are adjusted. The slippery slope is a little bit farther to the right in the IM model. This results from households using labor supply to self-insure, allowing for a greater labor taxation. Conversely, when debt is adjusted, the shape of the Laffer curve is dramatically affected: the Laffer curve now looks like an horizontal $S$. The result can be explained as follows. First, for a positive debt, the slippery curve moves to the right as in the case of lump-sum

1Obviously, in the CM setup, this distinction does not matter.
transfers adjustments. Second, for a negative public debt, government revenues increase with debt. As noticed previously, the labor tax rate changes non monotonically with debt in the IM setup. This implies that a given tax rate is compatible with either two debt levels or none (see Aiyagari and McGrattan, 1998 for a similar result). In terms of the Laffer curve, this implies that there can exist three tax rates compatible with the same level of fiscal revenues. When we consider capital taxes, the incomplete nature of financial markets deeply affects the results. First, when transfers are adjusted, the slippery slope of the Laffer curve in the IM setup moves to the right compared to the CM case. This is a consequence of the precautionary saving behavior: agents self insure against adverse productivity shocks by accumulating more assets, implying a less elastic saving behavior. Again, when debt is adjusted, the shape of the Laffer curve is deeply modified and the same level of fiscal revenues can be now compatible with three different capital income taxes. Finally, when consumption tax is considered, the CM and IM models exhibit broadly similar shapes when transfers are adjusted, each of them displaying no peak. Once again, when debt is adjusted instead, there exist two consumption taxes delivering the same level of fiscal revenues. This result comes again from the non-monotonic response of the consumption tax rate to changes in public debt in the IM setup.

This paper is related to the previous works of Aiyagari and McGrattan (1998), Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) that investigate taxation in an IM setup and compare them to the standard neoclassical growth model. Aiyagari and McGrattan (1998) have already obtained that a proportional income tax rate changes in a non-monotonic way with debt, but they did not explore their consequence for the shape of the Laffer curve. In addition, they did not investigate different types of distortionary taxes. Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) revisited in an IM setup the results of Prescott (2004), who raised the incentive issues of labor taxes. Ljungqvist and Sargent (2008) and Alonso-Ortiz and Rogerson (2010) compared the Laffer curves for the two model versions. They obtain that the slippery slope of the Laffer curve is weakly affected, but their modeling does not include a government debt at the stationary equilibrium and they only consider labor income taxes. Our paper complements these works by insisting more on the role of public debt and by considering various forms of distortionary taxes.

The rest of the paper is organized as follows. In section 2 we expound the IM model and define the stationary steady state under study. Section 3 is devoted to the quantitative results. We first discuss our calibration strategy and then explore the extent to which the Laffer curves change when computed in an IM model. The last section briefly concludes.
2. The Model

In this section, we describe the model economy used in our quantitative experiment. This model is basically a version of the one considered by Aiyagari and McGrattan (1998).

2.1. Environment

We consider a discrete time, deterministic economy, with time indexed by $t \in \mathbb{N}$. The final good $Y_t$, which we take as the numeraire, is produced by competitive firms, according to the Cobb-Douglas technology

$$Y_t = K_t^\theta (Z_t N_t)^{1-\theta},$$

where $\theta$ denotes the elasticity of production with respect to capital, $K_t$ and $N_t$ are the inputs of physical capital and efficient labor, respectively, and $Z_t$ is an exogenous technical progress index, evolving according to $Z_{t+1} = (1 + \gamma)Z_t$ with $Z_0 = 1$, $\gamma > 0$. Firms rent capital and efficient labor on competitive markets, at rates $r_t + \delta$ and $w_t$, respectively, where $\delta \in [0, 1]$ is the depreciation rate of physical capital, $w_t$ is the wage rate, and $r_t$ is the interest rate.

The economy is inhabited by a continuum of agents, of measure one. Each agent’s time endowment is normalized to 1 and can be allocated to market work $h_t$ or to leisure $1 - h_t$. Agents have preferences over consumption $c_t$ and leisure defined by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \right\}$$

with $c_t \geq 0$ and $0 \leq h_t \leq 1$. Here $\beta \in (0, 1)$ is the subjective discount factor, $E_0 \{ \cdot \}$ is the mathematical expectation conditioned on the initial individual state at date $t = 0$, and $u(c, 1 - h)$ is a well-behaved utility function, assumed to be homogeneous of degree $1 - \sigma_c$ in $c$.

Each period, households receive an uninsurable shock $s_t > 0$ to their labor productivity. These shocks are assumed to be i.i.d. across agents and evolve over time according to a Markov process, with bounded support $S$ and stationary transition function $Q(s, s^\prime)$. These idiosyncratic productivity shocks are normalized so that the unconditional mean of their logarithm is equal to zero, i.e. $E\{\log(s)\} = 0$. An individual agent’s efficient labor is thus $s_t h_t$, with corresponding labor earnings given by $(1 - \tau_N)w_t s_t h_t$, where $\tau_N$ denotes the labor income tax. Also, agents self-insure by accumulating $a_t$ units of assets which pay the after-tax rate of return $(1 - \tau_A) r_t$, where $\tau_A$ denotes the capital income tax. These assets can consist of units of physical capital and/or government bonds. Once

\[3\text{The transition } Q \text{ has the following interpretation: for all } s \in S \text{ and for all } S_0 \in \mathcal{S}, \text{ where } \mathcal{S} \text{ denotes the Borel subsets of } S, Q(s, S_0) \text{ is the probability that next period’s individual productivity lies in } S_0 \text{ when current productivity is } s.\]
arbitrage opportunities have been ruled out, each asset has the same rate of return. Also, agents must pay a sales tax $\tau_C$. Finally, they perceive transfers $T_t$. Thus, an agent’s budget constraint is

$$(1 + \tau_C)c_t + a_{t+1} \leq (1 - \tau_N)w_t h_t s_t + (1 - \tau_A)r_t a_t + T_t.$$ 

Borrowing is exogenously restricted by an “ad hoc” constraint

$$a_{t+1} \geq 0.$$ 

There is finally a government in the economy. The government issues debt $B_{t+1}$, collects tax revenues, rebates transfers, and consumes $G_t$ units of final good. The associated budget constraint is given by

$$B_{t+1} = (1 + r_t)B_t + T_t + G_t - (\tau_A r_t A_t + \tau_N w_t N_t + \tau_C C_t)$$

where $C_t$ and $A_t$ denote aggregate (per capita) consumption and assets held by the agents, respectively.

### 2.2. Equilibrium Defined

In the remainder of this paper, we focus exclusively on the steady state of an appropriately detrended version of the above economy. Growing variables are detrended by dividing them by $Y_t$. Detrended variables are referred to with a hat. In such an economy, detrended government expenditures $\hat{G}$ and transfers $\hat{T}$ are constant.

We let the joint distribution of assets $\hat{a}$ and individual productivities $s$ across agents be denoted $x(\hat{a}, s)$. Thus, for all $A_0 \times S_0 \in \mathcal{A} \times \mathcal{S}$, $x(A_0, S_0)$ is the mass of agents with assets in $A_0$ and idiosyncratic productivity in $S_0$, where $\mathcal{A} \times \mathcal{S}$ denotes the Borel subsets of $A \times S$.

We can now write an agent’s problem in recursive form

$$v(\hat{a}, s) = \max_{\hat{c}, h, \hat{a}'} \left\{ u(\hat{c}, 1 - h) + \beta \int_{S} v(\hat{a}', s')Q(s, ds') \right\}$$

s.t. \quad $$(1 + \tau_C)\hat{c} + (1 + \gamma)\hat{a}' \leq (1 - \tau_N)\hat{w}sh + (1 + (1 - \tau_A)r)\hat{a} + \hat{T},$$

$\hat{a}' \geq 0, \quad \hat{c} \geq 0, \quad 0 \leq h \leq 1,$

where $\beta \equiv \tilde{\beta}(1 + \gamma)^{1-\sigma_c}$ denotes the growth-adjusted discount factor.

For convenience, we restrict $\hat{a}$ to belong to the compact set $A = [0, \hat{a}_M]$, where $\hat{a}_M$ is a large number. We can thus define a stationary, recursive equilibrium in the following way.

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4In an alternative version of this setup, we consider an economy in which government expenditures and transfers are constant in level. This does not substantially change our quantitative results.

5$\hat{a}_M$ is selected so that the decision rule on assets for an individual with the highest individual productivity crosses the first bisectrice below $\hat{a}_M$. 

Definition 1—Given a vector of constant policy parameters \((\tau_C, \tau_A, \tau_N, \hat{T}, \hat{G}, \hat{B})\), a steady-state, recursive competitive equilibrium is a constant system of prices \(\{r, \hat{w}\}\), a value function \(v(\hat{a}, s)\), time-invariant decision rules for an individual’s assets holdings, consumption, and labor supply \(\{g_a(\hat{a}, s), g_c(\hat{a}, s), g_h(\hat{a}, s)\}\), a measure \(x(\hat{a}, s)\) of agents over the state space \(A \times S\), and aggregate quantities \(\hat{A} = \int \hat{a} dx, \hat{C} = \int g_c(\hat{a}, s) dx, N = \int s g_h(\hat{a}, s) dx, \) and \(\hat{K}\) such that:

1. The value function \(v(\hat{a}, s)\) solves the agent’s problem stated in eq. (1), with associated decision rules \(g_a(\hat{a}, s), g_c(\hat{a}, s), \) and \(g_h(\hat{a}, s)\);
2. Firms maximize profits and factor markets clear, so that
   \[
   \hat{w} = \frac{1 - \theta}{N}, \quad r + \delta = \frac{\theta}{\hat{K}},
   \]
3. Tax revenues equal government expenses
   \[
   \tau_N \hat{w} N + \tau_A r \hat{A} + \tau_C \hat{C} = \hat{T} + \hat{G} + (r - \gamma) \hat{B};
   \]
4. Aggregate savings equal firm’s demand for capital plus Government’s debt
   \[
   \hat{A} = \hat{K} + \hat{B};
   \]
5. The distribution of agents \(x\) is invariant
   \[
   x(A_0, S_0) = \int_{A_0 \times S_0} \left\{ \int_{A \times S} 1\{\hat{a}' = g_a(\hat{a}, s, x)\} Q(s, s') dx \right\} da' ds',
   \]
   for all \(A_0 \times S_0 \in \mathcal{A} \times \mathcal{S}\), where \(1\{\cdot\}\) is an indicator function taking value one if the statement is true and zero otherwise.

With a slight abuse of notation, we define the stationary level of output \(\hat{Y} = Y_t/Z_t\). It is linked to \(\hat{K}\) and \(N\) through \(\hat{Y} = \hat{K}^{\theta/(1-\theta)} N\).

From the government budget constraint, fiscal revenues (as a share of GDP) are then given by
\[
\hat{R} = \tau_N \hat{w} N + \tau_A r \hat{K} + \tau_C \hat{C}.
\]
\(\hat{R}\) is then converted to level according to \(R = \hat{R} \times \hat{Y}\). Notice that \(R\) is defined net of fiscal receipts from taxing public bond return.

In the remainder, we consider three Laffer curves, each relating \(R\) to one of the three tax rates \((\tau_N, \tau_A, \tau_C)\) considered here, holding the other two taxes constant. As argued by Trabandt and Uhlig (2009), this is the appropriate definition of the Laffer curve since it correctly takes into account the general equilibrium effects induced by a tax change. For example, a given change in \(\tau_N\) will modify \(x, g_a, g_h, \) and \(g_c,\) so that it will also impact on all the fiscal bases.

In equilibrium, we must always have
\[
\hat{R} = \hat{G} + \hat{T} + [(1 - \tau_A) r - \gamma] \hat{B},
\]
so that a given change in one of the three tax rates is associated with a corresponding adjustment in either $\hat{T}$ or $\hat{B}$. Depending on which variable is adjusted, we consider two sub-cases for each possible Laffer curve.

3. Quantitative Results

In this section, we calibrate the IM model in order to analyze quantitatively its predictions relative to the Laffer curves discussed above.

3.1. Calibration and solution method

A period is taken to be a year. For the most part, we follow the calibration strategy adopted by Trabandt and Uhlig (2009). The momentary utility function is

$$u(c, 1-h) = \eta \log(c) + (1-\eta) \log(1-h),$$

as is standard in the literature. This amounts to imposing $\sigma_c = 1$. Preferences are then described by two parameters, $\eta$ and $\beta$. We pin down $\eta$ so that aggregate hours worked $H \equiv \int g_h(a, s) \, dx$ equal 0.25. The subjective discount factor $\beta$ is set so that the after tax interest rate is equal to 4%.

The fiscal parameters $\hat{B}$ and $\hat{G}$ are set to match the debt-output ratio and the public expenditure-output ratio reported by Trabandt and Uhlig (2009), i.e. $\hat{B} = 0.63$ and $\hat{G} = 0.18$. The tax rates are calibrated to match estimates of effective tax rates computed using the methodology developed by Mendoza, Razin, and Tesar (1994). This yields $\tau_N = 0.28$, $\tau_A = 0.38$, and $\tau_C = 0.05$. Using these parameters, the transfer-output ratio $\hat{T}$ is endogenously computed so as to enforce the government budget constraint.

To calibrate the stochastic process $\{s_t\}$, we follow Heathcote (2005) and Domeij and Heathcote (2004). We assume that $\{s_t\}$, evolves over time according to a three-state Markov chain, with support $S = \{\bar{s}_1, \bar{s}_2, \bar{s}_3\}$ and transition matrix $Q$, where the element $Q_{ij}$ denotes the probability of reaching state $j$ from state $i$. We impose the following structure on $Q$

$$Q = \begin{pmatrix}
Q_{11} & 1 - Q_{11} & 0 \\
(1 - Q_{22})/2 & Q_{22} & (1 - Q_{22})/2 \\
0 & 1 - Q_{11} & Q_{11}
\end{pmatrix}.$$

Finally, as discussed in the previous section, we further impose the restriction $E\{\log(s_t)\} = 0$. Given the above restrictions, this leaves four free parameters to be calibrated: $\bar{s}_1$, $\bar{s}_2$, $Q_{11}$, and $Q_{22}$. We pin down their values by matching four calibration targets: the Gini coefficient of wealth distribution, the share of wealth held by the 40% poorest, $\rho(\log(s_t))$ the autocorrelation of $\log(s_t)$, and $\sigma(\log(s_t))^2$ the variance of $\log(s_t)$. The first two calibration targets are taken from Díaz-Giménez, Glover, and Ríos-Rull (2011). In particular, they report that the Gini index is equal to 0.816 and the share of aggregate wealth held by the 40% poorest amounts to 1.1%. The last two correspond to the values
Table I. Calibration Summary

<table>
<thead>
<tr>
<th>Pre­fer­ences</th>
<th>In­com­plete Mar­kets</th>
<th>Com­plete Mar­kets</th>
<th>Cal­i­bra­tion Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>η</td>
<td>0.3391</td>
<td>0.3057</td>
<td>H = 0.25</td>
</tr>
<tr>
<td>β</td>
<td>0.9683</td>
<td>0.9808</td>
<td>(1 − τA)r = 0.04</td>
</tr>
</tbody>
</table>

| Tech­no­logy  | θ                     | 0.3800             | Trabandt and Uhlig (2009) |
|               | δ                     | 0.0700             | Trabandt and Uhlig (2009) |

| Shocks        | s1                    | 0.2023             | Wealth held by 40% poorest |
|               | s2                    | 1.0184             | Gini wealth               |
|               | Q11                   | 0.9001             | ρ(log(s))                 |
|               | Q22                   | 0.9862             | σ(log(s))                 |

| Fiscal Block  | τN                    | 0.2800             | Trabandt and Uhlig (2009) |
|               | τA                    | 0.3600             | Trabandt and Uhlig (2009) |
|               | τC                    | 0.0500             | Trabandt and Uhlig (2009) |
|               | B                     | 0.6300             | Trabandt and Uhlig (2009) |
|               | G                     | 0.1800             | Trabandt and Uhlig (2009) |

*ρ(log(s)) stands for the first-order degree of serial correlation log(s1).

†σ(log(s)) stands for the standard error of log(s1).

reported by [Heathcote (2005)] and [Domeij and Heathcote (2004)]. In particular, we seek to match ρ(log(s1)) = 0.9 and σ(log(s1))² = 0.05/(1 − 0.92). The calibration is summarized in Table I.

For comparison purposes, we also consider a version of the previous model in which (i) we impose idiosyncratic labor income shocks st set to their average value and (ii) we relax the borrowing constraint. The model is recalibrated along the lines of Trabandt and Uhlig (2009), as described above. Importantly, the parameters η and β must be recalibrated to match the same calibration targets as those imposed in the IM economy. Notice that in this CM environment, the distinction between effective H and efficient N labor is no longer useful since both quantities coincide. We thus incorporate a productivity scale factor Ω in front of Nt in the production function to compensate the CM economy for the average labor productivity effect present in the IM economy (i.e. the difference between N and H). Doing so, we make sure that in the benchmark calibration described above, all economies share the same interest rate, the same effective labor H, and the same stationary production level Ŷ. Clearly, we have Ŷ = \( \hat{K}^{\theta/(1-\theta)}\Omega N \).

See appendix A for details on the solution.
The solution method adopted in the IM setup is now briefly described. After having postulated candidate values for the interest rate $r$ and the aggregate efficient labor $N$, we solve the government budget constraint for the transfer-output ratio $\hat{T}$. To do so, we use the representative firm’s first order conditions, which give us values for $\hat{K}$ and $\hat{w}$, and the aggregate resource constraint, from which we back out $\hat{C}$. Given these, we solve the agents problem using the endogenous grid method proposed by Carroll (2006), adapted to deal with endogenous labor supply, in the spirit of Barillas and Fernandez-Villaverde (2007). Using the implied decision rules, we then solve for the stationary distribution and use it to compute aggregate quantities. We then iterate on $r$ and $N$ and start the whole process all over again until the markets for capital and labor clear. For a given $N$, the interest rate is updated via a hybrid bisection-secant method. Once the clearing-market $r$ is found, $N$ is updated with a standard Newton scheme.

3.2. Preliminary Results in a Complete-Markets Economy

Here, we establish the claim made in introduction that classic Laffer curve calculations in a one-sector, CM neoclassical growth model can yield wild adjustments in the debt-output or the transfer-output ratios.

\footnote{See appendix B for further details.}

\footnote{In doing so, we exploit the special structure of the first order condition on $h$ induced by the specific functional form adopted for $u$.}
For the sake of parsimony, we restrict our attention to the labor income tax Laffer curve. Similar conclusions hold for the other two taxes. The Laffer curve is then the relation between the labor income tax $\tau_N$ and fiscal revenues $R$.

Figure 1 reports the simulation results. Panel (A) contains the Laffer curve on labor income taxation. This is more or less equivalent to what Trabandt and Uhlig (2009) obtain in the Cobb-Douglas utility case. Panel (B) reports the implied adjustment in the debt-output ratio, freezing transfers to their benchmark value. Similarly, panel (C) reports the implied adjustment in the transfer-output ratio, freezing public debt to its benchmark value. Recall that in the Ricardian setup considered in this exercise, both adjustments yield the exact same Laffer curve.

As panel (B) makes it clear, adjusting debt to make sure that the government budget constraint holds can imply wild debt-output ratios, approaching 1000% for sufficiently high labor income taxes or $-1000\%$ for sufficiently low taxes. Similarly, panel (C) makes clear that the steady-state transfer-output ratio also experiences large variations as the labor income tax varies. While in a Ricardian world such adjustments are irrelevant, they have potentially important consequences in a non-Ricardian setup. We next investigate this issue in our quantitative model.

3.3. Laffer Curves on Labor Income Taxes

Figure 2 reports three Laffer curves associated with variations in $\tau_N$, defined in the exact same way as before. The gray one corresponds to the CM economy, as above. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are adjusted to make the government budget constraint hold. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is adjusted instead.

When transfers are adjusted, the Laffer curve is computed in the following way. For each $\tau_N$ over a grid, we re-compute the economy’s equilibrium using the algorithm described above. This does not raise special difficulties. In contrast, when the debt-output ratio is adjusted, we can no longer wander over a pre-specified grid for $\tau_N$. As will become clear below, this approach would fail because $r\hat{B}$ can change sign and our algorithm is not well suited to deal with this. Instead, we impose a grid for the debt output ratio and back out the tax rate $\tau_N$ ensuring the government’s budget is balanced along a steady-state path.

In the case when transfers are adjusted, the Laffer curve associated with $\tau_N$ has the standard inverted-U shape. It clearly resembles the curve that would obtain in the CM economy, as shown in figure 2. The key difference appears in the high $\tau_N$ region of the graph. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of labor supply in the IM economy. As argued by Pijoan-Mas (2006), in this kind of economy, agents tend to supply more labor as part of their desire to self-insure. Put another way, the aggregate labor elasticity to taxation is lower in the IM economy than in the CM economy.
This translates into a tax rate maximizing revenues equal to $\tau_N^* = 0.5135$ in the IM economy and $\tau_N^* = 0.4864$ in the CM economy. This allows the government to raise 15.58% more revenues than in the benchmark calibration in the IM economy and only 13.78% in the CM economy. To sum up, when transfers are adjusted to make the steady-state government budget constraint hold, resorting to a CM model or to an IM model to characterize the shape and peak of the labor income tax Laffer curve can have important consequences. The maximum tax rate is higher by 2.7% in the IM economy. In turn, this implies that the government can permanently raise revenues higher by 1.8%.

This is further illustrated in figure 3, which reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate $((1 - \tau_A)r)$, effective hours ($H$), physical capital ($\hat{K}\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_N$ is varied (see the dark and gray, plain lines). As is clear from this picture, except for $r$, the variables considered look very similar in the IM and CM cases. This is particularly striking when it comes to the transfer-output ratio. Indeed, when transfers are adjusted, there is no discernible difference between the curves obtained in the IM and CM setups. This is reminiscent of the quasi-aggregation result obtained in Krusell and Smith (1998). More recently, the debate between Prescott (2004) and Ljungqvist and Sargent (2008) came to a similar conclusion: using an IM or a CM model does not change much the aggregate conclusion one draws from labor income tax experiments. Nevertheless, for a given tax rate, effective labor turns out to be higher in the IM economy than in the CM economy. Conversely, physical capital in the IM setup is slightly below the CM level for high tax rates. However, the difference is very modest. The reason why is that
transfers help people to self-insure, thus reducing the need to accumulate assets. As a consequence, the after tax interest rate \((1 - \tau_A) r\) rises with \(\tau_N\).

Finally, we report on figure 3 the decomposition by fiscal bases of the Laffer curve associated with \(\tau_N\). The figure makes clear that the similitude in shape of the curves in the IM and CM economies does not result from a composition effect. In both economies, the Laffer curve shape is dominated by the response of the labor income tax basis, as in Trabandt and Uhlig (2009).

In contrast, when the debt-output ratio \(\hat{B}\) is adjusted, we reach very different conclusions (see the dark, dashed curve in figure 2). Under this assumption, the Laffer curve looks like an S oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which continuously reaches the usual pattern as labor tax income taxes decrease. This junction takes place in what appears to be a minimum tax level which is close to 0.25. Interestingly, the minimum labor income tax obtains for a debt-output ratio close to \(-110\%\). Above this level, there can be one, two, or three tax rates associated with a given level of fiscal revenues. Also, in the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue maximizing labor income tax is \(\tau_N^\star = 0.5557\), allowing the government to raise 17.92\% more revenues than in the benchmark situation.

What explains the awkward shape of the Laffer curve in the left part of figure 2 when the debt-output ratio is adjusted? To gain an insight, imagine a simplified setting in which \(\tau_C = \tau_A = 0\), so
that, expressed as a share of GDP, fiscal revenues are \( \hat{R} = \tau_N (1 - \theta) \). The steady-state government budget constraint now writes

\[
\hat{R} = \hat{G} + \hat{T} + (r - \gamma) \hat{B}.
\]

Assuming differentiability with respect to \( \hat{B} \), one gets

\[
\frac{\partial \hat{R}}{\partial \hat{B}} = (r - \gamma) + \hat{B} \frac{\partial r}{\partial \hat{B}}.
\]

Now, since in this non-Ricardian economy, public debt crowds out capital in the household portfolio, we expect \( \partial r / \partial \hat{B} > 0 \). Indeed, as shown by Aiyagari and McGrattan (1998), when \( \hat{B} \) is large, \( \hat{K} \) gets smaller, which makes the equilibrium interest rate \( r \) increase. Conversely, when \( \hat{B} \) is negative and large in absolute value, private wealth \( \hat{A} \) shrinks and the aggregate level of capital \( \hat{K} \) increases, which makes the equilibrium interest rate decrease.

Thus the term \( \hat{B} \partial r / \partial \hat{B} \) changes sign when \( \hat{B} \) changes sign. For a sufficiently negative debt-output ratio, we can thus observe a change in the sign of \( \partial \hat{R} / \partial \hat{B} \) and, since \( \hat{R} = \tau_N (1 - \theta) \), correspondingly, a change in the sign of \( \partial \tau_N / \partial \hat{B} \). Clearly, this would be impossible in a CM economy since, there, \( \partial r / \partial \hat{B} = 0 \). By construction, this cannot happen either in the IM economy in which \( \hat{B} \) is constant and \( \hat{T} \) is adjusted.

In the general case, when \( \tau_C \) and \( \tau_A \) are non-zero, the above reasoning still holds but must also take into account the response of \( \hat{K} \) and \( \hat{C} \). These endogenous responses combine together to define the particular point at which fiscal revenues exhibit the awkward shape identified above. This also defines the minimal labor income tax. These responses are reported in figure 3.
As $\hat{B}$ gets smaller and smaller, the labor income tax $\tau_N$ gets higher and higher. This corresponds to the awkward part of the Laffer curve. In this part of the graph, physical capital $\hat{K}$ increases sharply (and concomitantly, private assets $\hat{A}$ decline). This leads to a marked decline in the real interest rate $r$. Since $\hat{K}$ increases, the level of output also increases, though at a slower pace. This is due to the fact that aggregate hours rise at a slower pace too and even decline for high enough a labor income tax. In the regular part of the Laffer curve, as $\hat{B}$ gets higher and higher, so too does the labor income tax. The increase in debt leads to a rise in private assets but to a decline in physical capital. Since the labor income tax increases, aggregate labor declines. In the end, output also declines.

Is there a limit to what the government can do by accumulating bigger and bigger assets? To answer this, imagine a limit situation in which $\hat{B}$ is so negative that $\hat{A}$ has been driven to zero. In this case, private agents no longer hold any assets and simply face static consumption and labor choices. Their income derives from wages and transfers only. From such a static program, one can easily establish that an individual labor supply $h$ is a decreasing function of $\tau_N$. Also, there is a tax rate above which agents no longer supply any labor, in which case, there is no production and the economy ceases to exist. Now, from the government budget constraint, one can see that $\tau_N$ decreases with $\hat{B}$ in this particular situation. Thus, as $\hat{B}$ gets smaller and smaller (i.e. as the government accumulates more and more physical capital), the labor income tax gets higher and higher. The limit to what the government can do, of course, is the particular $\hat{B}$ which leads to a $\tau_N$ so high that agents are no longer willing to supply any labor.

### 3.4. Laffer Curves on Capital Income Taxes

Figure 5 reports three Laffer curves associated with variations in $\tau_A$. As before, the gray one corresponds to the CM economy, as above. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are adjusted to make the government budget constraint hold. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is adjusted instead.

In the case when transfers are adjusted, the Laffer curve associated with $\tau_A$ has the standard inverted-U shape. It has the overall same shape as the curve that would obtain in the CM economy, as shown in figure 5. Once again, the key difference appears in the high $\tau_A$ region of the graph. Here, a given tax rate generates relatively more fiscal revenues than in the CM setup. This is clearly due to the relative inelasticity of agents saving behavior in the IM economy. As in [Aiyagari (1994)], in this kind of economy where contingent assets are ruled out, agents self-insure by accumulating relatively more assets (physical capital or public debt) than in a CM setup. This translates into a tax rate maximizing revenues equal to $\tau_A^* = 0.5773$ in the IM economy and $\tau_N^* = 0.5301$ in the CM economy. This allows the government to raise 3.11% more revenues than in the benchmark calibration in the
IM economy and only 2.0% in the CM economy. Notice that the difference is less marked than with $\tau_N$.

This is further illustrated in figure 6, which reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate ($(1 - \tau_A)r$), effective hours ($H$), physical capital ($\hat{K}\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_A$ is varied (see the dark and gray, plain lines). As is clear from this picture, except for $r$, the variables considered look similar in the IM and CM cases. For high tax rates, effective labor and physical capital turn out to be higher in the IM economy than in the CM economy. Notice that in this experiment, the after tax interest rate declines, in spite of an increase in transfers. This is because the rise in $\tau_A$ more than compensate the rise in $r$ consecutive to a decline in $\hat{K}$.

As in the previous section, when the debt-output ratio $\hat{B}$ is adjusted, we reach very different conclusions (see the dark, dashed curve in figure 5). Under this assumption too, the Laffer curve looks like an S oriented horizontally. In the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as capital income taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 0.25. Interestingly, the minimum capital income tax obtains for a debt-output ratio close to $-31.86\%$. Above this level, there can be one, two, or three tax rates associated with a given level of
fiscal revenues. Also, in the regular part of this Laffer curve (i.e. the part that is indeed inverted-U-shaped), the revenue maximizing capital income tax is $\tau^*_A = 0.5375$, allowing the government to raise 1.40% more revenues than in the benchmark situation.

Figure 6 helps understand what is happening in this case. When the debt-output ratio is increased (high $\tau_A$ region), physical capital is crowded-out by public debt, just as in the previous section. This implies an increase in the real interest rate, despite the increase in $\tau_A$. Thus, fiscal receipts from capital income taxation increase. At the same time, capital income taxation discourages individual labor supply through two channels. First, since the stock of capital declines, so too does the real wage. Second, the increase in public debt hampers softens the liquidity constraint and thus mitigates the need to self-insure through saving and working longer hours. Yet, because transfers (as a share of GDP) are held constant, agents work relatively harder than in the CM economy. As a consequence, fiscal receipts from labor income taxation decline. For sufficiently high tax rates on capital income, this decline more than compensates the rise in fiscal receipts from capital taxation.

In contrast, when the debt-output ratio is negative, the economy experiences a large inflow of physical capital. The real interest rate declines as capital increases (i.e. as the government accumulates more and more assets). Private wealth also shrinks, which forces private agents to work more. As a consequence, the capital income basis decreases while labor income basis increases. The combination of these effects implies that fiscal revenues increase when public debt is more and more negative. In
addition, for moderately negative debt-output ratio, the government rents physical capital to firms and rental revenues are sufficiently high that $\tau_A$ can decrease. When the government holds too many assets, rental revenues are no longer sufficient to cover transfers and final good expenditures. At this stage, the government increases $\tau_A$ again.

3.5. Laffer Curves on Consumption Taxes

Figure 7 reports three Laffer curves associated with variations in $\tau_C$, defined in the exact same way as before. As usual, the gray one corresponds to the CM economy, as above. The dark, plain line is the Laffer curve associated with the IM economy, when transfers $\hat{T}$ are adjusted to make the government budget constraint hold. Finally, the dark, dashed line is the Laffer curve in the IM economy, when the debt-output ratio $\hat{B}$ is adjusted instead.

As in Trabandt and Uhlig (2009), the Laffer curve associated with $\tau_C$ does not exhibit a peak, either in the CM setup or in the IM setup with adjusted transfers. In the latter, fiscal revenues are slightly higher than in the former. Figure 8 reports changes in steady-state output ($\hat{Y}$), (after tax) interest rate ($(1 - \tau_A)r$), effective hours ($H$), physical capital ($K\hat{Y}$), the debt-output ratio ($\hat{B}$), and the transfer-output ratio ($\hat{T}$) when $\tau_C$ is varied (see the dark and gray, plain lines). As is clear, the difference between the CM and IM economies is rather mild, even when it comes to the real interest rate. In both economies, as expected, output, hours, and capital decline when $\tau_C$ rises. However, for all $\tau_C$ considered, hours, capital, and output are slightly higher in the IM case than in the CM.
economy. Fundamentally, in both settings, taxing consumption is like taxing labor (both taxes show up similarly in the first order condition governing labor supply). A difference, though, is that in an IM economy such as ours, agents with a low labor productivity choose not to work whenever they hold enough assets. Clearly, those agents would not suffer from labor income taxation but do suffer from consumption taxes. Combined with the relative inelasticity of labor supply in the IM setup, this explains why the government can raise more revenues in this framework than in the CM setup.

As in the previous sections, when the debt-output ratio $\hat{B}$ is adjusted, we reach different conclusions (see the dark, dashed curve in figure 7). Under this assumption too, in the left part of the graph, for relatively low tax levels, the Laffer curve has an increasing branch which reaches the regular pattern as consumption taxes decrease. Once again, this junction takes place in what appears to be a minimum tax level which is close to 0.02, associated with a debt-output ratio close to $-63.09\%$. Above this level, there can be two tax rates associated with a given level of fiscal revenues.

4. Conclusion

In this paper, we inspect how allowing for liquidity-constrained agents and incomplete financial markets impacts on the shape of the Laffer curve. In doing so, we paid particular attention which of debt or transfers is adjusted to make the government budget constraint hold as taxes are varied.
While in a Ricardian framework, this does not matter, opting to adjust debt rather than transfers can potentially make a big difference in a non-Ricardian setup.

To address this question, we then formulate a neoclassical growth model with liquidity-constrained agents and incomplete financial markets along the lines of Aiyagari and McGrattan (1998). The model is then calibrated to the US economy to mimic great ratios as well as moments related to the wealth distribution. We then investigate how the Laffer curve changes shape.

Our main findings are the followings. When it comes to labor and capital income taxes, the benchmark and IM models deliver similar Laffer curves when transfers are adjusted. The slippery slope is a little bit farther to the right in the IM model. This results from households using labor supply and savings to self-insure, allowing for a greater levels of taxation. However, when debt is adjusted, the shape of the Laffer curve is dramatically affected: the Laffer curve now looks like an horizontal $S$. First, for a positive debt, the slippery curve moves to the right as in the case of lump-sum transfers adjustments. Second, for a negative public debt, government revenues increase with debt. This implies that there can exist three tax rates compatible with the same level of fiscal revenues. Finally, when consumption tax are considered, the CM and IM models exhibit broadly similar shapes when transfers are adjusted, each of them displaying no peak. Once again, when debt is adjusted instead, there exist two consumption taxes delivering the same level of fiscal revenues. This result comes again from the non-monotonic response of the consumption tax rate to changes in public debt in the IM setup.
Appendix A. The Complete-Markets Economy

The complete-markets economy is the representative-agent version of the IM framework, where we have eliminated idiosyncratic shocks. Once growing variables have been detrended by $Y_t$, the associated steady state is then solution to the system

$$\dot{C} + (\gamma + \delta) \dot{K} + \dot{G} = 1$$

$$(1 + \gamma) = (1 + (1 - \tau_A)r)\beta,$$

$$\dot{C} = \frac{\eta}{1 - \eta} \frac{1 - \tau_N}{1 + \tau_C} \dot{w}(1 - H)$$

$$\dot{w} = (1 - \theta)/H,$$

$$r + \delta = \theta/\dot{K}.$$

The first equation is the resource constraint. The second equation is the steady-state Euler equation on capital. The third equation is the first-order condition on labor. The last two equations are the representative firm’s first order conditions.

We recursively solve for the steady-state values in the standard way. First, notice that by combining the Euler equation on capital with the condition on optimal use of capital by firms, one gets

$$\dot{K} = \frac{\beta \theta (1 - \tau_A)}{1 + \gamma - \beta [1 - (1 - \tau_A)\delta]}.$$

Then, assuming the same $\dot{G}$ as in the IM model, we obtain

$$\dot{C} = 1 - [(\gamma + \delta) \dot{K} + \dot{G}].$$

Now, using the first order condition on labor supply together with the condition on optimal use of labor, one gets

$$H = \frac{1}{1 + \frac{1 - \eta}{(1 - \theta)\eta} \frac{1 + \tau_C}{1 - \tau_N} \dot{C}}.$$

Finally, we modify the production function to make sure that the level of stationary output $\hat{Y}$ in the CM economy coincides with that in the IM economy. To do so, define $\Omega = N_{IM}/H_{IM}$, where the subscript $IM$ refers to the IM setup variables. Then

$$Y_t = K_t^\theta (\Omega Z_t H_t)^{1 - \theta}.$$

$\Omega$ can be interpreted as a mean-preserving spread correcting for the average labor productivity effect present in the IM economy.
Appendix B. Solving for the Decision Rules with the Endogenous Grid Method

In this appendix, we describe how we implement the endogenous grid method to solve for the agents decision rules. At this stage, we assume that $r$ and $\hat{w}$ are known and take them parametrically, together with tax rates appearing in an individual’s budget constraint.

We set a grid of values for $\hat{a}'$, denoted by $G_a$. In practice, we select an exponential grid, with 2000 points. The algorithm is initialized by postulating an approximate decision rule for $\hat{a}''$, which we denote by $\hat{g}_a^{(0)}$. Also, we define a numerical tolerance parameter $\epsilon$; in practice $\epsilon = 1e-8$. We then implement the following steps:

1. Given $\hat{g}_a^{(i)}$, for each $(\hat{a}', s') \in G_a \times S$, compute
   - next period’s labor supply
     $$\hat{g}_h^{(i)}(\hat{a}', s') = \max \left\{ 0, 1 - (1 - \eta) \left[ 1 + \frac{[1 + (1 - \tau_A)r]\hat{a}' + \hat{T} - (1 + \gamma) \max\{0, \hat{g}_a^{(i)}(\hat{a}', s')\}}{(1 - \tau_N)\hat{w}s'} \right] \right\}.$$  
   - next period’s cash on hand
     $$m^{(i)}(\hat{a}', s') = (1 - \tau_N)\hat{w}s'\hat{g}_h^{(i)}(\hat{a}', s') + [1 + (1 - \tau_A)r]\hat{a}' + \hat{T},$$  
   - next period’s Lagrange multiplier
     $$\lambda^{(i)}(\hat{a}', s') = \frac{\eta}{m^{(i)}(\hat{a}', s') - (1 + \gamma) \max\{0, \hat{g}_a^{(i)}(\hat{a}', s')\}},$$

2. For each $(\hat{a}', s) \in G_a \times S$, compute
   - the current period Lagrange multiplier:
     $$\check{\lambda} = [1 + (1 - \tau_A)r]\frac{\beta}{1 + \gamma}E\{\lambda^{(i)}(\hat{a}', s')|s\},$$  
   - the current period consumption
     $$\check{c} = \frac{\eta}{(1 + \tau_C)\check{\lambda}},$$  
   - the current period cash on hand
     $$\check{m} = (1 + \tau_C)\check{c} + (1 + \gamma)\hat{a}'.$$

3. Using $\check{m}$, $m^{(i)}$ and $G_a$, update $\hat{g}_a^{(i)}$ via an interpolation procedure and thus compute $\hat{g}_a^{(i+1)}$.

4. If $||\hat{g}_a^{(i+1)} - \hat{g}_a^{(i)}|| < \epsilon$, stop, else go back to step 1.
References


Trabandt, M., and H. Uhlig (2009): “How far are we from the slippery slope? The Laffer curve revisited,” working paper, NBER.