Bank Capital, Credit Market Frictions and International Shocks Transmission

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Abstract

Recent empirical evidence suggests that the state of banks’ balance sheets plays an important role in the transmission of monetary policy and other shocks. This paper presents an international DSGE framework with credit market frictions and an active bank capital channel to assess issues regarding the transmission of domestic and foreign shocks. The theoretical framework includes the financial accelerator mechanism developed by Bernanke, Gertler and Gilchrist (1999), the bank capital channel and the exchange rate channel. Our simulations show that the exchange rate channel plays an amplification role in the propagation of shocks. Furthermore, with these three pass-through channels, domestic and foreign shocks have an important quantitative role in explaining domestic aggregates like output, consumption, inflation and total bank’s lending. In addition, results suggest that economies whose banks remain well-capitalized when affected by adverse shock experience less severe downturns. Our results highlight the importance of the bank capital in an international framework and can be used to inform the worldwide debate over banking regulation.

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## Contents

1 Introduction 3

2 The General Macroeconomic Environment 6
   2.1 The Optimal Financial Contract 8
   2.2 Households Workers 10
   2.3 Distribution and Good Production 15
      2.3.1 Final Good Production 16
      2.3.2 Domestic Composite Good Production 17
      2.3.3 Foreign Composite Good Production 18
   2.4 Intermediate Good Production 19
      2.4.1 Domestic Intermediate Good Production 19
      2.4.2 Foreign Intermediate Good Production 23
   2.5 Monetary Authorities and Government 24

3 Aggregation and Competitive Equilibrium 25

4 Model Calibration 27

5 Findings 29
   5.1 Bank Capital Channel 32
   5.2 International Transmission of Shocks 34
      5.2.1 Negative Technology Shock 34
      5.2.2 Expansionary Monetary Policy 35
   5.3 Transmission of International Shocks 35
      5.3.1 Foreign Demand Shock 35
      5.3.2 Foreign Monetary Policy Shock 36
   5.4 Sensitivity of the Bank Capital 37

6 Conclusion 37

7 Appendix 42
   7.1 Appendix 1: Business cycles comovement 42
   7.2 Appendix 2: Bank capital channel assessment 43
   7.3 Appendix 3: International transmission of domestic shocks 44
   7.4 Appendix 4: Transmission of international shocks 48
   7.5 Appendix 5: Properties of bank capital 50
1 Introduction

The high degree of globalization and interdependence between countries suggests strong international transmission of domestic and external shocks. This high interconnectedness between economic and financial markets may be viewed as a consequence of financial markets integration, the globalization of the trade market and the higher volume of cross-border assets held by economic agents. As an illustration, the recent financial turmoil which started with the meltdown of the U.S. subprime mortgage market has spread rapidly around the world, affecting the world’s economic system by a cross-country contagion mechanism. As consequence, GDP dropped around the world and led to the global malfunctioning of the real and financial markets. Figure 3 illustrates those recent global downturns in the United States, Canada, Japan and the United Kingdom.

Moreover, recent quantitative modelling has highlighted the importance of credit market imperfections with an emphasis on banks and corporates’ balance sheets channel in the mechanism of monetary policy transmission\(^1\) (Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2010), Gertler and Kiyotaki (2010), Meh and Moran (2010), and Dib (2010)). This evidence suggests that the financial health of banks and firms may significantly alter or magnify the transmission of monetary policy and others shocks.

This evidence underscores the need to develop a general equilibrium model with real-financial linkages and active banks’ balance sheets channel in an international framework. Understanding and quantifying these real-financial linkages is an important step towards the identification of the best policy response to international developments. For example, understanding these linkages would allow Canadians’ authorities to examine whether international trade in goods and financial markets can explain the observed spillover effects of U.S. business cycles on the Canadian economy. Moreover, a better knowledge of these linkages will allow central banks to assess the contribution of internal and external sources and to identify the channels of international transmission in OECD countries.

While the international transmission mechanism and the bank capita channel have both

\(^1\) Models including credit market imperfections can be categorized as two types: (i) corporate balance sheet (financial accelerator) channel models, which focus on the demand side of the credit market and (ii) bank balance sheet channel models, which focus on the supply side of the credit market (banks’ balance sheets).
generated a large body of research with well-established contributions, the analysis of these two issues simultaneously has received less attention. This paper aims to bridge this gap by proposing a medium sized Dynamic Stochastic General Equilibrium (DSGE) for a small open economy with an active bank balance sheets channel and the resulting model is calibrated to the Canadian economy. Specifically, we analyze the relative contribution of the bank balance sheets channel, the exchange rate channel, and the interest rate channel in the propagation of internal and external shocks. This paper contributes to the growing literature aimed at understanding how country specific react following to an adverse foreign shocks by assessing two major issues: first, how important is the banks’ balance sheets channel relative to the both interest rate and exchange rate channels. Second, how bank capital channel affects the international transmission mechanism.

The starting point of our model is the framework developed by Meh and Moran (2010), Gertler and Kiyotaki (2010) and Dib (2010), to which we include a cross-border goods distribution, the exchange rate channel and a government. The paper incorporates an active banking sector, the financial accelerator à la Bernanke, Gertler and Gilchrist (1999)(BGG (1999), hereafter), and the accumulation of capital in the spirit of Christiano, Motto and Rostagno(2005, 2010). Although the fact that credit conditions can affect the real economy is widely documented, incorporating credit market frictions in a quantitative general equilibrium started recently with the seminal contributions of Calstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999). These models highlight the link between the cost of borrowing and the net worth of the borrower, a link now widely referred to as the financial accelerator mechanism. However, the financial accelerator focuses on financial frictions caused by asymmetric information between investors and banks (on the demand side), but are silent about the effects of financial frictions on the supply side. As a result, the financial health of the banks does not affect its ability to collect deposits and lend.

Because evidence does suggest that the capitalization of banking system may affect the lending capacity of the financial sector. For example, a financial turbulence, by reducing the economic agents’ confidence in the financial market, leads to a strong deterioration in the economic outlook (quantitative modeling of this effect is undertaken in Gertler and Kiyotaki (2010), Meh
In contrast to this literature examining the role of bank capital in the business cycle fluctuations through a closed economy framework, this paper introduces the bank capital channel and the interest rate channel in a small open economy framework. The results of our simulations may be summarized as follow: 

(i) In the presence of the exchange rate channel, the propagation of both domestic and foreign shocks are amplified when comparing our baseline economy to a closed economy.

(ii) Depending of the size of the bank capital in the economy, shocks that originate domestically have an important quantitative role in explaining domestic output, investment, banks lending, entrepreneurs and banks net worth, inflation and interest rates.

(iii) External shocks (monetary policy shock and foreign demand shock) also contribute in explaining domestic aggregate fluctuations.

(iv) Economies whose banks remain well-capitalized when affected by adverse shock experience less severe downturns, i.e., when the bank capital channel is active, an economy with more bank capital has a better capacity to face against adverse shocks than an economy with less bank capital. This result, which remains valid for the transmission of international shocks, highlights the importance of the bank capital in an international framework and can be used to perform the worldwide debate over the banking regulation. Our simulations mimic most of evidences observed along the recent financial crisis and its heterogeneous impact among economies.

The rest of the paper is organized as follows. Section 2 describes the model and the financial contract is analyzed. In Section 3, we discuss aggregation and present the definition of the competitive equilibrium. Section 4 presents the calibration and describes the economy’s steady state. In Section 5, we discuss our findings and conduct a set of experiments related to the bank capital channel, the international transmission of domestic shocks and the transmission of international shocks. Section 5 concludes.

2Meh and Moran (2010) show that banks are also subject to financial frictions in the process of searching for loanable funds. Their model is a major contribution to that BGG (1999) and the results show that the friction in the supply of credit affects the propagation of shocks. B. Markovic (2006) developed a model very similar to Meh and Moran (2004) in which the bank must hold capital reserves and reduce the supply of loans to adjust to the required minimum capital regulations. Their results suggest that bank capital contributes significantly to the transmission mechanism of monetary policy.
2 The General Macroeconomic Environment

This section provides the main theoretical framework of the model, which is composed of households workers of mass $\eta^h$, entrepreneurs of mass $\eta^e$, bankers (financial intermediaries) of mass $\eta^b$, firms (domestic and foreign), domestic government and the monetary authority (domestic, foreign). At the beginning of each period, the households workers supply differentiated labor and receive money from the domestic monetary authority as lump-sum transfers and by renting physical capital. The total supply of households labor is assembled in a Dixit-Stiglitz form by an external firm that sells it to domestic intermediate goods producers. Households divide their high-powered money into currency, bank deposits and bonds$^3$. Currency pays no interest, and it’s held for the transactions services. Banks deposits pay interest as well as foreign and domestic bonds to households. In this model, bankers raise funds by issuing deposits liabilities to households workers and make loans to entrepreneurs in order to finance their entrepreneurship activities, to pay their wage bills, and to cover the capital rental costs.

The model includes an intermediate goods production sector, located in the domestic country$^4$. Moreover, firms in monopolistic competition use labor from households workers, bankers and entrepreneurs to produce the domestic intermediate goods. Then, a perfectly competitive representative firm produces both domestic and foreign composite goods. A part of the domestic composite goods produced is exported and the other part is combined to foreign composite goods to produce the final goods. Indeed, the final good is obtained by a constant-elasticity-of-substitution (CES) production function with both domestic composite goods and foreign composite goods as inputs. The final good is converted into consumption goods, investment goods and capital goods.

Entrepreneurs use their own resources and the bank loans to finance their entrepreneurship projects (of size $I_t$) in order to produce a new capital with a stochastic technology, $\tilde{\omega}_t$. Our model also includes the financial accelerator mechanism à la BGG (1999) because entrepreneurs

$^3$In this paper, we adopt a real money-in-the-utility-function approach to introducing currency, but a cash-in-advance version of the model yields qualitatively similar results.

$^4$The including of this sector provides a rich channel to capture the transmission of the technology shocks to the economy. It is worth mentioning that we do not permit an international mobility of labor between countries in this framework.
must obtain external fund from financial intermediaries to finance their investment projects\textsuperscript{5}. Besides to the credit channel introduced by the financial accelerator mechanism, the model includes the interest rate channel and the exchange rate channel. The interest rate channel results from the monetary authority response to economic shocks and the exchange rate arises because of the impact of real exchange rate to the cost of commodity imports and to the foreign demand for composite goods production firms.

The model discussion is organized into five subsections. The first subsection describes the informational environment and the optimal financial contract between entrepreneurs and bankers. Subsection 2 presents the preferences of households workers and subsection 3 describes the final goods production and its distribution. The fourth subsection provides the intermediate goods production. Finally, the fifth subsection describes government and monetary authorities. The following diagram (1) summarizes the general structure of the model.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{model_diagram.png}
\caption{General structure of the model}
\end{figure}

\textsuperscript{5}This highlights the credit channel because of asymmetric information between borrower and lenders.
2.1 The Optimal Financial Contract

Our financial contract model is built following Holmstrom and Tirole (1997), Chen (2001) and Meh and Moran (2010). The environment of the financial contract consists of a continuum of risk-neutral agents and involves three types of agents: households workers, entrepreneurs and bankers with a population of mass $\eta^h$, $\eta^e$ and $\eta^b$, respectively. In the model, entrepreneurs are finitely-lived with a probability $(1 - \tau^e)$ of dying each period. Entrepreneurs have access to a stochastic investment technology which has the final good $I_t$ as an input. Bankers finance the entrepreneurs with an aggregate capital which is the sum of households deposits and the banker’s equity commonly called bank capital. The bank capital can yield a grow return $R^a_t = (1 + r^a_t)$ per unit of investment if this capital is invested in a safe technology. Entrepreneurs with a net worth $N_t$ have a project that requires $I_t$ units of funds to generate a stochastic payoff with $I_t > N_t$. Entrepreneurs have a continuum of projects which produce the same public return $R_t$ units of capital when the project succeeds an zero when it fails. The returns from entrepreneurial projects are publicly observable and the success probability of the projects depends on an unobservable action taken by the entrepreneurs. If an action $a^h$ is undertaken, the probability of success is $\alpha^h$ and if an action $a^l$ is undertaken, the probability of success is $\alpha^l$ (with $\alpha^h > \alpha^l$). The success of the project differs in the action undertaken by the entrepreneurs and therefore in the probability of success. Entrepreneurs will enjoy a private benefit $b$ from choosing an action $a^l$ and zero from choosing the action $a^h$. This behaviour introduces a moral hazard problem. Henceforth, without a proper incentive, entrepreneurs may deliberately choose an action $a^l$ with low probability of success and high private benefit. Under this moral hazard problem and, in the way to reduce the entrepreneurs’ incentives to choose an action $a^l$, bankers’ have access to an imperfect monitoring technology, which can enforce entrepreneurs to choose a socially preferable action $a^h$. Therefore, if banker occurs a private monitoring cost $\mu$, this will reduce the private benefit to entrepreneur from $b$ to 0. The introduction of this monitoring cost is consistent to a costly state verification (CSV) literature (Carlstrom and Fuerst (1997), Bernanke et al. (1999) among others). The project unit return, $R_t$, is distributed among the entrepreneur ($R^e_t$), the banker ($R^b_t$) and the households ($R^h_t$). All agents receive nothing when the project fails. The optimal financial contract will allow us to determine the optimal level of
the aggregate variables of entrepreneurial net worth, investment, bank loans, deposits and bank capital.

Assumptions: [1] We assume that excluding the private benefit, the high probability action $a^h$ is socially preferable and optimal:

$$\alpha^h R^h_t I_t > (1 + r^d_t) D_t > \alpha^l R^h_t I_t,$$

(1)

[2] In this model, households are assumed to be neither able to monitor the activity of entrepreneurs nor of enforcing the financial contracts with entrepreneurs. Therefore, we not permit to households to lend directly to entrepreneurs, i.e., there is no feasible financial contract between entrepreneurs and households.

$$\alpha^h \left( R^h_t - \frac{b}{\Delta \alpha} \right) I_t < (1 + r^d_t) D_t,$$

(2)

where $\Delta \alpha = \alpha^h - \alpha^l$. We also assume that in the case of banks monitoring, there may exist a feasible financial contract between entrepreneurs and households workers, i.e., workers have an inventive to participate in the financial contract.

$$\alpha^h R^h_t I_t > (1 + r^d_t) D_t,$$

(3)

[3] We impose an incentive compatibility condition for a banker to allow a finance intermediation service. This condition requires that the expected return of the banker for the socially optimal action ($a^h$), net of monitoring cost, should be greater than or equal to the expected return of when the non-socially optimal action ($a^l$) is chosen.

$$\alpha^h R^b_t I_t - \mu I_t \geq \alpha^l R^b_t I_t$$

(4)

[4] We also impose that the entrepreneur have inventive to choose a socially optimal action when bankers provide a monitoring service. Then,

$$\alpha^h R^c_t I_t \geq \alpha^l R^c_t I_t + b I_t$$

(5)
Definition 1 (Optimal financial contract): The optimal financial contract consists of a maximization of the entrepreneur expected return, given to incentive compatibility and capital requirement constraints (1), (2), (3), (4) and (5). With \( R^e_t + R^b_t + R^h_t \leq R_t \), the entrepreneur maximization program can be written as

\[
V^e(A_t, D_t) = \max_{I_t, R^e_t, R^b_t, R^h_t} \alpha^h Q_t R^e_t I_t
\]

s.t.:

\[
\begin{align*}
\alpha^1 Q_t R^e_t I_t + b Q_t I_t & \leq \alpha^h Q_t R^e_t I_t \\
\alpha^1 Q_t R^b_t I_t & \leq \alpha^h Q_t R^b_t I_t - \mu I_t \\
(1 + r^a_t) A_t & \leq \alpha^h Q_t R^h_t I_t \\
(1 + r^d_t) D_t & \leq \alpha^h Q_t R^h_t I_t \\
L^s_t & \leq A_t + D_t - \mu I_t \\
L^d_t & \leq I_t - N_t
\end{align*}
\]

Proposition 1 (Optimal financial contract): Solving entrepreneur’s maximization program yields: \( R^e_t = \frac{b}{\Delta \alpha} ; R^b_t = \frac{\mu}{Q_t \Delta \alpha} ; R^h_t = R - \frac{b}{\Delta \alpha} - \frac{\mu}{Q_t \Delta \alpha} \). The amount of bank capital and households deposits, and investment level in equilibrium are given given by \( A_t = \frac{\alpha^h}{\Delta \alpha} \mu I_t / (1 + r^a_t) \Delta \alpha, D_t = \frac{\alpha^h Q_t}{1 + r^a_t} \left( R_t - \frac{b}{\Delta \alpha} - \frac{\mu}{Q_t \Delta \alpha} \right) I_t \) and \( I_t = (N_t + A_t) / \left( 1 + \mu - \frac{\alpha^h Q_t}{1 + r^a_t} \left( R_t - \frac{b}{\Delta \alpha} - \frac{\mu}{Q_t \Delta \alpha} \right) \right) = (N_t + A_t) / \text{Lev}_t \), where \( \text{Lev}_t \) is the bank’s total leverage.

2.2 Households Workers

The model is composed of a continuum of infinitely-lived households workers of mass \( \eta^h \) indexed by \( i \in (0, \eta^h) \). We assume that, households workers in the domestic country have a same level of wealth. This assumption allows us to analyze the behaviour of a representative household. A representative household worker maximizes a utility function that depends positively on consumption and negatively on work effort, which is positively related to output. In addition, household derives utility from holding currency, which they optimally choose along with their current level of labor supply and consumption. Households supply differentiated labor inputs used by domestic firms and set nominal wage using Calvo’s partial indexation. There is habit
persistence in consumption to respect stylized observed facts in consumption. The preferences of the representative household \( i, i \in (0, \eta^h) \) are given by the following lifetime utility function, which is separable into per consumption, real money balances and hours worked. The period utility function \( U(\cdot) \) has a constant-relative-risk aversion (CRRA) form. The consumption basket and the labour supply at period \( t + l \) is represented by \( C^h_{t+l}(i) \) and \( L_{t+l}(i) \). \( P_{t+l} \) is the aggregate level of domestic country price. \( M_{t+l}(i)/P_t \) denotes the real money balance at the end of period \( t + l \).

\[
U^h_t(i) \equiv E_t \left\{ \sum_{l=0}^{\infty} \beta^{t+l} U \left( C^h_{t+l}(i) - \gamma C^h_{t-1}(i), \frac{M_{t+l}}{P_{t+l}}(i), L_{t+l}(i) \right) \right\},
\]

where \( \beta \in (0, 1) \) is the household’s discount factor and \( \gamma \in (0, 1) \) is the parameter that controls habit persistence. In this expression, \( E_t \) is the conditional expectation operator evaluated at time 0. Households workers own all domestic firms and bankers hold the financial intermediation system. Households receive dividend payments, from firms and also earn income from holding domestic bonds issued by government, \( B^d_t \), and foreign bonds, \( B^f_t \). Domestic bonds yields a nominal return \( r^b_t \) and foreign bonds produces \( r^f_t \) as a nominal return. Households pay taxes on their wage and the taxes rate imposed by the government is given by \( \tau_w \). In this framework, bank has a special expertise in credit analysis, so it monitors borrowers more efficiently than households. For this reason, households can not directly lend to entrepreneurs. Even with single borrower, this assumption creates a role for financial intermediation system, and the bank advantage becomes greater as the number of entrepreneurs need to be financed increases.

Households face a capital utilization rate decision. Thus, at the beginning of each period, households decide to offer capital services \( u_t K^h_t \), where \( u_t \) is the utilization rate. The rental income from capital is given by \( r^h_t u_t K^h_t \) and the convex utilization cost is represented by \( v(u_{t+1}) \).

\( ^6 \)This kind of capital utilization function is used in Meh and Moran (2010), wherein more informations about the theoretical explanation are given.
The typical household’s budget constraint is thus given by

\[
C^h_{t+1}(i) + Q_{t+1}P^i_{t+1}(i) + D_{t+1}(i) + \frac{M_{t+1}(i)}{P^i_{t+1}} + B^d_{t+1}(i) + s_{t+1}B^f_{t+1}(i) \\
= (1 + r^d_{t+1})D_{t+1-1}(i) + (1 + r^b_{t+1})B^d_{t+1-1}(i) + s_{t+1}\kappa_{t+1}(e_{t+1}b^f_{t+1}, \epsilon^{f}_{t+1})(1 + r^f_{t+1})B^f_{t+1-1}(i) + \frac{M_{t+1-1}(i)}{\pi_{t+1}} + \Pi_{t+1}(i) + X_{t+1}(i),
\]

where, \(X_{t+1}(i)\) denotes a lump-sum money transfer from the monetary authority receive by a representative household and \(D_{t+1}(i)\) is the real bank deposits. The nominal wage and the rental rate of deposits are given by \(W_{t+1}\) and \(r^d_{t+1}\), respectively. \(\Pi_{t+1}(i)\) is the aggregate dividend payments from firms. Denote by \(s_t = e_tP^*_t/P_t\), the real exchange rate, where \(e_{t+1}\) represents the nominal exchange rate expressed in terms of the domestic currency per unit of foreign currency. \(P^*_t\) is the aggregate foreign price level. \(Q_{t+1}\) is price of the capital associated to the investment level \(I_{t+1}\). The function \(\kappa(.)\) represents the premium associated with buying foreign bonds and it captures the costs (or benefits) for households of undertaking positions in the international asset market. In the case that the amount of debt issued by a foreign country is greater than its steady state value, then households are charged a premium on the foreign interest rate. As in Schmitt-Grohé and Uribe (2003), we assume that this function depends on the per capita holding of foreign bonds in the entire economy with respect to nominal output of the composite domestic goods, \(B^f_{t+1} \equiv \int_0^{\infty} B^f_{t+1}(i)di\). Formally, we assume that the premium associated to the foreign interest rate function is given by \(\log(\kappa_{t+1}) = \varpi\epsilon^{f}_{t+1} \left[ \exp \left( \frac{s_{t+1}B^f_{t+1}}{\Pi_{t+1}} \right) - 1 \right]\), where \(\varpi\) is a parameter that captures the risk level of foreign bonds. Computationally, a premium on the foreign interest rate function is introduced to help the system to have a well-defined steady state. In addition, we add a time-varying shock, \(\epsilon^{f}_{t}\), to the risk premium function to capture the changes in the foreign debt. We assume that this shock follows a random walk with drift in logs:

\[
\log(\epsilon^{f}_{t}) = \rho_{\epsilon} \log(\epsilon^{f}_{t-1}) + \epsilon^{f}_{t}, \quad \text{where} \quad \epsilon^{f}_{t} \sim \mathcal{N}(0,1).
\]

We include a quadratic adjustment cost function on the changes of the level of investment.
Denote by \( \delta \), the depreciation rate of the capital, then the law of motion of capital accumulation is given by: 
\[
I_t^h = K_t^h - (1 - \delta)K_{t-1}^h + \Psi(K_t^h, K_{t-1}^h), \quad \text{where } \Psi'' > 0 \text{ and } \Psi(1) = 0, \text{ where the functional } \Psi(.,.) \text{ capture the adjustment costs } ( \Psi(k_t^h, k_{t-1}^h) = \frac{\phi}{2} \left( \frac{k_t^h}{k_{t-1}^h} - 1 \right)^2, \quad \phi > 0. 
\]
Given the preferences of a representative household and its budget constraints, the Lagrangian function associated is

\[
\mathcal{L}_t(.) = E_t \sum_{l=0}^{\infty} \beta^l \left\{ \log \left( C_{t+l}(i) - \gamma C_{t+l-1}(i) \right) + \psi \log \left( 1 - L_{t+l}(i) \right) + \zeta \log \left( \frac{M_{t+l}(i)}{P_{t+l}} \right) - \Lambda_{t+l}(i) \left\{ C_{t+l}(i) + Q_{t+l}I_{t+l}(i) + D_{t+l}(i) + \frac{M_{t+l}(i)}{P_{t+l}} + B_{t+l}^d(i) + s_{t+l}B_{t+l}^f(i) \right\} \\
- (1 + r^{d}_{t+l})D_{t+l-1}(i) - (1 - \tau_w)\frac{W^h_{t+l}}{P_{t+l}}L_{t+l}(i) - \frac{M_{t+l-1}(i)}{P_{t+l}} - \Pi_{t+l}(i) \right\},
\]
where \( \Lambda_{t+l}(i) \) is the Lagrange multiplier associated with the budget constraint.

**Definition 2 (the maximization problem of the household):** The household’s optimization problem consists of choosing \( \{ C_{t}^h(i), M_{t}(i), B_{t}^d(i), B_{t}^f(i), D_{t}(i), K_{t}^h \} \) for all \( t \in [0, \infty) \) to maximise lifetime utility function given a budget constraint.

**Proposition 2 (the maximization problem of the household):** Profits from firms are paid at the end of each period, therefore, they do not affect household’s optimization problem. Household’s first-order conditions (except for labor and wages) are given by

\[
\frac{1}{C_t^h(i) - \gamma C_{t-1}^h(i)} - \beta \gamma E_t \left[ \frac{1}{C_{t+1}^h(i) - \gamma C_t^h(i)} \right] = \Lambda_t(i); \tag{11}
\]

\[
\frac{\zeta}{M_t(i)/P_t} + \beta E_t \left[ \frac{\Lambda_{t+1}(i)}{\pi_{t+1}} \right] = \Lambda_t(i); \tag{12}
\]

13
In this framework, households supply differentiated labour inputs used by intermediate good producers and set nominal wage using Calvo’s partial indexation. We assume that the aggregate labour is supplied by a representative competitive firm that hires the labor supplied by each household. The differentiated labour inputs supplier aggregates labour using a constant-elasticity-of-substitution (CES) function given by

\[ L_t = \left( \int_0^{\eta h(0)} W_t(i) \left( \frac{K_t(i)}{K_t(i)} - 1 \right) \right)^{\frac{\xi_w}{\xi_w - 1}} \]

where \( 0 \leq \xi_w \leq +\infty \) is the elasticity of substitution among different types of labour. The differentiated labour inputs supplier maximizes profits subject to the production function given all differentiated labor wages, \( W_t(i) \), and the aggregate wage, \( W_t \).

The first order condition leads to

\[ L_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\xi_w} L_t, \]

and using the zero profits condition implied by perfect competition, we are: \( W_t = \left( \int_0^{\eta h} W_t(i) \left( 1 - \xi_w \right) di \right)^{\frac{1}{1-\xi_w}} \).

Following the assumption by Calvo (1983), we include a nominal rigidities on households’ wage setting. Thus, in each period, a fraction \( 1 - \phi_w \) can change their wages, i.e., households only reset optimally the wage contract in states of nature with a constant probability \( 1 - \phi_w \). All others are not able to lay out the optimal wage contract. In that case, they can only partially index their wages to past inflation of the composite domestic goods. The level of indexation is
captured by a parameter \( \chi_w \in (0, 1) \). This nominal rigidity implies that if the household cannot change its wage for \( k \) periods, then, its normalized wage is given by \( \prod_{s=1}^{k} \frac{\pi_{t+s-1}^{\chi_w} W_t(i)}{\pi_{t+s} P_t} \). The part of Lagrangian function used to set the optimal hours worked, \( L_t(i) \), and wage, \( W_t(i) \), is given by

\[
\max_{W_t(i)} \mathbb{E}_t \sum_{k=0}^{\infty} \phi_w^k \beta^k \left\{ \psi \log(1 - L_{t+k}(i)) + \Lambda_{t+k}(i) \prod_{l=1}^{k} \frac{\pi_{t+l-1}^{\chi_w}}{\pi_{t+l}} (1 - \tau_w) \frac{W_t(i)}{P_t} l_{t+k}(i) \right\} ,
\]

subject to the following constraint: \( L_{t+k}(i) = \left( \prod_{l=1}^{k} \frac{\pi_{t+l-1}^{\chi_w}}{\pi_{t+l}} W_t(i) \right)^{-\xi_w} L_{t+k} \). The first-order condition of the representative household problem leads to:

\[
\tilde{W}_t = \frac{\xi_w}{\xi_w - 1} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \phi_w)^k \frac{\psi}{1 - L_{t+k}} L_{t+k} \left( \prod_{l=1}^{k} \frac{\pi_{t+l-1}^{\chi_w}}{\pi_{t+l}} \right)^{1-\xi_w} W_{t+k}^{\xi_w}. \tag{19}
\]

This expression can be written as a geometric average of past real wage and the new optimal wage: \( \pi_{t}^{1-\xi_w} W_t^{1-\xi_w} = \eta^h (1 - \phi_w) \pi_{t}^{1-\xi_w} \tilde{W}_t^{1-\xi_w} + \phi_w W_{t-1}^{1-\xi_w} \pi_{t-1}^{1-\xi_w} \)

### 2.3 Distribution and Good Production

The distribution sector is composed of intermediate and final good producers. The intermediate good domestic producers is consisted of domestic and foreign firms in a monopolistic competition. Output produced by the intermediate good producers is converted into composite domestic good and composite foreign good by a perfectly competitive firm. The domestic composite output is combined with imported foreign good to produce a final output. In the final step, the final good is transformed into consumption good, investment good and good used up in capital utilization and in bank monitoring. The general structure of good distribution is set out in the following graphics.
2.3.1 Final Good Production

At the final step of the distribution chain, final good, $Z_t$, is produced by a domestic competitive firm using a CES technology. The representative firm assembles the domestic composite good ($Y_t^d$) with imported composite good ($Y_t^f$) to produce consumption good and investment good. The functional form of final good production technology is given by

$$Z_t = \left( \omega_d R^\frac{\lambda_z,t-1}{\lambda_z,t} (Y_t^d) \right)^{\frac{1}{\lambda_z,t}} + \left( 1 - \omega_d \right)^{\frac{1}{\lambda_z,t}} (Y_t^f)^{\frac{\lambda_z,t-1}{\lambda_z,t}} \right)^{\frac{1}{\lambda_z,t}},$$

(20)

where $0 < \omega_d < 1$ denotes the part of domestic good in the final good production process. $\omega_d$ can also interpreted as the steady state degree of openness. The elasticity of substitution between domestic composite good and imported good is assumed to follow a mean reverting stochastic process given by

$$\log(\lambda_z,t) = \rho_z \log(\lambda_z,t-1) + \epsilon^z_t, \quad \epsilon^z_t \sim \mathcal{N}(0,1).$$

(21)
The final good is distributed into household workers consumption, entrepreneurs consumption, bankers consumption, government spending, investment good and good used up in capital utilization and in bank monitoring. The final good producer maximizes its profits subject to the production function and taking as given the price of the domestic composite good \( P^d_t \), the price of the imported composite good \( P^f_t \) and the price of the final good \( P_t \). The maximization program is given by

\[
\max_{Y^d_t, Y^f_t} \left\{ P_t Z_t - P^d_t Y^d_t - P^f_t Y^f_t \right\},
\]

(22)

Solving the maximization program, we obtain the demand functions for the domestic composite good and the imported good.

\[
Y^d_t = \omega_d \left( \frac{P^d_t}{P_t} \right)^{-\lambda_{x,t}} Z_t,
\]

(23)

\[
Y^f_t = (1 - \omega_d) \left( \frac{P^f_t}{P_t} \right)^{-\lambda_{x,t}} Z_t,
\]

(24)

the null-profits condition yields to the determination of the price of the final good.

\[
P_t = \left[ \omega_d (P^d_t)^{1-\lambda_{x,t}} + (1 - \omega_d) (P^f_t)^{1-\lambda_{x,t}} \right]^{\frac{1}{1-\lambda_{x,t}}}.
\]

(25)

### 2.3.2 Domestic Composite Good Production

The domestic composite good, \( Y^d_t \), is produced by a continuum of domestic firms in perfect competitive market using domestic intermediate goods, \( Y^d_t(j) \), as inputs. The domestic composite good producer aggregates domestic intermediate goods using the Dixit-Stiglitz aggregator function given by

\[
Y^d_t = \left( \int_0^1 Y^d_t(j) \xi_{d,t}^{-1} \xi_{d,t} d_j \right)^{\xi_{d,t}};
\]

(26)

where \( \xi_{d,t} \) denotes the elasticity of substitution across intermediate goods used to produce a domestic composite good. The elasticity of substitution, \( \xi_{d,t} \), is assumed to follow a mean
reverting process given by.

\[
\log(\xi_{d,t}) = \rho_d \log(\xi_{d,t-1}) + \epsilon^d_t, \quad \epsilon^d_t \sim \mathcal{N}(0, 1). \tag{27}
\]

In order to determine the contribution of each intermediate goods, the domestic composite good maximizes its profits subject to the aggregation function. The maximization program is given by

\[
\max_{Y^d(j)} \left\{ P^d_t Y^d_t - \int_0^1 P^d_t(j) Y^d_t(j) dj \right\} \quad \text{s.c.} \quad Y^d_t = \left( \int_0^1 Y^d_t(j)^{\xi_{d,t-1}} \xi_{d,t} \right)^{\xi_{d,t}^{-1}} \tag{28}
\]

Taking as given all intermediate domestic goods prices \(P^d_t(j)\), the contribution of each intermediate good of type \(j\) and the price of the domestic composite good are given by

\[
Y^d_t(j) = Y^d_t \left( \frac{P^d_t(j)}{P^d_t} \right)^{-\xi_{d,t}}, \quad \tag{29}
\]

\[
P^d_t = \left( \int_0^1 P^d_t(j)^{1-\xi_{d,t}} dj \right)^{\frac{1}{1-\xi_{d,t}}}. \tag{30}
\]

### 2.3.3 Foreign Composite Good Production

The foreign composite good, \(Y^f_t\), is also assembled by a continuum of foreign firms in perfect competitive market using domestic intermediate goods, \(Y^f_t(j)\), as inputs. Similarly to the domestic composite good, the foreign composite good producer aggregates foreign intermediate goods using a Dixit-Stiglitz form given by

\[
Y^f_t = \left( \int_0^1 Y^f_t(j)^{\xi_{f,t-1}} \xi_{f,t} \right)^{\frac{\xi_{f,t}}{\xi_{f,t} - 1}}, \quad \tag{31}
\]

where the elasticity of substitution across foreign intermediate goods is denoted by \(\xi_{f,t}\) and assumed to follow an \(AR(1)\) process.

\[
\log(\xi_{f,t}) = \rho_f \log(\xi_{f,t-1}) + \epsilon^f_t, \quad \epsilon^f_t \sim \mathcal{N}(0, 1). \tag{32}
\]
The foreign composite good producer chooses the part of each intermediate good in order to maximizes its profits. Taking all intermediate foreign goods prices $P^f_t(j)$ as given, the demand of each intermediate good as well as the price the foreign composite good are given by

$$Y^f_t(j) = Y^f_t \left( \frac{P^f_t(j)}{P^f_t} \right)^{-\xi_{f,t}} ,$$

(33)

$$P^f_t = \left( \int_0^1 P^f_t(j)^{1-\xi_{f,t}} dj \right)^{\frac{1}{1-\xi_{f,t}}} .$$

(34)

### 2.4 Intermediate Good Production

#### 2.4.1 Domestic Intermediate Good Production

Intermediate goods are produced by monopolistically competitive firms with nominal rigidities à la Calvo (1983). The domestic intermediate good producer $j$ combines a purchased capital stock $K_t(j)$ with the households labour supply, bankers labour supply and entrepreneur labour supply to produce a differentiated intermediate good $Y_t(j)$. A part of this intermediate good is used for producing the composite domestic good and the remaining part $Y^x_t(j)$ is exported. Each intermediate good producer $j$ has access to a production function represented by

$$Y_t(j) = \begin{cases} 
A^Z_t K_t(j)^{\theta_k} L_t(j)^{\theta_h} H^e_t(j)^{\theta_e} H^b_t(j)^{\theta_b} - \Theta_t & \text{if } A^Z_t K_t(j)^{\theta_k} L_t(j)^{\theta_h} H^e_t(j)^{\theta_e} H^b_t(j)^{\theta_b} \geq \Theta_t \\
0 & \text{otherwise} \end{cases} ,$$

(35)

where $K_t(j)$ and $L_t(j)$ denote the services of capital and aggregate households workers labour supply. $H^e_t(j)$ and $H^b_t(j)$ denotes respectively entrepreneurs and bankers labour supply. The non-negative parameter, $\Theta$, represents the fixed costs of production and calibrated to guarantee that economic profits of entrepreneurs are roughly equal to zero in the equilibrium. $A^Z_t$ is a non-serially correlated technology shock that follows the stochastic process given by

$$\log(A^Z_t) = (1 - \rho_a) \log(A^Z_0) + \rho_a \log(A^Z_{t-1}) + \epsilon_A Z_t .$$

(36)

The total cost function $TC_t(j)$ is defined as
The optimization problem of production is standard and given by

\[
\min_{K_t(j), L_t(j), H_t^c(j), H_t^b(j)} \left\{ T C_t = r_t^k K_t(j) + W_t^h L_t(j) + W_t^e H_t^c(j) + W_t^b H_t^b(j) \right\}
\]

s.c. \( Y_t(j) = A_t^Z K_t(j)^{\theta_k} L_t(j)^{\theta_h} H_t^c(j)^{\theta_e} H_t^b(j)^{\theta_b} - \Theta_t \),

where \( r_t^k \) denotes the rental rate on capital services. The nominal wage on labour services received by households workers, entrepreneurs and bankers are respectively denoted by \( W_t^h \), \( W_t^e \) and \( W_t^b \). Let \( mc_t \) be the Lagrange multiplier associated with the production function constraint which can be interpreted as the real marginal cost of producing one unit of output \( Y_t(j) \). The first order conditions of a minimizing problem are given by

\[
\begin{align*}
  r_t^k &= mc_t A_t^Z \theta_k A_t^Z K_t(j)^{\theta_k - 1} L_t(j)^{\theta_h} H_t^c(j)^{\theta_e} H_t^b(j)^{\theta_b}, \\
  W_t^h &= mc_t A_t^Z \theta_h A_t^Z K_t(j)^{\theta_k} L_t(j)^{\theta_h - 1} H_t^c(j)^{\theta_e} H_t^b(j)^{\theta_b}, \\
  W_t^e &= mc_t A_t^Z \theta_e A_t^Z K_t(j)^{\theta_k} L_t(j)^{\theta_h} H_t^c(j)^{\theta_e - 1} H_t^b(j)^{\theta_b}, \\
  W_t^b &= mc_t A_t^Z \theta_b A_t^Z K_t(j)^{\theta_k} L_t(j)^{\theta_h} H_t^c(j)^{\theta_e} H_t^b(j)^{\theta_b - 1}.
\end{align*}
\]

At the end the intermediate goods production, a part of this intermediate good is used for producing the composite domestic good and the remaining part \( Y_t^x(j) \) is exported.

\[
Y_t(j) = Y_t^d(j) + Y_t^x(j).
\]

For price setting, we assume that in each period, a fraction \( 1 - \phi_d \) of domestic firms can change their prices. When allowed to do so, firm chooses the price of its output, \( \tilde{P}_t^d(j) \), in order to maximize its discounted real profits. All other firms can only index their prices to past inflation of the composite domestic good price. Indexation is controlled by a parameter \( \chi_d \in (0, 1) \), where \( \chi_d = 0 \) is no indexation case and \( \chi_d = 1 \) is perfect indexation. We assume that the intermediate good producer \( j \) chooses the optimal price \( \tilde{P}_t^d(j) \) at the time \( t \) and after \( l \)
periods with no reoptimizing, a firm’s $j$ price would be

$$P_{t+l}^d(j) = (\pi_{t+1}^d)^{x_d} \times (\pi_{t+2}^d)^{x_d} \times \cdots \times (\pi_{t+l-1}^d)^{x_d} \times P_t^d(j) = \prod_{s=1}^{l-1} (\pi_{t+s}^d)^{x_d} P_t^d(j), \quad (41)$$

where $\pi_{t+l}^d = P_{t+l}^d / P_{t+l-1}^d$. The problem of the firm $j$ is then:

$$\max_{P_t^d(j)} \mathbb{E}_t \sum_{l=0}^{\infty} (\beta \phi_d)^0 \Lambda_{t+l} \{ \left( \prod_{s=1}^{l-1} (\pi_{t+s}^d)^{x_d} \frac{\bar{P}_t^d(j)}{P_t^d(j)} - mc_{t+l} \right) Y_{t+l}(j) \}$$

s.c. $Y_{t+l}(j) = \left( \prod_{s=1}^{l-1} (\pi_{t+s}^d)^{x_d} \frac{\bar{P}_t^d(j)}{P_t^d(j)} \right)^{-\xi_{d,t}} Y_{t+l}, \quad (42)$

where $\Lambda_{t+l}$ is the marginal utility of wealth for a firm $j$ after $t + l$ periods. The first order conditions imply that

$$\xi_{d,t} \times \bar{N}_t^d = \bar{P}_t^d(j) \times (\xi_{d,t} - 1) \times \bar{D}_t^d,$$

where the functions $\bar{N}_t^d$ and $\bar{D}_t^d$ are given by:

$$\bar{N}_t^d = \sum_{l=0}^{\infty} (\beta \phi_d)^0 \Lambda_{t+l} mc_{t+l} Y_{t+l}(j),$$

$$\bar{D}_t^d = \sum_{l=0}^{\infty} (\beta \phi_d)^0 \Lambda_{t+l} \left( \prod_{s=1}^{l-1} (\pi_{t+s}^d)^{x_d} \right) Y_{t+l}(j) / P_{t+l}^d.$$ 

Let $\bar{p}_t^d = \bar{P}_t^d / P_t$ and assuming that all firms of type $j$ adopt a same strategy, then the first order condition relates the optimal price of a domestic intermediate good $j$.

$$\bar{p}_t^d = \frac{\xi_{d,t}}{\xi_{d,t} - 1} \frac{\mathbb{E}_t \sum_{l=0}^{\infty} (\beta \phi_d)^0 \Lambda_{t+l} mc_{t+l} \left( \prod_{s=1}^{l-1} \frac{\pi_{t+s}^d}{\pi_{t+s+1}^d} \right)^{-\xi_{d,t}} Y_{t+l}(j)}{\mathbb{E}_t \sum_{l=0}^{\infty} (\beta \phi_d)^0 \Lambda_{t+l} \left( \prod_{s=1}^{l-1} \frac{\pi_{t+s}^d}{\pi_{t+s+1}^d} \right)^{1-\xi_{d,t}} Y_{t+l}(j)}, \quad (43)$$

This expression can be used to derive a New Keynesian Phillips curve given by
\[(p_t^d)^{1-\xi_{d,t}} = \phi_d(p_{t-1}^d)^{1-\xi_{d,t}} + (1-\phi_d)(\bar{p}_t^d)^{1-\xi_{d,t}}. \tag{44}\]

Domestic composite output, \(Y_t\), is divided into domestic use, \(Y_t^d\), and exports, \(Y_t^x\). Foreign demand is aggregated by a monopolistically competitive firms using a following functional form:

\[Y_t^x = \left( \int_0^1 Y_{t}^x(j) \frac{\zeta_{y,t}^{-1}}{\zeta_{y,t}} dj \right)^{\frac{\zeta}{\zeta_{y,t}^{-1}}}, \tag{45}\]

where the elasticity of substitution between exported goods is denoted by \(\zeta_{y,t}\). We assume that this elasticity of substitution follows a stochastic process defined by:

\[\log(\zeta_{y,t}) = \rho_d \log(\zeta_{y,t-1}) + \epsilon_t^{yx}, \quad \epsilon_t^{yx} \sim \mathcal{N}(0,1). \tag{46}\]

In this model, domestic producers are not able to discriminate between the price of a good that will be used for domestic production and the price of a good to be exported. Solving a maximization program, the functional form of the foreign demand of a good \(j\) is given by:

\[Y_t^x(j) = \left( \frac{P_t^d(j)}{P_t^d} \right)^{-\zeta_{y,t}} Y_t^x. \tag{47}\]

Since our model is focused on a small open economy and in order to close our model, we assume that the foreign demand of a domestic good is proportional to a foreign GDP. Therefore, following Ambler et al. (2004), the functional form of the foreign demand is given by

\[Y_t^x = \left( \frac{P_t^d}{e_t^* P_t^*} \right)^{-\tau} Y_t^* = s_t^* Y_t^x \quad \text{where} \quad s_t = \left( \frac{e_t P_t^*}{P_t^d} \right), \tag{48}\]

In this expression, the parameter \(\tau (\tau > 0)\) describes the elasticity of demand for domestic good and \(s_t\) denotes the real exchange rate. The foreign price, \(P_t^*\), is an exogenous process and the foreign GDP, \(Y_t^*\), is assumed to follow a mean reverting stochastic process given by:

\[\log(Y_t^*) = (1 - \rho_y^*) \log(\bar{Y}^*) + \rho_y^* \log(Y_{t-1}^*) + \epsilon_t^y, \]
where $Y^*$ is a steady-state foreign production and $\epsilon_t^{y^*}$ is a zero-mean serially uncorrelated shock that affect the foreign production.

### 2.4.2 Foreign Intermediate Good Production

In a process of the final good production, the domestic economy imports foreign intermediate goods. These intermediate goods are produced by a continuum of intermediate good-importing firms indexed by $j \in (0, 1)$. There is a monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, $Y^f_t$, produced by a representative competitive firm. For price setting, we assume a nominal rigidity à la Calvo in the imported goods market\(^7\). Then, in each period, a fraction $1 - \phi_f$ of firms can change their prices. When allowed to do so, firm chooses the price of its output, $P^f_t(j)$, in order to maximize its discounted real profits. All other firms can only index their prices to past inflation of the composite domestic good price. The representative firm solves a following program:

$$
\max_{P^f_t(j)} E_t \sum_{t=0}^{\infty} (\beta \phi_f)^t \Lambda_{t+l} \frac{\Omega^f_{t+l}}{P^d_{t+l}},
$$

with

$$
\Omega^f_{t+l} = \left( \frac{P^f_t(j)}{P^d_{t+l}} - e_{t+l} P^* \right) \left( \frac{P^f_t(j)}{P^d_{t+l}} \right)^{-\xi_{f,t}} Y^f_{t+l},
$$

$$
P^f_t(j) = \frac{\xi_{f,t} E_t \sum_{k=0}^{\infty} (\beta \phi_f)^k \Lambda_{t+k} Y^f_{t+k}(j) s_{t+l}}{1 - \xi_{f,t} E_t \sum_{k=0}^{\infty} (\beta \phi_f)^k \Lambda_{t+k} Y^f_{t+k}(j) / P^d_{t+k}},
$$

where $\xi_{f,t}$ represents the elasticity of substitution between differentiated imported goods. As in the domestic intermediate good production case, equation (49) can be used to derive a New-Keynesian Phillips curve relationship for the rate of change of intermediate input prices given by:

\(^7\)This Calvo-type staggered price setting in the imported goods market allows to capture the incomplete exchange rate pass-through to import and imports prices.
\[(P^f_t)^{1-\xi_{f,t}} = \phi_f(P^f_{t-1})^{1-\xi_{f,t}} + (1 - \phi_f)(\tilde{P}^f_t)^{1-\xi_{f,t}}. \quad (50)\]

### 2.5 Monetary Authorities and Government

Monetary policy is conducted by the central bank, which manages the short-term nominal interest rate \( R^b_t = (1 + r^b_t) \), in response to fluctuations in the domestic GDP, \( Y_t \), and in consumer price index, CPI, inflation \( \pi_t \). This managing rule, also called the Taylor’s rule, allows the central bank to set the nominal interest rate through open market operations using a following Taylor’s rule:\(^8\)

\[
\log \left( \frac{R^b_t}{\bar{R}^b} \right) = \lambda_r \log \left( \frac{R^b_{t-1}}{\bar{R}^b} \right) + (1 - \lambda_r) \left( \lambda_\pi \log \left( \frac{\pi_t}{\bar{\pi}} \right) + \lambda_y \log \left( \frac{Y_t}{\bar{Y}} \right) \right) + \rho_\mu \log \left( \vartheta_t \right), \quad (51)
\]
with \( \lambda_r \in (0, 1) \), \( \bar{\pi} \) and \( \bar{Y} \), respectively, represent the target level of inflation and the target level of output of the domestic economy. The term \( \vartheta_t \) denotes a random shock to monetary policy that follows a first-order autoregressive process given by:

\[
\log(\vartheta_t) = \rho_{mp} \log(\vartheta_{t-1}) + \epsilon^{dmp}_t, \quad (52)
\]
with \( \epsilon^{dmp}_t \sim N(0, 1) \).

Foreign monetary policy variables, controlled by the interest rate paid for holding foreign bonds, \( R^f_t \), and the foreign inflation, \( \pi^*_t \), are both exogenous\(^9\). Formally, we use two stochastic processes to capture the quantitative impact of the foreign monetary policy shock. Thus,

\[
\log(R^f_t) = (1 - \rho_{R^f}) \log(\bar{R}^f) + \rho_{R^f} \log(R^f_{t-1}) + \epsilon^{fmp}_t, \quad (53)
\]
where \( \rho_{R^f} \in (0, 1) \) denotes the persistence of the foreign monetary policy shock and \( \bar{R}^f \) denotes the targeted steady-state value of the foreign interest rate, \( R^f_t \). The stochastic process using to

---

\(^8\)The use of the previous period interest rate allow us to match the smooth profile of the observed interest rate in the data

\(^9\)This assumption is consistent to a small open economy assumption in which the foreign monetary variables are exogenously determined
capture the change in the foreign price is likewise given by

\[ \log(\pi_t^*) = (1 - \rho_{\pi^*})\log(\pi^*) + \rho_{\pi^*}\log(\pi_{t-1}^*) + \epsilon_t^* . \]  

(54)

The government budget constraint is given by

\[ G_t + (1 + r^b_t) \frac{B_{t-1}^d}{\pi_t} + \frac{M_{t-1}}{P_t} + X_t = B_t^d + \tau_w W_t H_t + \frac{M_t}{P_t}, \]  

(55)

The right side of the budget constraint represents the government income which consisted of a new debt issued, \( B_t^d \), tax revenue paid by households workers, \( \tau_w W_t H_t \) and money creation, \( M_t - M_{t-1} \). The left side describes uses of government revenue which is composed of a final good spending, \( G_t \), money transfers \( X_t \) and debt repayments, \((1 + r^b_t)B_{t-1}^d\). Government spending is exogenous and determined using the following stochastic equation.

\[ \log(G_t) = (1 - \rho_g)\log(\bar{G}) + \rho_g \log(G_{t-1}) + \epsilon_t^g, \]  

(56)

where \( \bar{G} \) denote the targeted steady-state value of government spending.

The government faces a no-ponzi constraint that involves the actual value of foreign debt equals to a trade balance. This constraint is given by

\[ B_t^f + \kappa_t(1 + r^f_t) \frac{B_{t-1}^f}{\pi_t^*} = \frac{Y_t^x}{s_t} - Y_t^f. \]  

(57)

3 Aggregation and Competitive Equilibrium

In what follows, the total investment function, \( \bar{I}_t \), is defined as a sum of all individual investment projects, \( I_t \), in the economy and is given by

\[ \bar{I}_t = \frac{\bar{A}_t + \bar{N}_t}{G_t}, \]  

(58)

where \( 1/G_t \) denotes the bank leverage that occurs in the optimal financial contract. \( \bar{A}_t \) and \( \bar{N}_t \) respectively denote the total amount of bank capital and the entrepreneur’s net worth. The
capital stocks held by each group of agents are given by: \( \tilde{K}^h_t \) for the households workers, \( \tilde{K}^b_t \) for the bankers and \( \tilde{K}^e_t \) for the entrepreneurs. The aggregation condition is then given

\[
\tilde{K}^h_t = \eta^h K^h_t, \quad \tilde{K}^b_t = \eta^b K^b_t, \quad \tilde{K}^e_t = \eta^e K^e_t,
\]

where \( \eta^h \), \( \eta^b \) and \( \eta^e \) respectively represent the population masses of households workers, households bankers and entrepreneurs in the economy. Following the financial contract equilibrium, the dynamic of bank capital, \( \tilde{A}_t \), and entrepreneur’s net worth, \( \tilde{N}_t \), are given by\(^{10}\)

\[
\tilde{A}_t = (\gamma^b_t + Q_t(1-\delta))\tilde{K}^b_t + \eta^b W^b_t \quad \text{and} \quad \tilde{N}_t = (\gamma^e_t + Q_t(1-\delta))\tilde{K}^e_t + \eta^e W^e_t.
\]

As bankers and entrepreneurs are both assumed risk-neutral agents, the capital accumulation at the beginning of the period \( t + 1 \) can be written as\(^{11}\)

\[
\tilde{K}^b_{t+1} = \tau^b \alpha^g R^b_t \tilde{I}_t \quad \text{and} \quad \tilde{K}^e_{t+1} = \tau^e \alpha^g R^e_t \tilde{I}_t,
\]

with \( \tau^b \) and \( \tau^e \), the survival probability of bankers and entrepreneurs. These equation describes the inter-periods evolution of bank total assets and entrepreneur net worth. with a probability of \( 1 - \tau^b \), bankers exist the economy and become households workers. Similarly, entrepreneurs exist economy with a probability of \( 1 - \tau^e \) and also become households. This circular relationship between workers, entrepreneurs and bankers allows us to keep household’s total mass to 1. Exiting banks and entrepreneurs consume the value of their available wealth. This implies the following for aggregate consumption of entrepreneurs, bankers and workers:

\[
\tilde{C}^b_{t+1} = (1 - \tau^b)\alpha^g R^b_t \tilde{I}_t, \quad \tilde{C}^e_{t+1} = (1 - \tau^e)\alpha^g R^e_t \tilde{I}_t \quad \text{and} \quad \tilde{C}^h_t = \eta^h C^h_t.
\]

\[
\tilde{C}_t = \tilde{C}^h_t + \tilde{C}^e_t + \tilde{C}^b_t;
\]

**Definition 3 (Competitive equilibrium)** A competitive equilibrium is defined as a set of

---

\(^{10}\)In a symmetric equilibrium, all households bankers are identical as well as households entrepreneurs.

\(^{11}\)successful entrepreneurs and banks survive to the next period with probability \( \tau^e \) and \( \tau^b \), respectively. These agents save all their wealth, because of risk-neutral preferences and the high return on internal funds.
functions for (i) households’ policies $C_h(i)$, $I_h(i)$ and $K_h(i)$ that solve the maximization problem of the household; (ii) firms’ policies $K_f(j)$, $L_f(j)$, $H_f(j)$, $W_f(j)$, $W^h$ and $W^b(i)$ that solves firms maximization problem; (iii) optimal financial contract $I_t$, $R_t$, $R^b_t$, $R^h_t$, $A_t$, $D_t$ and $N_t$ that solve the maximization problem associated with the financial contract; (iv) aggregate prices $P^d_t$, $P_f^t$ and $P_t$ and (v) saving and consumption decision rules for bankers and entrepreneurs.

For equilibrium in the goods markets we require production to be equal to aggregate demand, that is

$$Z = \bar{C}_t + Q_t \bar{I}_t + G_t + \mu Q_t \bar{I}_t; \quad (64)$$

The market-clearing conditions are given by:

$$\tilde{K}_t = v_t \tilde{K}_t^h + \tilde{K}_t^c + \tilde{K}_t^b; \quad (65)$$

$$L_t = \int_0^{\eta_h} L_t(i) di; \quad (66)$$

$$H_t^c = \int_0^{\eta_c} H_t^c(j) dj; \quad (67)$$

$$H_t^b = \int_0^{\eta_b} H_t^b(j) dj; \quad (68)$$

$$H_t = L_t + H_t^b + H_t^c; \quad (69)$$

Equation (65) defines the total capital stock as the holdings of households, entrepreneurs and banks. Equation (69) requires that the total supply of labor input produced equals total demand by intermediate-good producers.

4 Model Calibration

To evaluate the relative contribution of the bank capital, the exchange rate and the interest rate channels in the mechanism of shocks propagation, we set the parameters of our model to reflect the key features of a small open economy such as Canada. The parameters values are generally consistent to those in the financial frictions literature as in Christiano, Motto and Rostagno (2010), Dib (2010) and Meh and Moran (2010). In the representative household’s
utility function, the elasticity of labour supply $\psi$ is set to 9.05, which leads to the steady-state value of household’s work effort equals to 30% of available time. Following results in Christiano, Motto and Rostagno (2010) and Meh and Moran (2010), the parameter of habits formation, $\gamma$, is fixed to 0.65 and $\zeta = 0.0001832219$. The value of $\zeta$ is set in order to match the steady-state of the model to the average ratio of $M1$ and $M2$ in a small open economy.

The household’s discount factor, $\beta$, is set to 0.99, implying a long-run nominal interest rate of 4% annually. The share of capital in the production function of intermediate goods producers, $\theta^k$, is set to 0.36 and the depreciation rate of the capital is fixed to 0.025, which is a widely used values in the New Keynesian literature. As we want to reserve a small role in the production for the hours worked by bankers and entrepreneurs, we set the share of the labour input of the households workers, $\theta^h$, to 0.6399. Then we choose $\theta^b = \theta^e = 0.00005$, reflecting an equal contribution of bankers and entrepreneurs in the intermediate goods production and allowing entrepreneurs and bankers to always have non-zero net worth.

The capital utilization parameters are set as follows: we impose that $u = 1$ and $v(1) = 0$ in the steady state, which ensures that the steady state is independent of $v(.)$. The parameter capturing the fixed costs in the production function, $\Theta$, is set to ensure that the steady state value of profits equals to zero. The persistence of the technology shock, $\rho_a$, is set to 0.95 and its standard deviation is 0.0015, which ensure that the model’s simulated output volatility equal that of observed aggregate data.

The nominal price rigidity parameter as well as the nominal wage-setting parameter are set following Calvo’s model of staggered price and wage adjustment. As in Christiano, Motto and Rostagno (2005), the probability of not reoptimizing for price and wage setters in the domestic country, $\phi_d$ and $\phi_w$, are fixed to 0.75 and 0.64, respectively. The elasticity of of substitution between domestic intermediate goods, $\xi_d$, and the elasticity of substitution between domestic labour types, $\xi_w$, are set to 8 and 21, respectively. These values are estimated in Christensen and Dib (2008) for the U.S. economy and are commonly used in the literature. Correspondingly, the probability of not reoptimizing for foreign price setters, $\phi_f$, is set to 0.5, while the elasticity of substitution between foreign intermediate goods production, $\xi_f$, is calibrated to 8.

Domestic monetary policy parameters $\lambda_r$, $\lambda_\pi$ et $\lambda_y$ are set of 0.8, 1.5 and $1/4$, respectively.
These values satisfy the Taylor principle and are consistent to those estimated in Clarida et al. (2000). The standard deviation of both domestic and foreign monetary policy shocks is fixed to 0.0016, $\rho_{mp} = \rho_{RF} = 0.0016$, which ensures that a one-standard deviation shock moves the interest rate by 0.6 percentage points. This value is consistent to the empirical estimates reported in Christiano et al. (2005).

In the financial market, the parameters are related to capital production and the optimal financial contract between bankers and entrepreneurs are set following, Carlstrom and Fuerst (1997), Bernanke et al. (1996), and Meh and Moran (2010). Accordingly, the steady state value of the bank’s capital asset ratio and the monitoring cost are respectively set to 14% and 2.5%. We set the probability of default in the loan contract in the event that action $a^H$ is undertaken to 1%. Thus, the probability of success is fixed to 99%, consistent to the results in Carlstrom and Fuerst (1997). The gap between the probability of success of the socially preferable action, $a^H$, and the free riders action, $a^L$, is set to 24%, consistent to the results in Meh and Moran (2010). The remaining parameters and steady-state ratios of the model are set in order to ensure that our model’s steady state match to standard New-Keynesian model. Thus, households consumption to GDP ratio is calibrated to 76%. Also, investment to GDP ratio and Capital to GDP ratio are set of 20% and 12. Finally, domestic good to final good ratio and imported good to final ratio are set of 70% and 30%. Table 1 and table 2 report the calibration and the steady-state values of some key variables.

5 Findings

To assess the relative contribution of the bank capital channel in international framework, we focus on the impulse functions of some key variables in response of a variety of structural shocks for three types of models: the first model, model (1), describes the full small open economy model with active bank capital channel and nominal rigidities (bank capital SOE, hereafter). The second model, model (2), is a closed economy with active bank capital channel à la Meh and Moran (2010) (bank capital CE, hereafter). The third model, model (3), is a variant of the first model in which an exogenous endowment is arbitrary added to surviving and newborn
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Habit formation</td>
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<tr>
<td>$\psi$</td>
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<tr>
<td>$\zeta$</td>
<td>Elasticity of money demand</td>
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<tr>
<td>$\theta_k$</td>
<td>Capital share</td>
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<tr>
<td>$\theta_h$</td>
<td>Workers labor share</td>
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</tr>
<tr>
<td>$\theta_e$</td>
<td>Entrepreneur labor share</td>
<td>0.00005</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Bankers labor share</td>
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</tr>
<tr>
<td>$\omega_d$</td>
<td>Share of domestic good in final good</td>
<td>0.7</td>
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<tr>
<td>$\lambda_z$</td>
<td>Elasticity of domestic good</td>
<td>0.59</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate of capital</td>
<td>0.02</td>
</tr>
<tr>
<td>$\alpha^d$</td>
<td>High probability of success</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha^l$</td>
<td>Low probability of success</td>
<td>0.75</td>
</tr>
<tr>
<td>$b$</td>
<td>Private benefit</td>
<td>0.16</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Monitoring cost</td>
<td>0.025</td>
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<tr>
<td>$\tau_e$</td>
<td>Entrepreneur’s death probability</td>
<td>0.78</td>
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<tr>
<td>$\tau_b$</td>
<td>Banker’s death probability</td>
<td>0.72</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Elasticity of labor supply</td>
<td>21</td>
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<tr>
<td>$\xi_d$</td>
<td>Elasticity of substitution for domestic goods</td>
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<tr>
<td>$\xi_f$</td>
<td>Elasticity of substitution for foreign goods</td>
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<tr>
<td>$\phi_w$</td>
<td>Wage reoptimization probability</td>
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<tr>
<td>$\phi_d$</td>
<td>Domestic price reoptimization probability</td>
<td>0.75</td>
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<tr>
<td>$\phi_f$</td>
<td>Foreign price reoptimization probability</td>
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<td>$\lambda_r$</td>
<td>Taylor rule: Interest smoothing</td>
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<tr>
<td>$\lambda_r$</td>
<td>Taylor rule: inflation coefficient</td>
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<tr>
<td>$\lambda_y$</td>
<td>Taylor rule: GDP coefficient</td>
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Table 2: Steady-state values and ratios: Baseline model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
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</thead>
<tbody>
<tr>
<td>π</td>
<td>Inflation</td>
<td>1.021/4</td>
</tr>
<tr>
<td>R</td>
<td>Gross real interest rate of investment projects</td>
<td>1.2118</td>
</tr>
<tr>
<td>( \overline{R} )</td>
<td>Gross real interest rate of domestic bonds</td>
<td>1.015</td>
</tr>
<tr>
<td>Rd</td>
<td>Gross real interest rate of deposits</td>
<td>1.0101</td>
</tr>
<tr>
<td>1/G</td>
<td>Bank leverage</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Steady-state ratios

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{C}_h/Y )</td>
<td>Households consumption to GDP ratio</td>
<td>76%</td>
</tr>
<tr>
<td>( \tilde{C}_b/Y )</td>
<td>Bankers consumption to GDP ratio</td>
<td>0.57%</td>
</tr>
<tr>
<td>( \tilde{C}_e/Y )</td>
<td>Entrepreneurs consumption to GDP ratio</td>
<td>2.76%</td>
</tr>
<tr>
<td>( \tilde{I}/Y )</td>
<td>Investment to GDP ratio</td>
<td>20%</td>
</tr>
<tr>
<td>( \tilde{K}/Y )</td>
<td>Capital to GDP ratio</td>
<td>12</td>
</tr>
<tr>
<td>( Y^d/Z )</td>
<td>Domestic good to final good ratio</td>
<td>70%</td>
</tr>
<tr>
<td>( Y^f/Z )</td>
<td>Imported good to final good ratio</td>
<td>30%</td>
</tr>
</tbody>
</table>

bankers. This economy is called an economy with more bank capital (economy with more bank capital, hereafter). Model (3), which describes an economy where banks are well-capitalized, is introduced to assess the role of the bank capital in the mechanism of shocks propagation. For this purpose, we plot on the same graph the impulse responses of some key variables for both model (1) and model (3). The method used to build our third model consists to provide a surplus of capital, \( e^b \), to survival and newborns banks. This surplus of capital, which is set to 0 in the baseline model, \( e^b = 0 \), may be viewed as an exogenous endowment offer to existent and new banks. We calibrate the value of \( e^b \) in order to ensure that the banker’s asset-capital ratio in model (3) is 20% more than its correspondent steady-state value in the baseline model. In the economy with more bank capital, only the following equations differ to those in the baseline economy.

\[
\tilde{A}_t = (r^k_t + Q_t(1 - \delta))\tilde{K}^b_t + \eta^b W^b_t + \eta^b e^b, \\
Z + \eta^b e^b = \tilde{C}_t + Q_t \tilde{I}_t + G_t + \mu Q_t \tilde{I}_t. 
\]

Figure 4 displays the impulse responses of key domestic aggregates following to a negative bank capital shock. This first empirical exercise is to investigate whether or not the bank
capital channel affects the mechanism of shocks propagation. Figures 5 and 6 depict impulse responses following to a negative technology shock and figures 7 and 8 display responses to a domestic monetary policy by emphasizing on model (1) and (2). Figures 9 and 10 assess the relative contribution of the bank capital in the transmission of international shocks by comparing our baseline model to model (3). Finally, figures 11, 12 and 13 display the impulse responses following to a domestic technology shock, domestic monetary policy and government spendings shock by assessing the role of the bank capital in the mechanism of international transmission of domestic shocks. Each variables’s response is expressed as the percentage deviation from its steady-state level.

5.1 Bank Capital Channel

Empirical evidence suggests that the bank capital as well as the entrepreneur’s net worth have a quantitative impact on the propagation mechanism. Thus, besides the financial accelerator mechanism (entrepreneurs’ net worth effects), our model includes the bank capital channel that may be interpreted as a financial accelerator mechanism on the supply side (interaction between households savers and banks). The purpose of this subsection is to investigate the relative contribution of the bank capital channel through which real and financial shocks are propagated. Figure 4 displays the impulse responses following to a negative bank capital shock by emphasizing on model (1) and model (2). Following the empirical evidences of Holmstrom and Tirole (1997) and Meh and Moran (2010), this shock may be interpreted as a ‘credit crunch’ caused by a sudden deterioration in the banks’ balance sheet and leads to exogenous declining in the banks net worth. Correspondingly, banks’ capitalization decreases alongside economic activity. An excellent way to capture the impact of a sudden deterioration in the bank’s net worth is to assume that the bank capital is subjected to continuous episodes of accelerated depreciation depicted by the following equations:

$$\tilde{A}_t = (r_t^k + Q_t (1 - \delta \tilde{A}_t)) \tilde{K}_t^b + \eta^b W^b_t,$$

(72)

where \(\eta^b_t\) is characterized by an AR(1) process and given by.
With (72), a positive shock on the depreciation rate of bank’s net worth leads to an unexpected decreasing in the value of bank’s total assets, which appears to be consistent to what happened during the recent financial crisis. In the event of existence of the bank capital channel, this negative shock on the bank capital should quantitatively affect the available loan supply and then the banking system’s capacity to lend. Conversely, in a Modigliani-Miller world, i.e., in a frictionless economic environment, this negative shock on the bank capitalization should be compensated by an increasing of households’ deposits. As result, this crowding out effect will counteract the negative effect of the bank capital on economic activity. However, with our financial frictions DSGE model, a decreasing in the bank’s capitalization leads to a credit rationing policy from banking system’s and then to a negative impact on the bank’s total lending. The downward effect is much more important in the model (2) (3.5%) than in the baseline model (by 2.5%). With the financial accelerator mechanism, a sudden scarcity in the lending market drives down entrepreneur’s net worth by around 10%, which moves up the external financial premium and domestic prices. As consequence, aggregate investment declines by 2% in the baseline model and by 3% in the closed economy. As result, exchange rate appreciates and imports react positively while exports move in opposite directions. However, the magnitude of declining in exports is less than the rise in imports. As result, output and aggregate investment decrease by much more in the baseline model than in the closed economy. Output drives down by more than 0.4% in the model (2) and by 0.2 in the baseline model\textsuperscript{12}. Except consumption, following to a negative bank capital shock, keys economic aggregates react negatively and a part of this negative effect is transferred to the foreign economy through the exchange rate channel. The results points out the dampening role of the exchange rate channel in propagation mechanism.

\[ \log(\tilde{\vartheta}_t^\delta) = \rho \log(\tilde{\vartheta}_{t-1}^\delta) + \epsilon_t^\delta. \] (73)

\textsuperscript{12}Aikman et Paustian (2006) shown that a 10% decreasing in bank’s capital asset ratio leads to a 0.6% decreasing in output
5.2 International Transmission of Shocks

5.2.1 Negative Technology Shock

Figures 5 and 6 display responses of both bank capital SOE (model 1) and bank capital CE (model 2) models following a 1% negative technology shock. The model (1) and the model (2), which displays some common characteristics, react negatively following this technology shock. Indeed, with a negative technology shock, the realized capital return is less than expected, leading to higher loan default rate than expected. This generates a negative effect of firms’ net worth and leverage ratio increases, causing higher agency cost in the financial contract. As consequence, the external finance premium increases and creates a negative retain on the credit demand side. Nevertheless, following a negative technology shock, nominal interest rate and inflation increase, which drives down the real cost of repaying existing debt and causing a debt-inflation effect. Under these circumstances, firms’ net worth declines and lower net worth pushes up the external finance premium.

On the other hand, a negative technology shock generates unexpected loss on the loan portfolio by weakening bank’s capital position. The deterioration in bank’s balance sheets, produce a negative signal to households about bank’s financial health. As consequence, households expected a higher banks default rate in the next period and are not willing to make deposits at lower rate of return. This increases bank’s external cost of funding and therefore creates a negative impact on the credit supply side. Given these two negative retains, aggregate lending declines, which pushes down investment and output.

Although model (1) and model (2) display almost common characteristics, responses for the small open economy are more amplified than responses for the closed economy, especially for aggregate output and consumption. Effects are smaller in the no exchange rate channel model than in the model with exchange rate channel. As illustration, following a negative technology shock, domestic prices drive up, which makes domestic good more costly than foreign goods. Following this shock, exchange which depreciates suddenly, appreciates persistently and a few quarters later. The appreciation of the exchange rate leads to a rising in imports and a decreasing in exports. With a negative technology shock, our model highlights both wealth and substitution effects. Substitution effect, which is a consequence of the appreciation of the
exchange rate, leads to an increasing in imported goods. Wealth effect leads to a reduction of household’s consumption in short run. All things considered, aggregate output, investment and consumption decrease sharply. Following a negative technology shock, exchange rate channel plays an amplification role in the mechanism of shocks propagation.

5.2.2 Expansionary Monetary Policy

Figures 7 and 8, which display impulse responses of a 1% positive domestic monetary shock, points out that monetary policy has direct effects on aggregate spending and output that operate through interest rate channel and exchange rate channel. This expansionary monetary policy could illustrate the recent quantitative easing undertaken by the Federal Reserve and by the most of others central banks. A decreasing in the domestic interest rate drives down the cost of deposits and bank credit supply increases. Households consumption as well as bankers’ and entrepreneurs’ consumption, aggregate investment move up. Exchange rate depreciates, which followed by an increasing in the exports goods and in a decreasing in the imports goods.

5.3 Transmission of International Shocks

With the contagion phenomena of the recent financial crisis, the knowledge of the effects of the transmission of international shocks has became one of the most important issues for both academicians and central banks. This section illustrates the contribution of the banking sector, especially the role of the bank capital in the transmission of international shocks. For this task, we focus on the impact of foreign monetary policy and a foreign demand shocks in both model (1) and model (3). As indicated before, model (3) illustrates a small open economy with a strong bank capital regulation, which is an economy with much more bank capital and model (1) is our baseline model. Figures 9 and 10 display impulse responses functions for a tightening foreign monetary policy and a negative foreign demand shocks, respectively.

5.3.1 Foreign Demand Shock

Figure 9, which gives the impulse responses to a negative foreign demand shock leads to a decreasing in exports by 1%, a decreasing in output by 0.8% and a decreasing in households’
consumption by 0.15%. The decrease in the domestic output produces a negative output gap and a rise in the prices, which creates an inflationary pressure. As result, central bank will react by a tightening monetary policy, which leads to higher interest rate and higher cost of deposits. Bank credit supply declines and loan default goes up, which deteriorates banks’ balance sheets. As consequence, banks’ net worth falls as well as entrepreneurs’ net worth, which produces a rise of the leverage ratio. Overall, a negative foreign demand shock has a negative impact on the aggregate lending, which drives down investment in equilibrium by 4%. Higher interest prices and interest rates are followed by an appreciation of the real exchange rate, which leads to a decrease in imports by substituting foreign factors by domestic factors of production. In absolute value, exports decrease by more than imports following the negative foreign demand shock, leading to a negative impact on the current account. The negative impact of the foreign demand shock is smaller in the model with more bank capital than in the baseline model. Also, In the model with more bank capital, return to equilibrium is faster than in the baseline model. These results highlight the dampening effect of a strong bank capital regulation and is consistent to that found by Meh and Moran (2010) and Dib (2010) in closed economy.

5.3.2 Foreign Monetary Policy Shock

Figure 10 depicts the impulse responses following to a tightening foreign monetary policy. This shock could be interpreted as a policy originating by the foreign central to regulate foreign economy. Foreign monetary policy affects domestic aggregates through the exchange rate channel. An increase in the foreign interest rate increases foreign prices, which depreciates domestic real exchange rate in short run. As consequence, imports decrease and exports increases, which positively affects the current account. However, the decreasing in imports is much more important than the increasing in exports, leading by a decreasing in domestic output and investment. In short term, households wealth will increase, which produces an increase in household’s consumption. As in the case of negative foreign output shock, return to equilibrium is faster in the model with more bank capital than in the baseline model. However, effects following to a tightening foreign monetary policy are small than those of a negative foreign demand shock.
5.4 Sensitivity of the Bank Capital

The aim of this section is to highlight the relative contribution of bank capital in the dynamic of international shocks transmission. For this purpose, we compare the impulse responses of the model (1) and the model (3) following to a negative domestic technology shock, tightening domestic monetary policy and negative government spendings shock. Figures 11, 12 and 13 display the impulse responses of key domestic aggregates following to these aforementioned shocks, respectively. Overall, results suggest that bank capital plays a crucial role in the transmission of shocks as well as in the velocity of return to equilibrium. These results, which are consistent to those highlighted by Meh and Moran (2010) and Dib (2010) show that following a negative shock, bank capital plus a dampening effect. However, following a positive shock, bank capital plays an amplification role in the dynamic of shocks propagation. An economy with more bank capital has a better capacity to face against adverse shocks than an economy with less bank capital. This result, which remains valid for both the transmission of international shocks, highlights the importance of bank capital.

6 Conclusion

Recent empirical evidence suggests that the state of banks’ balance sheets plays an important role in the transmission of monetary policy and other shocks. This paper presents an international DSGE framework with credit market frictions and an active bank capital channel to assess issues regarding the transmission of domestic and foreign shocks. The theoretical framework includes the financial accelerator mechanism developed by Bernanke, Gertler and Gilchrist (1999), the bank capital channel and the exchange rate channel. This paper addresses the role played by the bank capital channel, the exchange rate channel, and the interest rate channel in the mechanism of both domestic and foreign shocks transmission. Specifically, we analyze the relative contribution of the bank balance sheets channel, the exchange rate channel, and the interest rate channel in the propagation of internal and external shocks. The starting point of our model is the microfounded framework developed by Meh and Moran (2010), Gertler and Kiyotaki (2010) and Dib (2010), to which we include a cross-border goods distribution, the
exchange rate channel and a government. The paper incorporates an active banking sector, the financial accelerator à la Bernanke, Gertler and Gilchrist (1999) (BGG (1999), hereafter), and the accumulation of capital in the spirit of Christiano, Motto and Rostagno (2005, 2010). The resulting model is calibrated to the Canadian economy.

The results of our simulations may be summarized as follow: 

(i) In the presence of the exchange rate channel, the propagation of both domestic and foreign shocks are amplified when comparing our baseline economy to a closed economy. 

(ii) Depending of the size of the bank capital in the economy, shocks that originate domestically have an important quantitative role in explaining domestic output, investment, banks lending, entrepreneurs and banks net worth, inflation and interest rates. 

(iii) External shocks (monetary policy shock and foreign demand shock) also contribute in explaining domestic aggregate fluctuations. 

(iv) Economies whose banks remains well-capitalized when affected by adverse shock experience less severe downturns, i.e., when the bank capital channel is active, an economy with more bank capital has a better capacity to face against adverse shocks than an economy with less bank capital. This result, which remains valid for the transmission of international shocks, highlights the importance of the bank capital in an international framework and can be used to perform the worldwide debate over the banking regulation. Our simulations mimic most of evidences observed along the recent financial crisis and its heterogeneous impact among economies.

Finally, future research could allow the model to take into account the heterogeneity in banks’ capitalization that characterizes banking sectors by developing a quantitative two-country model with financial frictions and endogenous portfolio choice in both domestic and foreign economy. The present model does not include such heterogeneity, but important further insights and policy implications could potentially be gained from such model.
References


7 Appendix

7.1 Appendix 1: Business cycles comovement

Figure 3: Business cycles synchronization
7.2 Appendix 2: Bank capital channel assessment

Figure 4: Responses to a negative bank capital shock (in percent deviations from steady state)
7.3 Appendix 3: International transmission of domestic shocks

Figure 5: Responses to a negative productivity shock (in percent deviations from steady state)
Figure 6: Responses to a negative productivity shock (in percent deviations from steady state)
Figure 7: Responses to a monetary policy shock (in percent deviations from steady state)
Figure 8: Responses to a monetary policy shock (in percent deviations from steady state)
7.4 Appendix 4: Transmission of international shocks

Figure 9: Responses to a negative foreign output shock: Baseline Economy vs Economy with more bank capital.
Figure 10: Responses to a negative foreign monetary policy shock: Baseline Economy vs Economy with more bank capital
7.5 Appendix 5: Properties of bank capital

Figure 11: Responses to a negative technology shock: Baseline economy vs Economy with more bank capital

- Output
- Consumption
- Investment
- Entp_NetWorth
- Bank_NetWorth
- Bank_Lending
- Export
- Import
- Exchange_Rate

Steady state line Baseline economy Economy with more bank capital
Figure 12: Responses to a positive monetary policy shock: Baseline Economy vs Economy with more bank capital
Figure 13: Responses to a negative government spending shock: Baseline Economy and Economy with more bank capital

- **Output**: The graph shows the output response over time, with the baseline economy (solid line) and the economy with more bank capital (dashed line). The output decreases in the short run and then returns to the steady state.

- **Consumption**: Consumption decreases significantly in the short run and then recovers, with the baseline economy showing a larger initial decrease compared to the economy with more bank capital.

- **Investment**: Investment also decreases in the short run, with the baseline economy showing a sharper decline.

- **Entp_NetWorth**: Entrepreneur net worth decreases initially in both economies but recovers more quickly in the economy with more bank capital.

- **Bank_NetWorth**: Bank net worth decreases sharply in both economies, with the baseline economy showing a more pronounced decrease.

- **Bank_Lending**: Bank lending decreases sharply in both economies, with the baseline economy showing a more pronounced decrease.

- **Export**: Exports decrease initially, with the baseline economy showing a larger initial decrease.

- **Import**: Imports decrease initially, with the baseline economy showing a larger initial decrease.

- **Exchange_Rate**: The exchange rate decreases initially in both economies, with a more pronounced decrease in the baseline economy.

Legend:

- **Steady state line**
- **Baseline economy**
- **Economy with more bank capital**