Nominal Rigidities and Exchange Rate Pass-Through in a Structural Model of a Small Open Economy*

Steve Ambler† Ali Dib‡ Nooman Rebei§

September 2003

Abstract

We analyze exchange rate pass-through in a structural model of a small open economy with rigid wages and prices. The model is estimated using quarterly Canadian and U.S. data. It predicts a remarkably similar dynamic relationship between the nominal exchange rate and prices in response to the model’s eight different structural shocks: the nominal exchange rate overshoots its long-run level, and changes in the nominal exchange rate are passed through slowly to the domestic price level. Pricing to market is necessary for slow pass-through to the prices of imported goods, but not for slow pass-through to the overall price level. Sticky wages by themselves generate slow pass-through, even when the prices of imported goods adjust instantaneously to changes in the exchange rate.

JEL classification: F2, F31, F33

Key words: Business fluctuations and cycles; Economic models; Exchange rates; Inflation and prices; International topics

---

*We thank Brian Doyle, Hafedh Bouakez, and seminar participants at the Bank of Canada and the Canadian Economics Association annual meetings for helpful comments. The corresponding author is Ambler.
†CIRPEE, Université du Québec à Montréal, C.P. 8888 Succ. Centre-Ville, Montréal, QC, CANADA H3C 3P8, url http://www.er.uqam.ca/nobel/r10735/, e-mail ambler.steven@uqam.ca, tel. (514) 987-3000 ext.8372, fax 987-8494.
‡Monetary and Financial Analysis Department, Bank of Canada. 234 Wellington St. Ottawa, ON, K1A 0G9, url http://www.bankofcanada.ca/adib/, e-mail adib@bank-banque-canada.ca, tel. (613) 782-7851.
§Research Department, Bank of Canada. 234 Wellington St. Ottawa, ON, K1A 0G9, url http://www.bankofcanada.ca/nrebei/, e-mail nrebei@bank-banque-canada.ca, tel. (613) 782-8871.
1. Introduction

The pass-through of exchange rate changes to import prices and to domestic inflation is of obvious interest to central banks with inflation targets. Slow pass-through also has important consequences in theoretical models of optimal monetary policy. A large literature analyzes optimal monetary policy in the context of the New Open-Economy Macroeconomics (NOEM), a class of open-economy dynamic general-equilibrium models with explicit microfoundations, nominal rigidities, and imperfect competition.\(^1\) For example, Smets and Wouters (2002) show that optimal monetary policy with sticky domestic- and imported-goods prices involves the minimizing of a weighted average of domestic and import price inflation.\(^2\) This result provides a rationale for exchange rate stabilization, and qualifies the results of Galí and Monacelli (1999), who used a model with instantaneous pass-through of exchange rate changes to show that optimal monetary policy is identical in open and closed economies and involves stabilizing the overall price level, without regard to exchange rate fluctuations.

In this paper, we analyze exchange rate pass-through in a NOEM model of a small open economy with three types of nominal rigidity: wages and domestic and imported goods prices are all set in advance by monopolistically competitive agents. We attempt to answer three related questions raised by the recent theoretical and empirical literature. First, the joint dynamics of the exchange rate and of prices may differ in response to different structural shocks, while empirical studies of exchange rate pass-through typically focus on reduced-form equations\(^3\) or vector autoregressions with a limited number of error terms.\(^4\) This raises the question of whether this simplification is justified. We address this question by incorporating eight different structural shocks. Second, there competing definitions of pass-through in the literature. A narrow definition\(^5\) is the transmission of changes in the exchange rate to imported goods prices. A broader definition\(^6\) is the transmission of exchange rate changes to the overall price level, either the producer price index (PPI) or the consumer price index (CPI). This raises the question of the extent to which the dynamic responses of different price indexes are similar. We simulate the response of the import price index, the PPI

\(^1\)The NOEM literature, spawned by the pioneering work of Obstfeld and Rogoff (1995), has been successful in explaining phenomena such as high real exchange rate volatility and persistence and the strong impact of monetary policy shocks on real exchange rates. See Sarno (2001), Lane (2001), and Bowman and Doyle (2003) for recent surveys.

\(^2\)Similarly, Corsetti and Pesenti (2001) show in a similar model that it is optimal for the central bank to minimize a CPI-weighted average of markups charged in the domestic market by domestic and foreign producers.

\(^3\)Campa and Goldberg (2001), Bailliu and Fujii (2003).

\(^4\)For example, Kim (1998) and McCarthy (2000).

\(^5\)See for example Campa and Goldberg 2001.

\(^6\)See for example Devereux (2001) and Devereux and Yetman (2003)
and the CPI, along with the nominal exchange rate, to compare the joint dynamics of all four variables. Third, the recent theoretical literature on imperfect pass-through has stressed the role of pricing to market (PTM), which assumes that the prices of imported goods are set in advance in the local currency.\textsuperscript{7} It has been almost axiomatic that PTM is both necessary and sufficient to generate imperfect pass-through in the dynamic general-equilibrium model. This raises the question of whether other mechanisms can also generate slow pass-through. We investigate the roles of the three different types of nominal rigidity in generating slow pass-through.

Our main findings are as follows. The response of the exchange rate and prices to the model’s different structural shocks is remarkably similar. The nominal exchange rate overshoots, with its maximum response occurring during the period in which the shock hits. In contrast, import prices and the domestic price level adjust gradually. In most cases, the path followed by the price level is hump-shaped. The adjustment of the price level is more gradual than that of the price of imports. While PTM is necessary for slow pass-through to imported goods prices and sufficient for slow pass-through to the CPI, it is not necessary for the latter. Sticky wages and domestic prices also generate slow exchange rate pass-through to the CPI, even when the prices of imported goods adjust instantaneously to changes in the exchange rate.

We estimate most of the model’s structural parameters with Canadian and U.S. data using a combination of the generalized method of moments (GMM) and the simulated method of moments (SMM).\textsuperscript{8} The model passes a \textit{J}-test of its overidentifying restrictions. It can explain several other features of the data. It generates a large amount of persistence of the real exchange rate, of real variables such as output, and of inflation. The nominal and real exchange rates generated by the model are highly correlated, as in the data.

This paper is organized as follows. In section 2, we present the model. In section 3, we discuss the estimation strategy used to attribute values to the model’s structural parameters. Simulation results and sensitivity analyses are described in section 4. Section 5 offers some conclusions.

2. The Model

The economy faces fixed prices on world markets for imported goods. Its domestic output is an imperfect substitute for foreign goods, and it faces a downward-sloping demand curve for its output on world markets.\textsuperscript{7} Implicitly, this assumes that goods arbitrage is not feasible at least in the short run.\textsuperscript{8} Ambler, Guay, and Phaneuf (2003) use a similar methodology to estimate a closed-economy business cycle model.
It also faces an upward-sloping supply curve for funds on international capital markets.

Different labour types are associated with particular households that act as monopolistic competitors in the labour market. Differentiated intermediate goods are produced by monopolistically competitive domestic firms using labour and a final composite good as inputs. Differentiated intermediate goods are also imported by monopolistically competitive importers. Domestic and imported intermediate goods are aggregated by competitive firms to form a composite domestic and a composite imported good. Some of the composite domestic good is exported. The remainder is combined with the composite imported good to form the final good. As in McCallum and Nelson (1999, 2001), imports enter the production process rather than being consumed directly. The final good is used for consumption, government consumption, and as an input into the production of domestic intermediate goods.

There are therefore three sources of monopoly distortion and nominal rigidity. Households set wages in advance, and importers domestic intermediate goods producers set prices in advance. Following Calvo (1983), price and wage setters maintain constant prices and wages unless they receive a signal to revise them, which arrives at the beginning of each period with a constant probability. This assumption makes aggregation simple, allows us easily to vary the average duration of the nominal rigidities, and allows us to estimate the length of the nominal rigidities along with other structural parameters of the model.

### 2.1 Households

There is a continuum of different households on the unit interval, indexed by \( j \). The \( j \)th household’s preferences are given by:

\[
U_0(j) = E_0 \sum_{t=0}^{\infty} \beta^t u \left( C_t(j), \frac{M_t(j)}{P_t}, h_t(j) \right),
\]

where \( \beta \) is the discount factor, \( E_0 \) is the conditional expectations operator, \( C_t(j) \) is consumption, \( M_t(j) \) denotes nominal money balances held at the end of the period, \( P_t \) is the price level, and \( h_t(j) \) denotes hours worked by the household. The single-period utility function is:

\[
u(\cdot) = \frac{\gamma}{\gamma - 1} \log \left( C_t(j)^{\gamma - 1} + b_t^\lambda \left( \frac{M_t(j)}{P_t} \right)^{\gamma - 1} \right) + \eta \log (1 - h_t(j)) ,
\]

where \( \gamma \) and \( \eta \) are positive parameters. Total time available to the household in the period is normalized to one. The \( b_t \) term acts is a shock to money demand. It follows the first-order autoregressive process given

\footnote{Bergin (2002) and Kollmann (2002) develop similar models.}
by:

\[
\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt},
\]

where the serially uncorrelated shock, \(\varepsilon_{bt}\), is normally distributed with zero mean and standard deviation \(\sigma_b\). The household’s budget constraint is given by:

\[
P_tC_t(j) + M_t(j) + \frac{D_t^p(j)}{R_t} + \frac{e_tB_t^*(j)}{\kappa_t R_t^*} = (1 - \tau_t)W_t(j)h_t(j, \cdot) + M_{t-1}(j) + D_{t-1}^p(j) + e_{t-1}B_{t-1}^*(j) + T_t + D_t,
\]

Labour income is taxed at an average marginal tax rate, \(\tau_t\). \(B_t^*\) and \(D_t^p\) are foreign-currency and domestic-currency bonds purchased in \(t\), and \(e_t\) is the nominal exchange rate. Domestic-currency bonds are used by the government to finance its deficit. \(R_t\) and \(R_t^*\) denote, respectively, the gross nominal domestic and foreign interest rates between \(t\) and \(t+1\); \(\kappa_t\) is a risk premium that reflects departures from uncovered interest parity. The household also receives nominal profits \(D_t = D_t^d + D_t^m\) from domestic producers and importers of intermediate goods, and \(T_t\) in nominal lump-sum transfers from the government. The risk premium depends on the ratio of net foreign assets to domestic output:

\[
\log(\kappa_t) = \varphi \left[ \exp \left( \frac{e_tB_t^*}{P_t^d Y_t} \right) - 1 \right],
\]

where \(P_t^d\) is the GDP deflator or domestic output price index. The risk premium ensures that the model has a unique steady state. If domestic and foreign interest rates are equal, the time paths of domestic consumption and wealth follow random walks.\(^{10}\)

The foreign nominal interest rate, \(R_t^*\), evolves according to the following stochastic process:

\[
\log(R_t^*) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R_{t-1}^*) + \varepsilon_{R^*t},
\]

where the serially uncorrelated shock, \(\varepsilon_{R^*t}\), is normally distributed with zero mean and standard deviation \(\sigma_{R^*}\).

Household \(j\) chooses \(C_t(j), M_t(j), D_t^p(j),\) and \(B_t^*(j)\) (and \(W_t(j)\) if it is allowed to change its wage) to maximize the expected discounted sum of its utility flows subject to the budget constraint, equation (4), to

\(^{10}\)For an early discussion of this problem, see Giavazzi and Wyplosz (1984). Our risk premium equation is similar to the one used by Senhadji (1997). For alternative ways of ensuring that stationary paths exist for consumption in small open-economy models, see Schmitt-Grohé and Uribe (2003).
intermediate firms’ demand for their labour type, and subject to a transversality condition on their holdings of assets. Aggregate labour is given by:

$$\hat{h}_t = \left( \int_0^1 \hat{h}_t(j) \frac{e^{-\sigma}}{\hat{h}_t(j)} dj \right)^{\frac{1}{\sigma-1}},$$  

so that $\sigma$ is the elasticity of substitution between different labour skills. This implies the following conditional demand for labour of type $j$:

$$\hat{h}_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\sigma} \hat{h}_t,$$

where $\hat{h}_t$ is aggregate employment. $W_t$ is an exact average wage index given by:

$$W_t = \left( \int_0^1 W_t(j)^{1-\sigma} dj \right)^{\frac{1}{\sigma-1}}.$$

The household’s first-order conditions are:

$$\frac{C_t(j)^{1-\frac{1}{\sigma}}}{C_t(j)^{\frac{1}{\sigma-1}} + b_t^\frac{1}{\sigma} \left( \frac{M_t(j)}{P_t} \right)^{\frac{1}{\sigma-1}}} = \Lambda_t(j) \frac{P_t}{P_t^d};$$

$$\frac{b_t^\frac{1}{\sigma} \left( \frac{M_t(j)}{P_t} \right)^{\frac{1}{\sigma-1}} + b_t^\frac{1}{\sigma} \left( \frac{M_t(j)}{P_t} \right)^{\frac{1}{\sigma-1}}}{C_t(j)^{\frac{1}{\sigma-1}} + b_t^\frac{1}{\sigma} \left( \frac{M_t(j)}{P_t} \right)^{\frac{1}{\sigma-1}}} = \Lambda_t(j) - \beta E_t \left[ \frac{P_t^d}{P_{t+1}^d} \Lambda_{t+1}(j) \right];$$

$$\frac{\Lambda_t(j)}{R_t} = \beta E_t \left[ \frac{P_t^d}{P_{t+1}^d} \Lambda_{t+1}(j) \right];$$

$$\frac{\Lambda_t(j)}{\kappa_t R_t} = \beta E_t \left[ \frac{P_t^d}{P_{t+1}^d} \right] \Lambda_{t+1}(j),$$

where $\Lambda_t(j)$ is the Lagrange multiplier associated with the time $t$ budget constraint. With probability $d_w$ the household is allowed to set its wage. The first order condition is:

$$\tilde{W}_t(j) = \left( \frac{\sigma}{\sigma-1} \right) \frac{E_t \sum_{l=0}^\infty (\beta d_w)^l \frac{\partial \Lambda_{t+1}(j)}{\partial R_t}}{E_t \sum_{l=0}^\infty (\beta d_w)^l (1 - \tau_{t+1}) h_{t+1}(j) \Lambda_{t+1}(j)/P_{t+1}^d};$$

This first-order condition gives a New Keynesian Phillips curve for wage inflation (see section (2.6)). The wage index evolves over time according to:

$$W_t = \left[ d_w (W_{t-1})^{1-\sigma} + (1-d_w)(\tilde{W}_t)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where $\tilde{W}_t$ is the average wage of those workers who revise their wage at time $t$.  

2.2 Goods production

2.2.1 Domestic intermediate goods

Firms have identical production functions given by:

\[ Y_t(i) = X_t(i)^\phi (A_t h_t(\cdot, i))^{1-\phi}, \quad \phi \in (0, 1), \]

where \( h_t(\cdot, i) \) is the quantity of the aggregate labour input employed by firm \( i \) and \( X_t(i) \) is the quantity of the final composite good used by firm \( i \).\(^{11}\) \( A_t \) is an aggregate technology shock that follows the stochastic process given by:

\[ \log A_t = A + \log(A_{t-1}) + \varepsilon_{At}, \]

where \( \varepsilon_{At} \) is a normally distributed, serially uncorrelated shock with zero mean and standard deviation \( \sigma_A \). The firm chooses \( X_t(i) \) and \( h_t(\cdot, i) \) to maximize its stock market value. When allowed to do so (with probability \( d_p \) each period), it also chooses the price of its output, \( \tilde{P}^d_t(i) \). It solves:

\[
\max_{\{X_t(i), h_t(\cdot, i), \tilde{P}^d_t(i)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta d_p)^l \left( \frac{\Lambda_{t+l}}{\Lambda_t} \right) \frac{D^d_{t+l}(i)}{\tilde{P}^d_{t+l}} \right],
\]

where \( \Lambda_t \) is the marginal utility of wealth for a representative household, and

\[ D^d_{t+l}(i) = \tilde{P}^d_t(i) Y_{t+l}(i) - W_{t+l} h_{t+l}(\cdot, i) - P_{t+l} X_{t+l}(i), \]

where \( P_t \) is the price of the final output good, \( Z_t \). The maximization is subject to the firm’s production function and to the derived demand for the firm’s output (discussed in section (2.2.3)) given by:

\[ Y_{t+l}(i) = \left( \frac{\tilde{P}^d_t(i)}{\tilde{P}^d_{t+l}} \right)^{-\theta} Y_{t+l}, \]

where \( P^d_t \) is the exact price index of the composite domestic good. The elasticity of the derived demand for the firm’s output is \(-\theta\). The first-order conditions are:

\[ \frac{W_t}{\tilde{P}^d_t} = \xi_t(i) (1 - \phi) \frac{Y_t(i)}{h_t(\cdot, i)}; \]

\[ \frac{P_t}{\tilde{P}^d_t} = \xi_t(i) \phi \frac{Y_t(i)}{X_t(i)}; \]

\(^{11}\)We include \( X_t(i) \) in the production of domestic intermediates for two reasons. First, without \( X_t(i) \), the response of the real wage to demand shocks is too highly countercyclical. Second, as shown in similar models by McCallum and Nelson (1999, 2001), the presence of intermediates in the production function for domestic goods affects the correlation between the nominal exchange rate and domestic inflation.
\[ \tilde{P}^d_t(i) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{l=0}^{\infty}(\beta_t^d)^l \left( \frac{\Delta_t^{i+1}}{\lambda_t} \right) \xi_{t+1}(i) Y_{t+1}(i)}{E_t \sum_{l=0}^{\infty}(\beta_t^d)^l \left( \frac{\Delta_t^{i+1}}{\lambda_t} \right) Y_{t+1}(i)/P_{t+1}^d}, \]  

where \( \xi_t(i) \) is the Lagrange multiplier associated with the production function constraint. It measures the firm’s real marginal cost. The first-order condition with respect to the firm’s price relates the price to the expected future price of final output and to expected future real marginal costs. It can be used to derive a New Keynesian Phillips curve relationship for the rate of change of domestic output prices (see section (2.6)).

### 2.2.2 Imported intermediate goods

The economy imports a continuum of foreign intermediate goods on the unit interval. There is monopolistic competition in the market for imported intermediates, which are imperfect substitutes for each other in the production of the composite imported good, \( Y_t^m \), produced by a representative competitive firm. When allowed to do so (with probability \( d_m \) each period), the importer of good \( i \) sets the price, \( \tilde{P}_t^m(i) \), to maximize its weighted expected profits. It solves:

\[
\max_{\{P_t^m(i)\}} E_t \left[ \sum_{l=0}^{\infty} (\beta_t^m)^l \left( \frac{\Lambda_t}{\Lambda_t^l} \right) D_{t+1}^m(i) D_{t+1}^m(i) / P_{t+1}^d \right],
\]

where:

\[
D_{t+1}^m(i) = \left( \tilde{P}_t^m(i) - e_{t+1}P_{t+1}^* \right) \left( \frac{P_t^m(i)}{P_{t+1}^m} \right)^{-\theta} Y_{t+1}^m.
\]

For convenience, we assume that the price in foreign currency of all imported intermediates is \( P_t^* \), which is also equal to the foreign price level. The elasticity of the derived demand for the imported good, \( i \), is \(-\theta\).

The first-order condition is:

\[
\tilde{P}_t^m(i) = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{l=0}^{\infty}(\beta_t^m)^l \left( \frac{\Delta_t^{i+1}}{\lambda_t} \right) Y_{t+1}(i)e_{t+1}P_{t+1}^*/P_{t+1}^d}{E_t \sum_{l=0}^{\infty}(\beta_t^m)^l \left( \frac{\Delta_t^{i+1}}{\lambda_t} \right) Y_{t+1}(i)/P_{t+1}^d}.
\]

This equation can be used to derive a New Keynesian Phillips curve relationship for the rate of change of intermediate input prices (see section (2.6)).

### 2.2.3 Composite goods

The composite domestic good, \( Y_t \), is produced using a constant elasticity of substitution (CES) technology with a continuum of domestic intermediate goods, \( Y_t(i) \), as inputs:

\[
Y_t = \left( \int_0^1 Y_t(i) \frac{d(i)}{\lambda_t} \right)^{\frac{1}{\theta_t}}.
\]
It is produced by a representative competitive firm that solves:

\[
\max_{\{Y_t(i)\}} P_t^d Y_t - \int_0^1 P_t^d(i)Y_t(i)di,
\]  

subject to the production function (24). The first-order conditions yield the derived demand functions for the domestic intermediate goods given by (17). The exact price index for the composite domestic good is:

\[
P_t^d = \left( \int_0^1 P_t^d(i)^{1-\theta} di \right)^\frac{1}{1-\theta}.
\]  

The price level obeys the following law of motion:

\[
P_t^d = \left[ d_p(P_{t-1}^d)^{1-\theta} + (1 - d_p)(\tilde{P}_t^d)^{1-\theta} \right]^\frac{1}{1-\theta},
\]  

where \(\tilde{P}_t^d\) is the price index derived by aggregating over all firms that change their price at time \(t\).

Composite domestic output, \(Y_t\), is divided between domestic use, \(Y_t^d\), and exports, \(Y_t^x\). Foreign demand for domestic exports is:

\[
Y_t^x = \alpha_x \left( \frac{P_t^d}{\epsilon_t P_t^*} \right)^{-\varsigma} Y_t^*,
\]  

where \(Y_t^*\) is foreign output. The elasticity of demand for domestic output is \(-\varsigma\), and \(\alpha_x > 0\) is a parameter determining the fraction of domestic exports in foreign spending. Domestic exports form an insignificant fraction of foreign expenditures, and have a negligible weight in the foreign price index.

The foreign variables \(P_t^*\) and \(Y_t^*\) are both exogenous and, when stationarized, evolve according to

\[
\log(P_t^*/P_{t-1}^*) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(P_{t-1}^*/P_{t-2}^*) + \varepsilon_{\pi^*t},
\]  

and

\[
\log Y_t^* = (1 - \rho_{y^*}) \log(Y^*) + \rho_{y^*} \log(Y_{t-1}^*) + \varepsilon_{y^*t},
\]  

where \(\pi^*\) is steady-state foreign inflation, and \(\varepsilon_{\pi^*t}\) and \(\varepsilon_{y^*t}\) are zero-mean, serially uncorrelated shocks with standard errors \(\sigma_{\pi^*}\) and \(\sigma_{y^*}\), respectively.

The composite imported good, \(Y_t^m\), is produced using a CES technology with a continuum of imported-intermediate goods, \(Y_t^m(i)\), as inputs:

\[
Y_t^m \leq \left( \int_0^1 (Y_t^m(i))^{\frac{\varphi-1}{\varphi}} di \right)^\frac{1}{\varphi}.
\]  

---

12This condition can be derived from a foreign importing firm that combines non-perfectly substitutable imported goods.

13To ensure the existence of a balanced growth path for the economy, we assume that foreign output grows at the same trend rate as domestic output.
It is produced by a representative competitive firm. Its profit maximization gives the derived demand function for intermediate imported good \( j \) given by:

\[
Y_m^m(i) = \left( \frac{P_m^m(i)}{P_t^m} \right)^{-\theta} Y_t^m. \tag{32}
\]

The exact price index for the composite imported goods is given by:

\[
P^m = \left( \int_0^1 P_m(i)^{1-\theta} di \right)^{1/\theta}. \tag{33}
\]

The price index obeys the following law of motion:

\[
P^m_t = \left[ d_m (P^m_{t-1})^{1-\theta} + (1 - d_m) (\tilde{P}_t^m)^{1-\theta} \right]^{1/\theta}, \tag{34}
\]

where \( \tilde{P}_t^m \) is a price index derived by aggregating over all importers that change their price in time \( t \).

### 2.2.4 Final-goods production

The final good, \( Z_t \), is produced by a competitive firm that uses \( Y^d_t \) and \( Y^m_t \) as inputs subject to the following CES technology:

\[
Z_t = \left[ \alpha_d (Y^d_t)^{\frac{1}{\nu}} + \alpha_m (Y^m_t)^{\frac{1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{35}
\]

where \( \alpha_d > 0, \alpha_m > 0, \nu > 0, \) and \( \alpha_d + \alpha_m = 1 \). The final good, \( Z_t \), is used for domestic consumption, \( C_t \), as inputs to produce domestic intermediate goods, \( X_t \), and government purchases, \( G_t \). The final good is produced by a competitive firm that solves:

\[
\max_{Y^d_t, Y^m_t} P_t Z_t - P^d_t Y^d_t - P^m_t Y^m_t, \tag{36}
\]

subject to the production function (35). Profit maximization entails:

\[
Y^d_t = \alpha_d \left( \frac{P^d_t}{P_t} \right)^{-\nu} Z_t, \tag{37}
\]

and

\[
Y^m_t = \alpha_m \left( \frac{P^m_t}{P_t} \right)^{-\nu} Z_t. \tag{38}
\]

Furthermore, the final-good price, \( P_t \), is given by:

\[
P_t = \left[ \alpha_d (P^d_t)^{1-\nu} + \alpha_m (P^m_t)^{1-\nu} \right]^{1/(1-\nu)}. \tag{39}
\]
2.3 Monetary authority

Following Taylor (1993), Dib (2003) and Ireland (2003), the central bank manages the short-term nominal interest rate, $R_t$, in response to deviations of detrended output, $y_t = Y_t/A_t$; inflation, $\pi_t = P_t/P_{t-1}$; money growth, $\mu_t = M_t/M_{t-1}$; and the real exchange rate, $s_t = e_t P^*_t/P_t$. Its interest rate reaction function is given by:

$$\log\left(\frac{R_t}{R}\right) = \vartheta_y \log\left(\frac{y_t}{y}\right) + \vartheta_\pi \log\left(\frac{\pi_t}{\pi}\right) + \vartheta_\mu \log\left(\frac{\mu_t}{\mu}\right) + \vartheta_s \log\left(\frac{s_t}{s}\right) + \varepsilon_{Rt},$$

(40)

where $y, \pi, \mu,$ and $s$ are the steady-state values of $y_t, \pi_t, \mu_t,$ and $s_t$, where $R$ is the steady-state value of the gross nominal interest rate, and where $\varepsilon_{Rt}$ is a zero-mean, serially uncorrelated monetary policy shock with standard deviation $\sigma_R$.

2.4 The government

The government budget constraint is given by:

$$P_t G_t + T_t + D^g_{t-1} = \tau_t W_t h_t + M_t - M_{t-1} + \frac{D^g_0}{R_t}.$$  

(41)

The left side of (41) represents uses of government revenue: goods purchases, transfers, and debt repayments. The right side includes tax revenues, money creation, and newly issued debt. The government also faces a no-Ponzi constraint that implies that the present value of government expenditures equals the present value of tax revenue plus the initial stock of public debt, $D^g_0$.

Because households have infinite horizons, there is Ricardian equivalence, and we can simplify the budget constraint without loss of generality to:

$$P_t G_t + T_t = \tau_t W_t h_t + M_t - M_{t-1}.$$  

(42)

This implies that $D^g_t$ is zero in each period. Government spending and the tax rate are determined by:

$$\log\left(\frac{G_t}{A_t}\right) = (1 - \rho_g) \log(g) + \rho_g \log\left(\frac{G_{t-1}}{A_{t-1}}\right) + \varepsilon_{gt},$$

(43)

and

$$\log(\tau_t) = (1 - \rho_\tau) \log(\tau) + \rho_\tau \log(\tau_{t-1}) + \varepsilon_{\tau t}.$$  

(44)

Given these stochastic processes and that the nominal money stock is determined by money demand once nominal interest rate is set, lump-sum taxes are determined residually to balance the government’s budget.
2.5 Equilibrium

There are three different stochastic trends in the model. The first is in the foreign price level, and arises from the specification of the stochastic process for \( P_t^* \) in terms of rates of change in equation (29). The second is in the price of domestic output and all other domestic nominal variables, and arises from the fact that the monetary authority adjusts the domestic nominal interest rate as a function of inflation, rather than the price level according to equation (40). The third is in domestic output and other domestic real variables, such as consumption and real balances). It comes from the unit root in the technology process given in equation (15).

Solving the model involves using stationary transformations of variables with unit roots. We use the following transformations:

\[
p_t = \frac{P_t}{P^*_t}, m_t = \frac{M_t}{(A_t P_t)}, \quad p^m_t = \frac{P^m_t}{P^*_t}, \quad p^d_t = \frac{P^d_t}{P^*_t}, \quad \pi_t = \frac{P_t}{P_{t-1}}, \quad \pi^*_t = \frac{P_t^*}{P^*_t}, \quad s_t = \frac{e_t}{P^*_t}, \quad \lambda_t = A_t \Lambda_t.
\]

All the other variables are transformed according to the following formula:

\[
x_t = \frac{X_t}{A_t}.
\]

The complete system of equations in stationary variables that characterize the model’s equilibrium are given in Appendix C, available on request from the authors.

2.6 New Keynesian Phillips Curves

The price- and wage-setting equations cannot be used directly to simulate the model since they involve infinite summations. By linearizing these equations around the steady-state values of the variables, and assuming zero inflation in the steady state, we obtain three New Keynesian Phillips curves relationships that determine the rates of inflation of locally produced goods intermediates, imported intermediates, and the nominal-wage index. Defining \( \pi^m_t = \frac{P^m_t}{P^*_t}, \) and \( \pi^w_t = \frac{W_t}{W^*_t}, \) we get:

\[
\hat{\pi}_t^d = \beta \hat{\pi}_{t+1}^d + \frac{(1 - \beta d_p)(1 - d_p)}{d_p} \hat{\xi}_t ; \quad (45)
\]

\[
\hat{\pi}_t^m = \beta \hat{\pi}_{t+1}^m + \frac{(1 - \beta d_m)(1 - d_m)}{d_m} \hat{s}_t ; \quad (46)
\]

and

\[
\hat{\pi}_t^w = \beta \hat{\pi}_{t+1}^w + \frac{(1 - \beta d_w)(1 - d_w)}{d_w} \frac{1}{\left( \frac{h}{1-h} \right) \hat{h}_t + \left( \frac{\tau}{1-\tau} \right) \hat{\tau}_t - \hat{w}_t},
\]

\[14\]This includes foreign output, which must grow at the same trend rate as domestic output for the economy to be on a balanced growth path in the long run.
where hats over variables denote deviations from steady-state values. The New Keynesian Phillips curve for domestic output inflation is the same as in Galí and Gertler (1999). It relates inflation to expected future inflation and to the real marginal cost of output. The equation for import price inflation is analogous, with real marginal cost captured by the real exchange rate. The wage inflation equation is also analogous. The term in square brackets measures the marginal rate of substitution (the real marginal cost to workers of their work effort) minus the real wage. The household’s first-order condition for the nominal wage can be interpreted as a markup over the average marginal cost of work effort over the life of the wage contract.

3. Parameter Estimation and Simulation Methodology

The model cannot be solved analytically. Our results are derived by numerical simulations, which entails assigning numerical values to the model’s structural parameters. For this, we use a combination of estimation and calibration. A small number of parameters were impossible to identify econometrically. For these parameters, used calibrated values based on previous results in the literature. Conditional on these values, estimated the remaining structural parameters of the model using a combination of GMM and SMM techniques, using quarterly Canadian and U.S. data.\textsuperscript{15} We first describe our empirical methodology, and then discuss the parameter values themselves, which are summarized in Tables 1 through 3.

First step, we estimated the parameters of the exogenous stochastic processes involving observable variables, using standard GMM techniques. Then, we used SMM to estimate the remaining structural parameters.

The model has five exogenous stochastic processes involving observable variables: \( R^*_t, \pi^*_t, y^*_t, g_t, \) and \( \tau_t \). The stochastic processes for the logs of these variables all take the following form:

\[
\begin{align*}
\log z_t &= (1 - \rho_z)z + \rho_z z_{t-1} + \epsilon_t,
\end{align*}
\]

where \( z \) is the unconditional sample mean of the (log of) variable \( z_t \). To obtain precise estimates of the persistence parameters, \( \rho_z \), we use moment conditions of the following form:

\[
E[z_{t-1} (z_t - (1 - \rho_z)z - \rho_z z_{t-1})] = 0.
\]

We use the first lag of \( z_t \) as the only instrument to achieve exact identification. We use moment conditions

\textsuperscript{15}Data sources are described in Appendix A.
of the following form to estimate the unconditional means of the \( z \) variables:

\[
E \left[ 1 \left( z_t - z \right) \right] = 0,
\]

with a vector of ones \( (1) \) as the instrument. To obtain precise estimates of the variances of the innovations to the stochastic processes, we impose the following moment conditions:

\[
E \left[ \left( z_t - (1 - \rho_z)z - \rho_z z_{t-1} \right)^2 - \sigma_z^2 \right] = 0.
\]

Conditional on the calibrated parameter values and the GMM estimates of the parameters of the stochastic shock processes, the SMM estimator of the parameter vector \( (\Theta) \) is the solution to the following problem:

\[
\hat{\Theta}_T = \arg \min_{\Theta} \left( \frac{1}{T} \sum_{t=1}^{T} f(y_t, \Theta) \right)' W_T \left( \frac{1}{T} \sum_{t=1}^{T} f(y_t, \Theta) \right),
\]

where \( W_T \) is a random non-negative symmetric matrix, \( y_t \) is a vector containing a subset of the model’s variables, and \( f(y_t, \Theta) \) is a \( q \)-vector of unconditional moment restrictions. Most of the moment restrictions involve the difference between an unconditional moment predicted by the model and the corresponding moment in the data, where the predicted moments are calculated for given parameter values using the linearized version of the model without simulating. \( W_T \) is the inverse of the variance-covariance matrix of the moment conditions evaluated at a set of first-step estimates, in which \( W_T \) is set equal to the identity matrix. This matrix is consistently estimated following Newey and West (1994). Heuristically, it gives more weight to moments that are precisely estimated in the data.

This methodology has several attractive features. First, it allows flexibility in selecting the moments used in the estimation. These can include unconditional means, variances, covariances, and autocovariances of any of the variables in the model. Information from the data can be used in the estimation that cannot be used with some alternative methods. For example, matching the impulse-response functions of variables in the model with those of an estimated vector autoregression (VAR) reproduces as closely as possible the entire joint distribution of variables in the VAR. However, only as many variables can be included as there are structural shocks in the model: moments involving excluded variables cannot be used at all. Second, the method can rely on variables that are measured accurately. For instance, net foreign indebtedness is poorly measured in the data: moments involving this variable can be excluded from the estimation. Methods that use GMM directly to estimate the optimality or orthogonality conditions of structural models are forced to
use data on such poorly measured variables. Finally, when the dimension of the vector of moments \((q)\) is greater than the dimension of the vector of structural parameters, the overidentifying restrictions implied by the model can be tested formally using a \(J\)-test.\(^{16}\)

The parameters of the processes for \(A_t\) and \(b_t\) are estimated as part of our SMM procedure. Both processes involve unobserved variables, but can be expressed in terms of observable variables and one or more of the model’s structural parameters. Since they involve other structural parameters of the model, we could not estimate them in the first round by GMM along with the parameters of the other exogenous stochastic processes. In the case of the technology shock, we used the factor demand equations of the domestic intermediate-goods-producing firm, along with its production function, to solve for \(A_t\) in terms of observable variables. Using (14), (18), and (19) gives:

\[
\log(A_t) = \frac{1}{1 - \phi} \left[ \log(Y_t/h_t) - \phi \log(W_t/R_t) - \phi \log(\phi/(1 - \phi)) \right].
\]

Since technology follows a random walk with drift, this variable is non-stationary, but in first differences we have:

\[
\log(A_t) - \log(A_{t-1}) = A + \epsilon_{At}.
\]

We estimate \(A\) by adding the following moment condition:

\[
E \left[ \mathbb{1} \left( \Delta \log(A_t) - A \right) \right] = 0,
\]

and we estimate the variance of \(\epsilon_{At}\) using:

\[
E \left[ (\Delta \log(A_t))^2 - \sigma_A^2 \right] = 0.
\]

In the case of \(b_t\), we used (8), (9), and (10) to derive the following money-demand equation:

\[
\log(m_t/c_t) = \gamma \log(R_t/(R_t - 1)) + \log(b_t).
\]

Adding and subtracting the mean of the stochastic process generating \(\log(b_t)\), we get:

\[
\log(m_t/c_t) = \log(b) + \gamma \log(R_t/(R_t - 1)) + \log(b_t/b).
\]

\(^{16}\)Bergin (2002) estimates the parameters of a similar model using maximum-likelihood techniques. He tests its overidentifying restrictions by re-estimating an exactly identified version of the model and implementing a likelihood ratio test.
We treat $\log(b_t/b)$ as a mean-zero AR(1) error term. We estimated $\rho_b$ using:

$$E[R_{t-2} \left( \log(m_t/c_t) - \rho_b \log(m_{t-1}/c_{t-1}) - (1 - \rho_b) \log(b) + \gamma \log(R_t/(R_t - 1)) - \rho_b \gamma \log(R_{t-1}/(R_{t-1} - 1)) \right) \right] = 0.$$ 

We use the second lag of the nominal interest rate to achieve exact identification of $\rho_b$. We estimated the variance of the money-demand innovation using:

$$E \left[ \left( \log(m_t/c_t) - \rho_b \log(m_{t-1}/c_{t-1}) - (1 - \rho_b) \log(b) + \gamma \log(R_t/(R_t - 1)) - \rho_b \gamma \log(R_{t-1}/(R_{t-1} - 1)) \right)^2 \right] - \sigma^2_b = 0.$$ 

Finally, we estimated the constant term $\log(b)$ using the following moment condition:

$$E \left[ \log(m_t/c_t) - \gamma \log(R_t/(R_t - 1)) - \rho_b \gamma \log(R_{t-1}/(R_{t-1} - 1)) - \log(b) \right] = 0.$$ 

### 3.1 Parameter values

Table 1 summarizes the parameter values fixed by calibration. The subjective discount rate, $\beta$, is given a standard value, which implies an annual real interest rate of 4 per cent in the steady state. The weight on leisure in the utility function, $\eta$, is calibrated so that the representative household spends about one third of its total time working in the steady state. The foreign supply and demand parameters come from equation (28), which gives foreign demand for domestic exports, and from the risk premium equation (5), which relates to the elasticity of supply of foreign funds to the domestic economy: it also affects the economy’s net indebtedness in the long run. The $\alpha_x$ parameter is a normalization that ensures that the current account is balanced in the long run. The parameter $\varphi$ is set equal to -0.06, which gives an average risk premium of 93 basis points as in Clinton (1998).

The demand elasticities, $\sigma$, $\theta$, and $\vartheta$, influence the stochastic properties of the model in a very indirect way. After linearization, they no longer appear in the three Phillips curve equations. By influencing the size of the markups over marginal cost, they do influence the steady-state levels of the domestic production of intermediate goods, imported intermediate goods, and employment. Because certain coefficients in the linearized model depend on the steady-state levels of endogenous variables, the moments predicted by the model are related to these parameters. Unfortunately, the influence is so weak that it is impossible to estimate them precisely. The $\theta$ and $\vartheta$ parameters give the elasticity of substitution across different types
of intermediate goods in the production of the composite domestic good and the composite imported good. Setting \( \theta = \vartheta = 8.0 \) gives a steady-state markup of 14 per cent, which agrees well with estimates in the empirical literature of between 10 per cent and 20 per cent (see, for example, Basu 1995). The \( \sigma \) parameter gives the elasticity of substitution across different labour types in the production of individual domestic intermediate goods. The value of 6.0 corresponds to estimates from microdata in Griffin (1992).\(^{17}\)

Table 2 summarizes the parameters of the model’s exogenous stochastic processes. Apart from the parameters of the stochastic process for public spending, the individual parameter estimates are all significant at the 1 per cent level.\(^{18}\) Except for foreign inflation, the processes are persistent, with AR(1) parameters above 0.65. The standard deviations of the innovations to the processes vary widely in magnitude, ranging from 0.0016 in the case of foreign interest rate shocks to 0.0402 in the case of tax rate shocks. The volatility of foreign shocks is smaller than that of domestic shocks, which suggests the relative importance of domestic shocks for business cycle fluctuations in the Canadian economy.

Table 3 summarizes the parameters estimated by SMM. Appendix B summarizes the moments used to obtain these estimates. The model passes a test of its overidentifying restrictions at the 10 per cent level: the marginal significance level for the \( J \)-test is 13.18 per cent.\(^{19}\) For this subset of parameters, our estimates are very precise. In particular, the nominal rigidity parameters are highly significant (at the 0.1 per cent level). They are also of plausible magnitude and well within the range of values in previous empirical studies and in calibrated general-equilibrium models. The estimate of \( d_p \) implies that the prices of domestic intermediate goods remain fixed for, on average, slightly more than three quarters. The other prices are slightly more sluggish, but still well within the range of plausibility. Import prices remain fixed for, on average, about 3.4 quarters. Nominal wages remain fixed for, on average, slightly more than four quarters.

The \( \varrho \) parameters are from the interest rate reaction function. They are intended to capture a fairly standard Taylor-type rule. Since the sum of \( \varrho_x \) and \( \varrho_p \) is greater than unity, the long-run level of the inflation rate is determinate. For our base-case simulation results, we set \( \varrho_s = 0 \) to avoid some anomalous responses of some variables to shocks,\(^{20}\) but the coefficient is built into the structural model in order to allow for

\(^{17}\)It also agrees with the estimated value in Ambler, Guay, and Phaneuf (2003). They succeed in estimating the value of the equivalent parameter in their model by calibrating the equivalent of the \( d_w \) parameter.

\(^{18}\)The constants of the stochastic processes for foreign output and foreign inflation are normalized to equal zero, so estimates for these parameters do not appear in the table.

\(^{19}\)As described in the preceding section, the parameters of the interest rate rule, money-demand shock, and technology process are estimated using exactly identified GMM. We estimate them simultaneously with the other parameters in the table, but for the eighteen parameters unconditional moments are used to estimate the other eight parameters.

\(^{20}\)With \( \varrho_s \) set equal to its estimated value, the response of the exchange rate to several types of shocks is highly non-monotonic,
sensitivity analysis. We estimated $\varphi_y$ and found it to be close to zero and insignificant, so we set it to zero in our simulations, as in Dib (2003).

The $\nu$ and $\varsigma$ parameters, which capture the elasticities of demand for composite intermediate imports and for exports, would enter symmetrically into the domestic and foreign aggregate production functions for final goods, which suggests that their values should be similar. Their estimated values are close to the calibrated value of 0.6 set by McCallum and Nelson (2001) for the equivalents of both parameters. With our estimated values, the sum of import and export elasticities exceeds one, so the static Marshall-Lerner condition is satisfied by the model. The parameter $\alpha_d$ gives the relative weight on the composite domestic intermediate good in the production of the final good. The estimated value of 0.7413 is estimated precisely and reflects a degree of home bias in the demand for composite intermediate goods. $\phi$ gives the weight on the composite good used as an input in the production of domestic intermediate goods. The estimated value implies that labour accounts for just over 70 per cent of the value added in the intermediate-goods sector.

The money-demand process is highly persistent, with an autoregressive coefficient that is indistinguishable from a random walk, even though our theoretical model implies that the observable variables used to estimate the process should be stationary. This could reflect the fact that many empirical studies find it difficult to reject the null hypothesis of a unit root in short-term nominal interest rates. The standard deviation of the innovation to the money-demand shock process, 0.0610, is larger than the standard deviation of any other forcing process in the model. This, combined with the degree of persistence of the money-demand shock, means that the unconditional variance of money demand is very large compared with that of the other shocks in the model.

The drift term in the technology process is highly significant and entails an annual per-capita growth rate of 1.4 per cent, which is close to the per-capita growth rate of output in our sample period. The standard deviation of the innovation to the technology process is 0.0026, which is smaller than the volatility of any other domestic innovation. This suggests that technology shocks have not been the primary driving force of the Canadian business cycle.

The model is simulated by linearizing its equilibrium conditions around their deterministic steady-state values. This yields a set of saddlepoint-stable linear difference equations, which we simulate using standard

\[ \frac{\text{with the long-run change in the exchange rate being of a different sign than the initial change.}} {21} \text{Kollmann also uses values of 0.6 for the equivalents of both parameters in a similar model. He justifies these values as being approximately equal to the median values of the import and export elasticities estimated by Hooper and Marquez (1995).} \]
techniques described by Blanchard and Kahn (1980).

4. Simulation Results

4.1 Unconditional moments and impulse response functions

Table 4 summarizes some of the unconditional second moments predicted by the model. The second column gives the model’s prediction concerning the moment defined in the first column. The third column gives the estimated value (using exactly identified GMM) of the moment in our data set, and the fourth column gives the standard error of this estimate. To compute the model’s predictions, we use stochastic simulations with shocks generated using the standard deviations from Tables 2 and 3, and imposing independence across the different types of shocks. The calculated statistics, both for the model and the data, are for series measured in growth rates.

The volatility of output in the model exceeds that in the data. Consumption volatility in the model is also too high. This reflects the lack of capital in the model, which reduces the consumption-smoothing opportunities that are available to agents. This also leads to a relative volatility of hours that is too high when hours are measured in levels. The volatility of employment growth is, however, comparable with that in the data. Diminishing marginal returns to labour in the production functions for intermediate goods mean that, in response to any type of shock except a technology shock, hours must be more volatile than output. The real exchange rate is 2.15 times as volatile as output. This relative volatility is somewhat lower, but of the same order of magnitude, as in the data. The nominal and real exchange rates are highly correlated, which also corresponds well with what we observe in the data.

Figure 1 shows in graphical form some of the unconditional autocorrelations predicted by the model. The series are filtered using the Hodrick-Prescott filter, for comparability with results in the literature. Output and the nominal and real exchange rates are quite persistent, although somewhat less than in the data: the first-order autocorrelations in the Canadian data are, respectively, 0.77, 0.82, and 0.79. The model’s prediction concerning the, persistence of inflation is quite striking. Generating inflation persistence has been a strong challenge to recent business cycle models, both closed and open economy, as Fuhrer and Moore (1995) note. Although the normalized level of output is strongly autocorrelated, the first-order autocorrelation of

22 The predictions of the model concerning unconditional moments are calculated using the asymptotic variance-covariance matrices of the state variables and endogenous variables.
output growth (not shown) is only mildly positive (0.12), which means that the autocorrelation of output growth predicted by the model does not match that in the data. It is clear that reproducing this feature of the data would require adding features to the model such as the slow adjustment of employment.\footnote{Ambler, Guay, and Phaneuf (2003) find that employment adjustment costs were crucial in allowing a closed-economy business cycle model to reproduce the positive autocorrelations at low orders of output growth.}

The model exhibits prolonged output responses to most types of shocks, including nominal shocks.\footnote{The graphs of these impulse responses are available on request from the authors.} The maximum response of output to most types of shocks, however, occurs during the same period that the shock hits. Output has a hump-shaped response only in the case of a tax rate shock. In response to a foreign output shock, domestic output initially falls, but its response changes sign after the first period and then peaks after several periods. Only technology shocks have permanent effects on real variables in the model, because of the unit root in the technology process. On the other hand, because the interest rate rule responds to the rate of change of prices and money-supply growth, most shocks have permanent effects on the model’s nominal variables.

\subsection*{4.2 Exchange rate pass-through}

Figure 2 summarizes the model’s predictions concerning the effects of different shocks on the nominal exchange rate and on three different price indexes: $P_t$, the equivalent of the CPI; $P_t^d$, the equivalent of the PPI; and $P_t^m$, the price index of imported goods. The results are striking. In response to all types of shocks except technology shocks, the maximum response of the nominal exchange rate to the shock occurs immediately upon impact. As stated earlier, the model’s nominal variables have a common stochastic trend due to the interest rate rule that responds to inflation rather than to the price level. This implies that the exchange rate overshoots (its short-run response is greater than its long-run response) in response to each of the eight structural shocks in the model. The responses of import prices, the PPI, and the CPI are invariably gradual. Even in the case of technology shocks, the immediate response of the exchange rate is much more pronounced than that of the price level, and exceeds the response of the CPI at horizons up to 40 periods. Pass-through is also incomplete (to the PPI and the CPI) after 40 periods in response to each of the different structural shocks in the model, in spite of the fact that pass-through to the import price index is essentially complete after 10 periods in response to all shocks. The CPI has a hump-shaped response to all types of shocks except shocks to the domestic interest rate, where its response is a gradual descent to a plateau.
In the case of tax rate shocks, the impact effect on the nominal exchange rate is negative, whereas the impact effect on the overall price level is positive. This means that pass-through to the CPI is actually negative in the very short run in response to tax shocks. It takes approximately 10 periods for the price level to return to its initial level, so the degree of pass-through after 10 periods is approximately zero.

The impulse-response functions of the exchange rate and prices make it clear that exchange rate pass-through is a conditional phenomenon. The dynamics of pass-through depend on the type of shock that hits the economy. In response to all of the structural shocks in the model, however, pass-through is less than complete in the short run (negative in the case of tax rate shocks). The degree of pass-through in the medium run is quite consistent across different types of shocks. Pass-through to imported goods prices is close to complete after 10 quarters in response to all types of shocks. Omitting tax rate shocks, which are discussed in the preceding paragraph, after 10 quarters pass-through is lowest in the case of foreign output shocks: about 30 per cent of the change in the exchange rate has been passed through to the CPI. Pass-through is most complete after 10 quarters in response to foreign nominal interest rate shocks: more than 50 per cent of the change in the nominal exchange rate has been passed through to the CPI.

After 40 quarters, exchange rate pass-through to the CPI is close to complete in response to all shocks except foreign output shocks, in that the absolute differences between the values of the exchange rate and the price level are small. However, because the nominal exchange rate initially overshoots in response to the different structural shocks, the relative responses of the price level compared with that of the exchange rate remains quite small, even after 40 quarters. Figure 2 shows that exchange rate pass-through is complete in the long run in response to all of the types of shocks in the model. This reflects the fact that slow pass-through in the model is the result of short-term frictions (nominal rigidities). There is no strategic pricing in our model that would lead to long-run deviations from purchasing-power parity: models of international product differentiation with strategic pricing are popular micro-based explanations for slow and incomplete exchange rate pass-through. Ghosh and Wolf (2001) conclude that the empirical evidence supports the case for complete long-run pass-through and hence for macro-based models of sticky prices, rather than micro-based models of strategic pricing.

A VAR analysis of the bivariate dynamics of the nominal exchange rate and the price level generated by the model would have difficulty distinguishing among the large number of different structural shocks using long run identification restrictions, since most of the model’s shocks have permanent effects on the exchange
rate and the price level. They also have qualitatively similar effects on the exchange rate and the price level in the very short run (except for money-demand shocks), making identification using short run restrictions also difficult. However, the qualitative uniformity across shocks of the responses of the exchange rate and domestic prices should be picked up by VAR estimates applied to artificial data generated by the model.

4.3 Sensitivity analysis

In this section, we show how our results pertaining to pass-through are sensitive to the degree of nominal rigidity of both domestic goods prices and import prices. Figure 3 shows the impulse-response functions of the exchange rate and the overall domestic price level (CPI) to a domestic interest rate shock when domestic output prices are no longer sticky. For these simulations, we set $d_p = 0$, with all other parameters set equal to their values in the base-case scenario discussed in section 5.3. The exchange rate still overshoots in response to each of the eight structural shocks. There is still slow pass-through, even to the price of domestic output, despite the fact that, by assumption, the prices of domestic intermediate goods can adjust instantaneously to shocks, except in the case of technology shocks. In the latter case, a positive technology shock lowers the marginal cost of production by so much that the price of domestic output drops initially by more than the exchange rate. In the very short run, pass-through to the PPI is more than 100 per cent. For all of the other structural shocks, the degree of pass-through both in the short run and the medium run (after 10 quarters) is almost the same as in the base-case scenario.

The results with $d_p = 0$ suggest that nominal wage rigidity may be an important part of the explanation of slow pass-through. Figures 4 and 5 confirm this hypothesis. In Figure 4, we shut down both types of nominal price rigidity, so that the only nominal rigidity left in the model is wage rigidity. Since the prices of imported goods now adjust instantaneously, the import price index, $P^m$, tracks the nominal exchange rate quite closely, including in response to technology shocks (in this case, import prices respond slightly more strongly in the short run than the exchange rate). The exchange rate, however, still overshoots in response to each of the structural shocks and the CPI adjusts slowly. The quantitative measure of pass-through both in the very short run and after 10 quarters is again little changed from the base-case scenario.

Figure 5 shows the response of the nominal exchange rate and of three different price indexes when all three of the nominal rigidities are removed from the model. The exchange rate continues to overshoot in response to most types of structural shocks, although in response to interest rate shocks its response is essentially
flat after the first period. In response to domestic nominal shocks (interest rate shocks and money-demand shocks), exchange rate pass-through to the CPI is immediate. Even with no nominal rigidities, however, most real shocks (tax rate shocks, government spending shocks, and foreign shocks) lead the CPI to respond less than the nominal exchange rate does in the very short run. In response to tax rate shocks, the exchange rate and the CPI initially move in opposite directions. In contrast to the scenarios with at least one type of nominal rigidity, the response of the CPI is no longer hump-shaped. It responds either monotonically (foreign output shocks), is flat after the first period (interest rate shocks and technology shocks), or overshoots its long-run response (all other shocks).

The main conclusion that can be drawn from our sensitivity analyses is that, although pricing to market is sufficient to generate slow exchange rate pass-through, it is not necessary. Wage rigidity is an important structural feature of our model that leads to slow pass-through in response to structural shocks. Even with no nominal rigidities, real shocks can also lead to incomplete pass-through in the short run. These results qualify the conclusions of earlier theoretical models, such as those by Betts and Devereux (2000) and Smets and Wouters (2002), that pricing to market is crucial in generating slow exchange rate pass-through.

5. Conclusions

Our structural model can distinguish among the effects of a large number of different types of shocks. Because the exchange rate and prices are endogenous variables, exchange rate pass-through in the model is always conditional on the type of shock. However, there is a remarkable degree of uniformity in the dynamics of exchange rate pass-through across the different types of structural shocks in the model. The effect of shocks on the price level is much smaller than on the exchange rate. After 10 quarters, the degree of exchange rate pass-through varies between 50 and 75 per cent. After 40 quarters, the degree of exchange rate pass-through is greater than 90 per cent in response to all shocks except foreign output shocks. Our sensitivity analysis shows that sticky imported-goods prices (pricing to market) are sufficient to generate slow exchange rate pass-through, but that they are not necessary. Other structural features, such as nominal-wage rigidities, can also by themselves result in sluggish exchange rate pass-through. Even a model with no nominal rigidities can generate slow pass-through in response to real shocks.
Bibliography


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard deviation</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_g )</td>
<td>0.8548</td>
<td>0.7590</td>
<td>1.1262</td>
<td>0.2101</td>
</tr>
<tr>
<td>( \bar{g} )</td>
<td>6.2332</td>
<td>0.0729</td>
<td>85.3962</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \sigma_g )</td>
<td>0.0098</td>
<td>0.0131</td>
<td>0.7488</td>
<td>0.2994</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.2908</td>
<td>0.0096</td>
<td>30.3564</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \rho_\tau )</td>
<td>0.6729</td>
<td>0.1363</td>
<td>4.9386</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \sigma_\tau )</td>
<td>0.0402</td>
<td>0.0104</td>
<td>3.8509</td>
<td>0.0004</td>
</tr>
<tr>
<td>( \rho_{R^*} )</td>
<td>0.8102</td>
<td>0.0933</td>
<td>8.6845</td>
<td>0.0000</td>
</tr>
<tr>
<td>( R^* )</td>
<td>0.0150</td>
<td>0.0014</td>
<td>10.4156</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \sigma_{R^*} )</td>
<td>0.0016</td>
<td>0.0004</td>
<td>4.3298</td>
<td>0.0001</td>
</tr>
<tr>
<td>( \rho_{y^*} )</td>
<td>0.8835</td>
<td>0.0519</td>
<td>17.0074</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \sigma_{y^*} )</td>
<td>0.0059</td>
<td>0.0010</td>
<td>5.9069</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \rho_{\pi^*} )</td>
<td>0.4273</td>
<td>0.0907</td>
<td>4.7134</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \sigma_{\pi^*} )</td>
<td>0.0035</td>
<td>0.0005</td>
<td>7.0488</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Table 3: Parameter Estimation (SMM)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard deviation</th>
<th>t-stat</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal rigidity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_p$</td>
<td>0.6763</td>
<td>0.1200</td>
<td>5.6348</td>
<td>0.0000</td>
</tr>
<tr>
<td>$d_m$</td>
<td>0.7045</td>
<td>0.1078</td>
<td>6.5326</td>
<td>0.0000</td>
</tr>
<tr>
<td>$d_w$</td>
<td>0.7572</td>
<td>0.0652</td>
<td>11.6156</td>
<td>0.0000</td>
</tr>
<tr>
<td>Interest rate rule</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.0193</td>
<td>0.0013</td>
<td>14.3899</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\varrho_\pi$</td>
<td>0.7391</td>
<td>0.0538</td>
<td>13.7352</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\varrho_\mu$</td>
<td>0.5059</td>
<td>0.1421</td>
<td>3.5599</td>
<td>0.0011</td>
</tr>
<tr>
<td>$\varrho_s$</td>
<td>0.0525</td>
<td>0.0241</td>
<td>2.1803</td>
<td>0.0388</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>0.0023</td>
<td>0.0054</td>
<td>0.4354</td>
<td>0.3609</td>
</tr>
<tr>
<td>Foreign supply/demand</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varsigma$</td>
<td>0.5251</td>
<td>0.1166</td>
<td>4.5053</td>
<td>0.0001</td>
</tr>
<tr>
<td>Production</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5521</td>
<td>0.0738</td>
<td>7.4789</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_d$</td>
<td>0.7413</td>
<td>0.0839</td>
<td>8.8358</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.2966</td>
<td>0.1068</td>
<td>2.7757</td>
<td>0.0099</td>
</tr>
<tr>
<td>Money-demand shock</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>0.3820</td>
<td>0.1521</td>
<td>2.5111</td>
<td>0.0188</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>0.9999</td>
<td>0.0014</td>
<td>698.8693</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.0610</td>
<td>0.0279</td>
<td>2.1857</td>
<td>0.0383</td>
</tr>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.2485</td>
<td>0.1187</td>
<td>2.0937</td>
<td>0.0462</td>
</tr>
<tr>
<td>Technology process</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>0.0035</td>
<td>0.0014</td>
<td>2.4952</td>
<td>0.0195</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.0098</td>
<td>0.0026</td>
<td>3.7067</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

$J$-stat=12.4619, p-value=0.1318

Table 4: Standard Deviations and Correlations

<table>
<thead>
<tr>
<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>S.E.†</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\Delta y}$</td>
<td>0.0110</td>
<td>0.0074</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\sigma_{\Delta c}$</td>
<td>0.0126</td>
<td>0.0077</td>
<td>0.0012</td>
</tr>
<tr>
<td>$\sigma_{\Delta h}$</td>
<td>0.0083</td>
<td>0.0081</td>
<td>0.0016</td>
</tr>
<tr>
<td>$\sigma_{\Delta e}$</td>
<td>0.0279</td>
<td>0.0190</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\sigma_{\Delta s}$</td>
<td>0.0240</td>
<td>0.0177</td>
<td>0.0015</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
<td>0.0094</td>
<td>0.0192</td>
<td>0.0053</td>
</tr>
<tr>
<td>$\sigma(\Delta c_t, \Delta s_t)$</td>
<td>0.9345</td>
<td>0.9567</td>
<td>0.1548</td>
</tr>
</tbody>
</table>

†: standard error of value estimated from data
*: with $s_t \equiv c_t p_t^* / p_t$
Appendix A: Data and Data Sources

Our data set is available on request. The data are from Canada and the United States and are quarterly from 1981Q3 to 2001Q4. The Canadian data are from Bank of Canada Banking and Financial Statistics, a monthly publication by the Bank of Canada. Series numbers are indicated in brackets and correspond to Cansim databank numbers.

- Consumption, $c_t$, is measured by real personal spending on non-durable goods and services in 1997 dollars (non-durables [v1992047] + services [v1992119]).
- The CPI inflation rate, $\pi_t$, is measured by changes in the consumer price index, $p_t$ [v18702611].
- The PPI inflation rate, $\pi_t^d$, is measured by changes in the GDP implicit deflator, $p_t^d$ [v1997756].
- The short-term nominal interest rate, $R_t$, is measured by the rate on Canadian three-month treasury bills [v122531].
- The output-growth rate, $\Delta y_t$, is measured by changes in real per-capita GDP [v1992067].
- The money growth rate, $\mu_t$, is measured by changes in nominal per-capita M2 stock [v37124].
- Exports, $y_{xt}$, are measured by real per-capita exports of goods and services [v1997750].
- Imports, $y_{mt}$, are measured by real per-capita imports of goods and services [v1997753].
- The average nominal wage, $W_t$, is measured by average hourly labour earnings (wages and salaries [v498076] / total hours worked [v4391505]).
- Employment, $h_t$, is measured by average weekly hours worked (total hours worked [v4391505] / all employees [v2062811]).
- The nominal exchange rate, $e_t$, is average Canadian dollars per unit of U.S. dollars [v37426].
- Government spending, $G_t$, is measured by government expenditures on goods and services (total domestic demand [v1992068] − total personal expenditures [v1992115] − construction [v1992053 + v1992055] − machinery and equipment [v1992056]).
- The labour tax rate, $\tau_t$, is measured by the effective labour tax rate (calculated following the methodology of Jones 2002; and Mendoza, Razin, and Tezar 1994).
- The series in per-capita terms are obtained by dividing each series by the Canadian civilian population aged 15 and over (civilian labour force [v2062810] / labour force participation [v2062816]).

The U.S. data are from the Federal Reserve Bank of St. Louis, with the series numbers in brackets. The world series are approximated by some of the U.S. series.

- World output, $y_t^*$, is real U.S. GDP per capita in 1996 dollars [GDPC96] divided by the U.S. civilian non-institutional population [CNP16OV].
- The world nominal interest rate, $R_t^*$, is measured by the rate on U.S. three-month Treasury Bills [TB3MS].
- The world inflation rate, $\pi_t^*$, is measured by changes in the U.S. GDP implicit price deflator, $p_t^*$ [GDPDEF].
Appendix B: Moments Used to Estimate the Model

The set of unconditional moments used to estimate the structural parameters of the model are:

- $\text{Corr}(\pi_t, \pi_{t-i})$ for $i = 1, 2, 3$;
- $\text{Corr}(\pi^d_t, \pi^d_{t-i})$ for $i = 1, 2, 3$;
- $\text{Corr}(\pi^w_t, \pi^w_{t-i})$ for $i = 1, 2, 3$;
- $\text{Corr}(\Delta y_t, \Delta c_t)$;
- $\text{Corr}(\Delta y_t, \Delta y_x)$;
- $\text{Corr}(\Delta y_t, \Delta y_m)$;
- $\text{Corr}(\Delta e_t, \pi^d_t)$;
- $\sigma_{\Delta c_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta e_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta s_t}/\sigma_{\Delta y_t}$;
- $\sigma_{\Delta h_t}/\sigma_{\Delta y_t}$.

Appendix C: Equilibrium Conditions

The following system of equations defines the economy’s equilibrium:

\begin{align*}
\frac{c_{t+1}}{c_t} + \frac{b_{t+1}^{\frac{1}{\gamma}}}{b_t^{\frac{1}{\gamma}}} &= \lambda_t p_t; \quad (C.1) \\
\frac{b_{t+1}^{\frac{1}{\gamma}}}{c_t} + \frac{b_t^{\frac{1}{\gamma}}}{b_{t+1}^{\frac{1}{\gamma}}} &= \lambda_t P_t \left(1 - \frac{1}{R_t}\right); \quad (C.2) \\
\frac{R_t}{s_t R_t} &= E_t \left[\frac{s_t+\pi_{t+1}}{\pi_{t+1}}\right] \quad (C.3) \\
\frac{\lambda_t}{R_t} &= \beta E_t \left[\frac{1}{s_t^{\phi}} \exp(-A - \epsilon_{At+1})\right] \quad (C.4) \\
\tilde{w}_t = \left(\frac{\sigma}{\sigma - 1}\right) &\frac{E_t \sum_{l=0}^{\infty} (\beta d_w)^l \eta h_{t+l} / (1 - h_{t+l})}{E_t \sum_{l=0}^{\infty} (\beta d_w)^l (1 - \pi_{t+l}) \lambda_t h_{t+l} \prod_{k=0}^{l} (\pi_{t+k}^d)^{-1} \exp(-A - \epsilon_{At+k})}; \quad (C.5) \\
w_{t+1}^{1-\sigma} = d_w \left[\left(\frac{w_{t+1}^{1-\sigma}}{\pi_t^{1-\sigma}}\right) \left(\frac{w_{t+1}^{1-\sigma}}{\pi_t^{1-\sigma}}\right) + (1 - d_w)\tilde{w}_t^{1-\sigma}\right]; \quad (C.6) \\
y_t = x_t^{\phi} h_t^{1-\phi}; \quad (C.7)
\end{align*}
\[ w_t = (1 - \phi) \xi \frac{y_t}{x_t}; \quad (C.8) \]

\[ p_t = \phi \xi \frac{y_t}{x_t}; \quad (C.9) \]

\[ \tilde{p}_t^d = \left( \frac{\theta}{\theta - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d p)_t \lambda_t t y_t + t \xi_t + t}{E_t \sum_{l=0}^{\infty} (\beta d p)_t \lambda_t t y_t + t \prod_{k=0}^{\infty} (\pi_t^d + 1)}; \quad (C.10) \]

\[ 1 = d_p \left( \frac{1}{\pi_t^d} \right) (1 - \theta) + (1 - d_p) (\tilde{p}_t^d (1 - \theta)); \quad (C.11) \]

\[ \tilde{p}_t^m = \left( \frac{\vartheta}{\vartheta - 1} \right) \frac{E_t \sum_{l=0}^{\infty} (\beta d m)_t \lambda_t t y_t + t \pi_t^m + t \lambda_t + t \prod_{k=0}^{\infty} (\pi_t^m + 1)}{E_t \sum_{l=0}^{\infty} (\beta d m)_t \lambda_t t y_t + t \prod_{k=0}^{\infty} (\pi_t^m + 1)}; \quad (C.12) \]

\[ (p_t^m)^{(1 - \vartheta)} = d_m \left( \frac{p_t^m}{\pi_t^m} \right)^{(1 - \vartheta)} + (1 - d_m) (\tilde{p}_t^m (1 - \vartheta)); \quad (C.13) \]

\[ (p_t)^{(1 - \nu)} = \alpha d + \alpha_m (p_t^m)^{(1 - \nu)}; \quad (C.14) \]

\[ z_t = c_t + x_t + g_t; \quad (C.15) \]

\[ y_t = y_t^d + y_t^d; \quad (C.16) \]

\[ \bar{y}_t^d = \alpha d \left( \frac{1}{p_t^d} \right)^{-\nu} z_t; \quad (C.17) \]

\[ y_t^d = \alpha d \left( \frac{1}{p_t^d} \right)^{-\nu} z_t; \quad (C.18) \]

\[ y_t^m = \alpha_m \left( \frac{p_t^m}{\pi_t^m} \right)^{-\nu} z_t; \quad (C.19) \]

\[ \frac{b_t^*}{\kappa_t R_t} - \frac{b_t^* - 1}{\pi_t^d} \exp(-A - \varepsilon A_t) = y_t^d - s_t \bar{y}_t^d; \quad (C.20) \]

\[ \log(\kappa_t) = \varphi \left[ \exp \left( \frac{s_t b_t^*}{y_t} \right) - 1 \right]; \quad (C.21) \]

\[ \log(R_t/R) = \varphi_y \log(y_t/y) + \varphi_s \log(\pi_t/\pi) \]

\[ + \varphi_s \log(s_t/s) + \varepsilon R_t; \quad (C.22) \]

\[ \pi_t = \frac{m_t - 1}{m_t^*} \exp(A + \varepsilon A_t + \mu_t); \quad (C.23) \]

\[ \log A_t = (A) + \log(A_{t-1}) + \varepsilon A_t; \quad (C.24) \]
\[
\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \epsilon_b; \quad (C.25)
\]

\[
\log(g_t) = (1 - \rho_g) \log(g) + \rho_g \log(g_{t-1}) + \epsilon_g; \quad (C.26)
\]

\[
\log(\tau_t) = (1 - \rho_{\tau}) \log(\tau) + \rho_{\tau} \log(\tau_{t-1}) + \epsilon_{\tau}; \quad (C.27)
\]

\[
\log(R^*_t) = (1 - \rho_{R^*}) \log(R^*) + \rho_{R^*} \log(R^*_{t-1}) + \epsilon_{R^*}; \quad (C.28)
\]

\[
\log(\pi^*_t) = (1 - \rho_{\pi^*}) \log(\pi^*) + \rho_{\pi^*} \log(\pi^*_{t-1}) + \epsilon_{\pi^*}; \quad (C.29)
\]

\[
\log(y^*_t) = (1 - \rho_{y^*}) \log(y^*) + \rho_{y^*} \log(y^*_{t-1}) + \epsilon_{y^*}; \quad (C.30)
\]

where equation (C.20) gives the trade balance of the economy.
Figure 1: Autocorrelation Functions
Figure 2: Pass-Through

Technology Shock

Interest Rate Shock

Money Demand Shock

Government Spending Shock

Tax Rate Shock

Foreign Interest Rate Shock

Foreign Output Shock

Foreign Inflation Shock
Figure 3: Pass-Through with $d_p = 0$
Figure 4: Pass-Through with $d_m = d_p = 0$
Figure 5: Pass-Through with $d_m = d_p = d_w = 0$