Abstract

In this paper I characterize time consistent equilibrium in an economy with price rigidity and discretionary monetary policy. Firms have the option to increase their frequency of price change (at a cost) in response to higher expected inflation. As in previous studies – which assume a constant degree of price rigidity across inflation regimes – there exist two time consistent equilibria: an ‘optimistic’ equilibrium with low inflation, and a ‘pessimistic’ equilibrium with high inflation. But the nature of the pessimistic equilibrium in the current model is very different: for quantitatively reasonable specifications, the pessimistic equilibrium displays full price flexibility. As a result, the model’s implications for inflation and real output differ from one with exogenous price rigidity. I demonstrate this by characterizing the set of pessimistic equilibria that arise when the monetary authority can play both pure and mixed strategies. Finally, I modify the model to include an arbitrarily small cost of inflation that exists independently of rigid prices. I show that quantitative versions of this model display unique time consistent equilibrium.
1. INTRODUCTION

Central bank policy is best characterized as being set with discretion. That is, monetary policy makers do not simply implement policy plans determined in the past. So while it is crucial to characterize policy under commitment, it is equally important to understand what outcomes can arise when it is recognized that policy makers act in a discretionary manner. In this paper, I characterize time consistent equilibrium in a model with monetary discretion and an endogenously determined degree of price rigidity. The objective is to determine whether, for quantitatively reasonable specifications, the model generates multiple equilibria, and if so, to characterize outcomes in ‘high inflation’ equilibria.

Kydland and Prescott (1977) and Barro and Gordon (1983) describe economies in which the interaction between monetary discretion and a forward-looking private sector produce unique equilibrium. This equilibrium displays expected and realized inflation higher than that obtained under commitment. More recently, the issue of monetary discretion has been studied in dynamic, rational expectations, general equilibrium models. Chari et al. (1998) show that if agents play trigger strategies, equilibrium outcomes are no longer unique. In particular, expectation traps arise in which equilibria associated with expectations of low or high inflation become self-fulfilling. Hence, the inflation experience of the US during the 1970’s could be reasonably interpreted as a high inflation expectation trap. An important shortcoming of this explanation, however, is that the play of trigger strategies admits many possible equilibria. For example, in the analysis of Ireland (1997) – upon which Chari et al. is based – the same model that predicts expectation trap equilibria predicts the first-best, commitment solution as an equilibrium outcome as well.

Two recent papers – Albanesi et al. (2003) and King and Wolman (2004) – study discretionary policy when reputational mechanisms are ruled out. These papers show that multiplicity of time consistent equilibrium remains. Hence, periods of low and high inflation can still be expectations determined. In this paper I study an extension of the model presented by King and Wolman, and so I briefly summarize their framework here.

Firms are monopolistic and set staggered two-period prices. The presence of sticky prices
means that in setting policy, the monetary authority has an incentive to produce unanticipated inflation. Since the output of sticky-price firms is demand determined, this stimulates output and reduces the monopoly distortion. The presence of staggering implies that inflation is also costly since it generates relative price distortions across firms. Hence, the monetary authority has an incentive to generate positive, but finite, inflation. Forward-looking firms account for this in their price-setting decisions, resulting in sub-optimally high equilibrium inflation.

But the combination of price rigidity and monetary discretion has another important effect: since policy is accommodative, a strategic complementarity arises in firms’ price-setting decisions. An individual firm’s expectations about other firms’ actions, and policy’s response to those actions, will impact upon the individual’s decision. If firms coordinate expectations on low inflation occurring, they set accordingly low prices. Accommodation by the monetary authority delivers low (but sub-optimally positive) inflation, thus validating individual firms’ expectations and pricing decisions. However, if firms coordinate expectations on high inflation, they set accordingly high prices. If monetary policy is sufficiently accommodating, these expectations are confirmed, leading to high inflation in equilibrium.

Hence, accommodation results in the possibility of equilibrium being expectations determined. Accommodation is precisely the hallmark of discretionary policy determination: the monetary authority is unable to commit its future self to take into consideration the effect of its policy on current private sector expectations and decisions.\footnote{This succinct and focused statement of the time consistency problem is taken from Kydland and Prescott (1977), pg. 481.} If maximizing behavior on the part of the monetary authority requires it to be sufficiently accommodating, multiple equilibria exist.

A problem with this reasoning is that it relies heavily on the degree of price rigidity being exogenously determined. With sticky prices, a firm’s future price is not permitted to adjust for inflation that happens between now and then. Expectations of high inflation lead price-setting firms to set high prices now, thus compelling the monetary authority to generate high inflation. While considering exogenously rigid prices is a fruitful assumption
in monetary business cycle analysis, it seems problematic in trying to describe periods such as the 1970's experience. This is particularly true if exogeneity of price rigidity is central to the existence of high inflation equilibria.

Here, I modify the analysis of King and Wolman by endogenizing the degree of price rigidity. I do this by considering a generalization of a state-dependent pricing model, in which firms can choose to incur a fixed cost in order to increase their frequency of price change. If the degree of price rigidity is allowed to adjust across low and high inflation equilibria, this can have large effects on the nature of multiplicity.

I show that, as in previous work, there exist two time consistent equilibria: an ‘optimistic’ equilibrium with low expected and realized inflation, and a ‘pessimistic’ equilibrium with high inflation. But for quantitatively reasonable specifications of the model, the steady state of the pessimistic equilibrium displays full price flexibility. As a result, the implications for inflation and real output differ from the case with exogenous price rigidity. When prices are fully flexible, the monetary authority has no influence over real variables and is indifferent between all values of inflation. Hence, a rich set of inflation outcomes can be supported in the pessimistic equilibrium if the monetary authority’s policy rule allows for discontinuities or mixed strategies at full price flexibility. Moreover, real output is independent of inflation in the pessimistic equilibrium, and is in fact higher than in the optimistic equilibrium. Finally, I introduce an arbitrarily small cost of inflation into the model that is independent of the degree of price rigidity. I show that quantitative versions of the modified model display unique time consistent equilibrium; that is, this modification eliminates the pessimistic equilibrium.

Section 2 presents the model, and Section 3 characterizes equilibrium for arbitrarily determined monetary policy. Section 4 details the strategic complementarity in firm’s pricing decisions that is the source of multiplicity. Section 5 characterizes Markov perfect equilibrium in which the discretionary monetary authority maximizes private sector welfare. Section 6 analyzes Markov perfect equilibria and Section 7 considers an arbitrarily small perturbation of the model that generates unique equilibrium. Section 8 concludes.
2. THE MODEL

Here I present a generalized version of the model of King and Wolman (2004), hereafter KW. This is a perfect foresight, infinite horizon economy populated by a representative household, a representative final good firm, a continuum of monopolistically competitive intermediate good firms, and a discretionary monetary authority. Price rigidity among intermediate good firms admits non-neutral effects of monetary policy. Specifically, firms make a *pricing decision* every second period; half of firms do so in odd periods, the other in even, so that pricing decisions are staggered. To introduce endogenous price rigidity, this pricing decision is modeled as having two dimensions: the price(s) to charge and the frequency of price change. I elaborate on this below. The timing within a period is as follows: first, the monetary authority (hereafter MA) makes a decision on the growth rate of the money stock; after observing the MA’s decision, private sector decisions are made.

At the beginning of a period, the state observed by the MA is denoted \( s \in \sigma \), which I call the *MA state*. I focus on an environment in which reputational mechanisms are explicitly ruled out; attention is restricted to the play of Markov strategies, so that \( s \) contains only fundamental or ‘pay-off relevant’ information. The MA’s strategy is therefore conditioned only on pricing decisions made in the previous period, which will become obvious shortly. After observing \( s \), the MA chooses a gross money growth rate, \( X \). That is, if \( M_t \) is the money stock at date \( t \), \( X_t = M_t / M_{t-1} \). After observing \( (s, X) \), private sector decisions are made – call \( (s, X) \) the private sector or *PS state*. Since private sector agents are forward-looking and make intertemporal decisions, they must have beliefs or expectations of how policy will be chosen in the future. Since the MA at each date acts after observing \( s \), these beliefs are summarized as a money growth rule or *policy rule*, \( \chi(\cdot): \sigma \to \mathcal{R}_+ \).

2.1. Households

Households value consumption \((c)\) and labor \((h)\) according to:

\[
\sum_t \beta^t U(c_t, h_t), \quad 0 < \beta < 1,
\]
where

\[ U(c, h) = \log c - \psi h, \quad \psi > 0. \]

The household faces two sequences of constraints. The first is the flow budget constraint:

\[
M_t + B_t \leq R_{t-1}B_{t-1} + M_{t-1} - P_{t-1}c_{t-1} + (1 + \theta_{t-1})\left(W_{t-1}h_{t-1} + \int_0^{t-1} \Pi_{i,t-1}d\bar{t}\right) + T_t.
\]

This is relevant during securities trading in each period \( t \); trading occurs after observing the PS state. Here: \( B_t \) is the value of nominal bond holdings which pay a gross return of \( R_t \) upon maturity at date \( t + 1 \); \( M_t \) is the value of nominal money holdings; \( P_t \) is the consumption good price; \( W_t \) is the nominal wage rate; \( \Pi_{i,t} \) are nominal profits from monopolistic firm \( i \), \( i \in [0, 1] \); and \( T_t \) is a lump-sum transfer from the MA. Finally, \( \theta_t \) is a subsidy to production income. I let \( \theta_t \equiv R_t - 1 \) to eliminate the interest rate distortion on the intratemporal valuation of consumption versus labor, as well as the intertemporal valuation of profits (relevant in the firm’s problem below). I do this so that the only distortions affected by monetary policy are the monopoly distortion and the relative price distortion stemming from the specification of intermediate good production and price rigidity.

After securities trading, the household supplies labor at the wage \( W_t \), and buys consumption at the price \( P_t \). Consumption purchases are subject to a cash-in-advance constraint:

\[ M_t \geq P_tc_t, \quad \forall t. \]

Setting \( \theta \equiv R - 1 \), the first-order necessary conditions (FONCs) for the household are:

\[
\frac{1}{Pc} = \beta R \frac{1}{P'c'}, \\
W = \psi Pc,
\]

where primes (') denote one-period-ahead variables. In equilibrium, \( R \geq 1 \), so that the cash-in-advance constraint holds with equality. Substituting \( M = Pc \) into the FONCs delivers:

\[
\chi(s') = \frac{M'}{M} = \beta R, \\
W = \psi M.
\]
In equilibrium, the rate of nominal interest reflects the expected rate of money growth relative to time preference. Also, since production of output is specified as being linear in labor (see below), the growth rate of nominal marginal cost will be equal to that of money.

### 2.2. Final Good Production

Final good firms are perfectly competitive and produce output using intermediate goods as input. Final goods are consumed by households and cannot be stored. The representative final good firm’s problem is:

$$\max_{\{y_i\}} P \left[ \int_0^1 y_i^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)} - \int_0^1 P_i y_i di, \quad \lambda > 1.$$  

Here, $P$ is the current price of final output, $P_i$ is the input price set by intermediate good producer $i \in [0, 1]$, and $\lambda$ is the elasticity of substitution across intermediate goods. The FONC for this problem states the standard ‘demand as a function of relative price’ condition:

$$y_i = \left( \frac{P}{P_i} \right)^{\lambda} y, \quad y = \left[ \int_0^1 y_i^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)}.$$  

### 2.3. Intermediate Good Production

Intermediate good firms produce input goods using labor according to $y_i = h_i$. Labor is hired at the competitive wage $W$. The potential for price rigidity is introduced via the decision-making constraints of these firms. In the analysis of KW, firms have only one option in their pricing decision: choose a single price to charge in the current and following period after observing the PS state, $(s, X)$. This is the standard, two-period Taylor (1980) specification of price stickiness found in the monetary business cycle literature (see, for example, the textbook treatment of Romer, 2001).

In contrast, a firm making its pricing decision in the present model has two options: (a) be sticky, and choose a single price that must hold in the current and following period after observing $(s, X)$; or (b) be flexible, and choose one price for the current period after observing $(s, X)$, and another price in the next period after observing $(s', X')$. Choosing option (b) requires paying a fixed cost, though more generally, this could be interpreted as a
higher fixed cost than that associated with choosing option (a). This fixed cost corresponds to the decision-making, communication, and physical cost of charging an additional price.

2.3.1. Sticky prices If the firm chooses to be sticky (or is assumed to be, as in KW), it will choose a single price, $\bar{P}'$, to maximize two-period discounted profits:

$$\bar{\Upsilon} \equiv \max_{P'} \left[ \alpha \left( P^{\lambda} \bar{P}'^{1-\lambda} y - W P^{\lambda} \bar{P}'^{1-\lambda} y' \right) + \beta \alpha' \left( P'^{\lambda} \bar{P}'^{1-\lambda} y' - W' P'^{\lambda} \bar{P}'^{1-\lambda} y' \right) \right].$$

Here, the final good firm’s demand function has been substituted in, and $\alpha$ is the marginal value of current profit:

$$\alpha = \beta \frac{U_c}{P} (1 + \theta) = \frac{U_c}{P}.$$

In a symmetric equilibrium, all sticky price firms charge the same two-period price:

$$\bar{P}' = \hat{\lambda} \left( \frac{\alpha W P^{\lambda} y + \beta \alpha' W' P'^{\lambda} y'}{\alpha P^{\lambda} y + \beta \alpha' P'^{\lambda} y'} \right), \quad \hat{\lambda} = \frac{\lambda}{\lambda - 1}.$$ This states that the optimal price is a markup over the appropriately discounted sum of current and future marginal cost.

2.3.2. Flexible prices If the firm chooses to be flexible, it will choose a price to charge today, $\bar{P}$, and a price to charge tomorrow, $\bar{P}'$, according to:

$$\bar{P} = \hat{\lambda} \psi M,$$
$$\bar{P}' = \hat{\lambda} \psi M',$$

where I have used the fact that $W = \psi M$. Since these prices are chosen after observing the MA’s action, they are set optimally as a markup over observed marginal cost.

In order to set flexible prices, a firm must pay a fixed cost, i.e. the cost it incurs if it chooses option (b) instead of option (a). The CDF for the fixed cost among the measure of firms making their pricing decisions is given by $F(\varphi)$, with domain $[0, \varphi_{\text{max}}]$. These fixed costs represent the units of labor it will expend in the following period to set its price after observing $M'$. An individual firm will choose option (b) as opposed to option (a) if the
difference in discounted two-period profits is greater than the fixed cost. That is, firm $i$ will choose to be flexible if:

$$a\tilde{\Pi} + \beta\alpha' \left[ \tilde{\Pi}' - W'\varphi_i \right] \geq \tilde{\Upsilon}.$$  

Call this condition the cut-off condition. Here, optimal one-period profits, $\tilde{\Pi}$, are those earned when charging current price $\tilde{P}$ and selling current output of $\tilde{y} = \left( P'P \right)^{\lambda} y$:

$$\tilde{\Pi} = \left( \tilde{P} - W \right) \tilde{y} = \left[ \frac{1}{(\lambda - 1)^{\lambda}} \right] W^{1-\lambda}P^{\lambda}y.$$  

Denote the value of the fixed cost that satisfies the cut-off condition with equality as $\varphi^*$. That is, all firms that draw $\varphi_i \leq \varphi^*$ will choose to set flexible prices, while all others set sticky two-period prices. The fraction of firms currently making their pricing decisions which choose to be flexible is given by $z' = F(\varphi^*)$. If the cut-off condition holds with strict inequality at the maximal fixed cost, $\varphi_{\text{max}}$, then $z' = 1$.

2.4. Government Budget Constraint

The budget constraint faced by the MA is:

$$T_t = M_t - M_{t-1} - \theta_{t-1} \left( W_{t-1}h_{t-1} + \int_0^1 \Pi_{i,t-1}di \right) , \forall t.$$  

That is, the lump-sum transfer to the household finances the monetary injection, net of the subsidy to production income.

3. PRIVATE SECTOR EQUILIBRIUM

Ultimate interest is in characterizing Markov perfect equilibrium (MPE). These are equilibria in which: (i) taking past decisions and future policy as given, the MA’s choice for

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2Given perfect foresight and no uncertainty, the timing of the pricing decision generates identical results to an economy where the decision whether to be flexible is made in the second period of the price contract. The current interpretation allows for clearer model exposition.
current money growth maximizes the household’s lifetime utility from the current period forward; and (ii) reputational mechanisms are inoperational. To make progress, I first define a private sector equilibrium (PSE) in which the MA’s actions – both current \((X)\) and future \((\chi(.))\) – need not be welfare maximizing. In the definition, lower-case variables denote nominal variables determined in the current period normalized by the current period money stock, e.g. \(p \equiv P/M, \bar{p} \equiv \bar{P}/M, \bar{\bar{p}} \equiv \bar{\bar{P}}/M\), etc. I do this since all equilibria are neutral in the usual sense: if the initial money stock is doubled, a PSE exists in which all real allocations are identical and only nominal variables are doubled. Doing so also makes clear that when the MA is maximizing, the use of trigger strategies contingent on the level of the nominal money stock is ruled out. That is, in the analysis of time consistent equilibrium, attention is restricted to MPE.

In the following definition, I make explicit that current private sector decisions and equilibrium variables depend on the PS state, e.g. \(p \equiv p(s, X)\).

**Definition 1** Given beliefs (a policy rule) \(\chi : \sigma \rightarrow \mathcal{R}_+,\) for all PS states \((s, X) \in \sigma \times \mathcal{R}_+,\) a private sector equilibrium is a set of allocation rules \(\{c(s, X), h(s, X)\}\), pricing rules \(\{\bar{p}'(s, X), \bar{p}(s, X), \bar{z}'(s, X)\}\), and prices \(\{R(s, X), \bar{p}(s, X)\}\) such that: households are optimizing, prices are set optimally, \(z'(s, X)\) satisfies the cutoff condition, the goods and labor markets clear, \(b = 0\), and \(R(s, X) \geq 1\).

Note that by Walras’ Law, the money market clears.

In the rest of this section, I provide a more compact characterization of PSE. First, non-negative nominal returns implies that the cash-in-advance constraint holds with equality:

\[
c = 1/p. \tag{1}
\]

I suppress the arguments of the allocation and pricing rules for the sake of exposition. The household’s intratemporal FONC states that the nominal wage is proportional to the money stock, so the normalized wage is constant:

\[w = \psi.\]
The intertemporal FONC bounds the set of feasible PSE money growth rules:

\[ R = \chi(s') / \beta, \]

so that \( \chi(s) \geq \beta \) for all \( s \).

Optimality on the part of flexible price intermediate good firms implies that the normalized flexible price is a constant markup over the normalized wage:

\[ \tilde{p} = \tilde{p}' = \tilde{\lambda} \psi. \]

Optimality of final good firms implies the following normalized price level equation holds:

\[ p = \left\{ \frac{1}{2} \left[ (1-z) \left( \frac{\tilde{p}}{X} \right)^{1-\lambda} + (1-z')\tilde{p}'^{1-\lambda} + (z+z')\tilde{p}^{1-\lambda} \right] \right\}^{\frac{1}{1-\lambda}}. \] (2)

The labor market clearing condition is:

\[ h = \frac{p^{\lambda-1}}{2} \left[ (1-z) \left( \frac{X}{p} \right)^{\lambda} + (1-z') \frac{1}{p^{\lambda}} + (z+z') \frac{1}{p^{\lambda}} \right] + \frac{1}{2} \int_{0}^{F^{-1}(z)} \varphi dF(\varphi). \] (3)

Given values for \((\tilde{p}, z, X)\), equilibrium values for \(c, w, R, \tilde{p}, p, \) and \(h\) are given by the above equations once \(\tilde{p}'\) and \(z'\) are determined. These latter two values are determined from the FONC for sticky price-setting and the cutoff condition. Optimality on the part of sticky price firms implies that:

\[ \tilde{p}' = \tilde{\lambda} \psi \left( \frac{p^{\lambda-1} + \beta p'^{\lambda-1} \chi(s')^{\lambda}}{p^{\lambda-1} + \beta p^{\lambda-1} \chi(s')^{\lambda-1}} \right). \] (4)

Note that the direct effect of inflation between the current and following period, \(\chi(s') \equiv M'/M\), on the current normalized sticky price, \(\tilde{p}'\), is increasing. This is due to the fact that marginal cost growth, \(W'/W\), is exactly equal to \(\chi(s')\). As discussed below, it is this, as well as the equilibrium effect of \(\chi(s')\) on (normalized) prices, \(p\) and \(p'\), that gives rise to strategic complementarity in pricing. Finally, the cutoff condition states that the fraction of firms choosing flexibility, \(z' = F(\varphi^*)\), satisfies:

\[ \frac{p^{\lambda-1}}{\tilde{p}^{\lambda}} (\tilde{p} - \psi) + \frac{p'^{\lambda-1}}{\tilde{p}'^{\lambda}} (\tilde{p}' - \psi) - \psi \varphi^* \geq \frac{p^{\lambda-1}}{\tilde{p}^{\lambda}} (\tilde{p}' - \psi) + \beta p'^{\lambda-1} \left( \frac{\chi(s')^{\lambda}}{\tilde{p}'} \right)^{\lambda} \left( \frac{\tilde{p}'}{\chi(s') - \psi} \right). \] (5)
This cut-off equation holds with strict equality whenever $\varphi^* < \varphi_{\text{max}}$, and with weak inequality whenever $\varphi^* = \varphi_{\text{max}}$. Hence, given $(\bar{p}, z, X)$ and the policy rule, $\chi(\cdot)$, conditions (4) and (5) characterize PSE values for $\bar{p}'$ and $z'$. Remaining PSE values are determined from the conditions given above. This simplifying result is summarized as follows:

**Proposition 2** Given beliefs $\chi : \sigma \to \mathcal{R}_+$, a PSE is characterized as decision rules, $\bar{p}' = P(s, X)$ and $z' = Z(s, X)$, such that for all $(s, X)$, equations (4) and (5) are satisfied.

Recall that ultimate interest is in characterizing MPE in which the MA’s policy rule and private sector decision rules are conditioned on only pay-off relevant information. From the equilibrium conditions given above, it is clear that in MPE the MA state is $s = (\bar{p}, z)$. That is, the only fundamental state variables inherited by the current MA are the previous period’s pricing decisions.\(^3\)

Moreover, when the MA inherits full price flexibility, current money growth is neutral. To see this, note that when $z = 1$, current money growth has no direct influence on the normalized price level, $p$, via the erosion of sticky prices (see equation (2)), and no indirect influence on $p$ via the current decision rules, $P(s, X)$ and $z' = Z(s, X)$ (see equations (4) and (5)). Hence, there is no effect on consumption or hours worked (see equations (1) and (3)). That is, when $z = 1$ current money growth has no effect on the monopoly distortion or the relative price distortion, since this influence requires the presence of sticky prices.

### 4. MULTIPLECTY OF EQUILIBRIUM

Given the characterization of PSE, it is possible to illustrate the potential for multiplicity when monetary policy is determined with discretion. Specifically, the potential for multiple PSE arises whenever money growth, $\chi(\cdot)$, is increasing in the normalized sticky price, $\bar{p}$. When $\partial \chi / \partial \bar{p}$ is positive, this generates a strategic complementarity in the price-setting

\(^3\)Accordingly, the PS state is $(s, X) = (\bar{p}, z, X)$, and private sector expectations of future money growth is given by $\chi(\bar{p}', z')$. 

12
decision rule, $\bar{p}' = P(s, X)$, of sticky price firms.\footnote{Following Cooper and John (1988), I refer to strategic complementarity as a situation in which the increase in one player’s strategy increases the optimal strategy of others.}

This result is not new to this paper, and is first illustrated by KW. I discuss this strategic complementarity here for the sake of completeness. To do so, it is easiest to work with the KW model, which is a special case of that presented in this paper.\footnote{For obvious reasons, the exposition closely follows that of KW.} In the KW model, choosing to increase the frequency of price change (i.e., choosing to charge one price for each period, as opposed to charging a sticky price for two periods) is infinitely costly. As a result, all firms act as sticky price firms. With exogenous price rigidity, $z \equiv 0$ in PSE, and so the decision rule, $z' = Z(s, X)$, is dropped along with the corresponding characterizing equation, (5). The MA and PS states are reduced to $s = \bar{p}$ and $(s, X) = (\bar{p}, X)$, respectively.

### 4.1. Strategic Complementarity in Price-Setting

To understand the complementarity, consider the following rewriting of the FONC for sticky price-setting:

$$\bar{p}' = \hat{\lambda} \left[ (1 - \gamma) \psi + \gamma \chi (\bar{p}') \psi \right].$$

This states that the optimal two-period price is a markup over a weighted average of current ($\psi$) and future marginal cost ($\chi (\bar{p}') \psi$). The weight on future marginal cost is given by:

$$\gamma = \frac{\beta p'^{\lambda - 1} \chi (\bar{p}')^{\lambda - 1}}{p^{\lambda - 1} + \beta p'^{\lambda - 1} \chi (\bar{p}')^{\lambda - 1}} \in (0, 1).$$

In PSE:

$$p^{\lambda - 1} = \left\{ \frac{1}{2} \left[ \left( \frac{\bar{p}}{X} \right)^{1 - \lambda} + \bar{p}'^{1 - \lambda} \right] \right\}^{-1},$$

$$p'^{\lambda - 1} = \left\{ \frac{1}{2} \left[ \left( \frac{\bar{p}'}{X (\bar{p}')} \right)^{1 - \lambda} + \bar{p}''^{1 - \lambda} \right] \right\}^{-1}.$$
Following KW, I interpret equation (6) as the best response function for an individual firm: given \((\bar{p}, X)\), this equation maps out the optimal price for firm \(i, \bar{p}_i^0\), as a function of the price set by all other price-setting firms, \(\bar{p}_j^0\), for all \(j \neq i\). To be clear, rewrite (6) as:

\[
\bar{p}_i^0 \equiv f(\bar{p}_j^0) = \hat{\lambda} \left[ (1 - \gamma(\bar{p}_j^0)) \psi + \gamma(\bar{p}_j^0) \chi(\bar{p}_j^0) \psi \right],
\]

where dependence on \((\bar{p}, X)\) is suppressed for exposition. The individual firm’s optimal price depends on the price set by all other firms through two channels. The first is through the value of future marginal cost, via the effect of \(\bar{p}_j^0\) on future money growth, \(\chi(\bar{p}_j^0)\). The second is through the relative weight placed on future marginal cost, via the effect of \(\bar{p}_j^0\) on \(\gamma(\bar{p}_j^0)\). In PSE, it must be that \(\bar{p}_i^0 = \bar{p}_j^0\).

If \(f(.)\) is increasing in \(\bar{p}_j^0\), then there exists a strategic complementarity between firms’ price-setting decisions; that is, if \(\partial f/\partial \bar{p}_j^0 > 0\), the higher is the price set by other firms, the higher is the optimal price for any individual firm. If this complementarity is sufficiently strong, there may be multiple \(\bar{p}\) values that satisfy equation (7), and hence, multiple PSE. Whether this is the case depends crucially on the nature of the policy rule, \(\chi(.)\). To see this, I plot the best response function for various specifications of \(\chi(.)\).

As a benchmark, consider the case when \(\chi(\bar{p}) \equiv 1\); in this case the MA delivers zero money growth irrespective of the pricing behavior of firms. This corresponds to the first-best policy achieved under commitment (see King and Wolman, 1999). Future money growth and marginal cost is non-responsive to current price-setting, so that the individual firm’s best response is simply \(\bar{p}_i^0 = \hat{\lambda} \psi\). Regardless of the price set by other firms, the optimal price is a markup over marginal cost which is constant across periods \(\chi(\bar{p}_j^0) \psi = \psi\). With zero money growth there is no complementarity, and the best response function crosses the 45°-line only once at \(\bar{p}' = \hat{\lambda} \psi\). This is displayed in Figure 1.

As a second example, consider the case when money growth is an increasing, linear function of prices, \(\chi(\bar{p}) = a_0 + a_1 \bar{p}\). In this case, prices are strategic complements. As firms set higher prices, \(\bar{p}_j^0\), one-period ahead marginal cost rises. Moreover, as \(\bar{p}_j^0\) rises, \(\gamma(\bar{p}_j^0) \to 1\), so that the weight on future marginal cost relative to current marginal cost in (7) rises. The optimal price for an individual firm is increasing in the price set by other
firms. In Figure 2, I illustrate this for \( \chi (\bar{p}) = 0.302 \times \bar{p} \).

At low values of the sticky price, it is optimal for an individual firm to set a price higher than that of others: \( \bar{p}_i' = f (\bar{p}_j') > \bar{p}_j' \). As well, the slope of the best response function, \( \partial f (.) / \partial \bar{p}_j' \), is initially less than one so that the best response first crosses the 45°-line from above. However, as \( \bar{p}_j' \) increases so too does the slope \( \partial f (.) / \partial \bar{p}_j' \). In the example, the slope eventually exceeds one, so that there is a second crossing of the 45°-line from below. Because \( \gamma (\bar{p}_j') \rightarrow 1 \) as \( \bar{p}_j' \) rises, and because \( \partial \chi (.) / \partial \bar{p} = a_1 \) in this example:

\[
\lim_{\bar{p}_j' \to \infty} \frac{\partial f (.)}{\partial \bar{p}_j'} = \hat{\lambda} \psi a_1.
\]

Hence, when the MA’s policy rule is linear, a necessary condition for multiplicity is that \( \hat{\lambda} \psi a_1 > 1 \). Note that this is not sufficient since it is possible that \( \bar{p}_i' = f (\bar{p}_j') \) lies everywhere above the 45°-line, so that no PSE exist. Moreover, when multiple PSE exist, there are exactly two of them: an ‘optimistic’ equilibrium in which expectations are coordinated on low inflation (and actions are coordinated on low price-setting), and a ‘pessimistic’ equilibrium in which expectations (actions) are coordinated on high inflation (high price-setting).

Finally, Figure 3 illustrates that the exact number of crossings depends on the shape of the policy rule \( \chi (.) \). Here, \( \partial^2 \chi (.) / \partial \bar{p}^2 \) is initially positive, but beyond an inflection point, \( \partial^2 \chi (.) / \partial \bar{p}^2 < 0 \). As a result, the number of crossings is three.

4.2. Discussion

Before proceeding, I provide a few comments on the source of complementarity. First, when attention is turned to monetary policy that is set with discretion, the policy rule will be an increasing function of the normalized sticky price. That is, a benevolent MA will find it optimal to ‘accommodate’ the private sector’s expectations and pricing decisions.

\(^6\)Here are some details with additional discussion contained in Appendix A. The model is calibrated with: \( \beta = 0.98, \lambda = 11, \) and \( \psi \) set so that \( h_{ss} = 0.3 \) in the flexible price version (or, alternatively, the zero inflation version) of the economy. In the figure, \( \bar{p} \) and \( X \) are set at their steady-state PSE values. As I vary \( \bar{p}_j' \) in \( f (.) \), I calculate the PSE values for \( \chi (\bar{p}_j'), \bar{p}, \) and \( p' \) to evaluate the best response, \( \bar{p}_i' \). To anticipate results, whenever multiple equilibria exist, I use the lower of the two equilibrium \( \bar{p}' \) values to derive \( p' \).
Second, it is useful to understand the differences between the strategic complementarity highlighted here and that discussed in Ball and Romer (1991). There are three key differences. First, in Ball and Romer, the complementarity arises due to the decision of firms to alter prices in a state-contingent manner. Here, the complementarity operates through the two-period, sticky price firms decide to set; this is obvious since the discussion above is based on the KW model in which prices are exogenously rigid. Second, Ball and Romer’s model is static, and the strategic complementarity operates through a feedback of current price-setting by firms, through current marginal cost, into the pricing decision of individual firms. In the present analysis, current (normalized) marginal cost is pinned down as $\psi$, but future marginal cost, $\chi(\bar{p})\psi$, does respond via the increasing function $\chi(.)$. Hence, the feedback operates through future marginal cost, so that the complementarity is dynamic in nature. The last difference is that in Ball and Romer, (current) marginal cost is responsive to pricing decisions because of real rigidity. Here, (future) marginal cost is responsive because of policy accommodation.

Finally, the discussion of the previous subsection emphasizes that a key element to the complementarity is the effect of price-setting on the relative weight placed on future marginal cost in (7). As firms raise prices future marginal cost rises, and an individual firm cares more about the future in its price-setting. This is because the firm’s profit function is asymmetric across having a relative price that is too high versus a relative price that is too low. This asymmetry is discussed in detail in Devereux and Siu (2004), and can be understood through the following extreme, but intuitive, thought experiment.

Consider a situation in which future money growth, $\chi(\bar{p}_j)$, is extremely high. The individual firm must decide whether to set its price to be ‘in line’ with current monetary and marginal cost conditions, or with one-period ahead conditions. As an example, suppose it is deciding between setting its price as a markup over current marginal cost, $\bar{p}_i = \hat{\lambda}\psi$, or future marginal cost, $\bar{p}_i = \hat{\lambda}\chi(\bar{p}_j)\psi$. In either case, it will earn statically optimal profits in either the first or second period of the price contract.

By choosing the latter option, the firm earns optimal profits in the second period, but its current price will be high relative to firms which set their price in the previous period.
As a result, the firm’s first period demand and profit will be low, but bounded above zero. In contrast, by pricing to current marginal cost it earns optimal profits in the current period, but its one-period ahead relative price will be low. This implies that the firm’s second period demand will be high – in the same period when the firm’s profit margin is potentially negative (at least for sufficiently high money growth). Hence, in setting its two-period price, the firm prefers to set a price that is too high relative to one that is too low; the firm sacrifices current profit to ensure non-negative profits in the future. It is this relative price effect on a firm’s demand that generates the positive relationship between $\bar{p}_0^j$ and the relative weight on the future, $\gamma \left( \bar{p}_j^j \right)$.

Of course, this example is too extreme since any firm, given the opportunity, would shut down rather than meet demand at a negative profit margin. However, the mechanics of the intuition hold for any positive value of money growth (again, see Devereux and Siu, 2004, for detailed discussion). As long as the policy rule, $\chi (\cdot)$, is increasing in $\bar{p}$, the asymmetry in firm profits acts as a force for strategic complementarity in price-setting behavior.

4.3. The Case with Endogenous Price Rigidity

When changing the frequency of price change is infinitely costly, the only defensive action firms can take to guard against future inflation is to set a high price in its current two-period price contract. However, it is plausible to think that there are other defensive actions firms can take when faced with pessimistic expectations of high inflation. One of these is to reset prices more frequently so that at any point in time, prices are more ‘in line’ with current aggregate conditions. In this subsection, I show how this consideration affects the nature of equilibrium.

To illustrate this, I again plot a firm’s best response function as a function of the price set by other sticky-price firms, $\bar{p}_j^j$. Specifically, let the MA’s policy rule be $\chi (\bar{p}, z) = 0.302 \times \bar{p}$, and let the distribution of fixed costs be uniform, $\varphi \sim U [0, \varphi_{\text{max}}]$. I sort the measure of firms making pricing decisions in the current period on the unit interval according to the size of their fixed cost; that is, firm $i$’s fixed cost of price change is given by $\varphi_i = \varphi_{\text{max}} \times i$,
for $i \in [0, 1]$. Given $(\bar{p}, z, X)$ and $\bar{p}'$, I use the cutoff condition (5) to determine the fraction of firms, $z'$, that prefer to incur the fixed cost and charge flexible prices, $\bar{p}$ and $\bar{p}'$, as opposed to charging the sticky price, $\bar{p}'$. Using $(\bar{p}, z, X)$, $\bar{p}'$ and $z'$, the FONC (4) determines the optimal price, $\bar{p}'$, for the individual firm that chooses to charge a sticky price. Additional detail regarding the construction of these functions is contained in Appendix A.

Figure 4 plots the best response functions for two values of $\varphi_{\text{max}}$: the first column sets the maximal fixed cost to 18% of firm revenue in the steady-state of the flexible price version of the model, and the second column sets this to 9%.7 Allowing for endogenous price rigidity does not qualitatively change the sticky price firm’s best response, $\bar{p}_i = f \left( \bar{p}_j \right)$. Indeed, the quantitative impact is small so long as $\varphi_{\text{max}}$ is large and full price flexibility is not reached. In the first column, there are two crossings where $\bar{p}_i = \bar{p}_j$: the first featuring optimistic expectations, low inflation, and a small fraction of firms choosing price flexibility, and the second with pessimistic expectations, high inflation, and a large degree of flexibility.

But, as the support of the fixed cost distribution becomes smaller, an increasing number of firms choose to be flexible relative to being sticky. When the maximal fixed cost is sufficiently small, as displayed in the second column, the best response sticky price is not well defined for all values of $\bar{p}_j$. For sufficiently high values of $\bar{p}_j$ and future inflation, all firms making their pricing decision choose flexibility, so that $z' = 1$. At this point the best response price, $\bar{p}_i$, ceases to be ‘relevant’ since no firms choose to set sticky prices.

In the rest of the paper, I characterize time consistent equilibrium in which strategies are Markov. Hence, the MA’s policy rule will be of the form $\chi (\bar{p}, z)$, and optimality on the part of the MA will mean that $\partial \chi / \partial \bar{p} > 0$. This implies that the strategic complementarity discussed here will be operative. For quantitatively relevant specifications of $\varphi_{\text{max}}$, the high inflation, pessimistic MPE displays fully flexible prices. The nature of this equilibrium depends on the specification of the policy rule at $z = 1$. Moreover, the implications regarding the effect of monetary policy on real outcomes in the pessimistic equilibrium are very different from those derived when prices are exogenously rigid.

7 Quantitative specification of the remaining parameter values is as given in Subsection 4.1.
5. A MAXIMIZING MONETARY AUTHORITY

The MA’s objective is to maximize the present discounted value of household utility from the current period forward through the choice of current money growth, $X$. In making its decision, the MA takes its future incarnation’s policy rule, $\chi(.)$, as given. This is the expression of the time-consistency problem, as articulated by Kydland and Prescott (1977): the current MA is unable to compel the future MA to appropriately account for its policy’s effect on current expectations. The current MA’s only influence on the future is through the value of the future MA state, $s' = (\bar{p}', z')$, it induces.

The MA’s problem can be stated as follows:

$$\max_{\chi} \left[ U(s,X) + \beta U(s',\chi(s')) + \beta^2 U(s'',\chi(s'')) + \ldots \right], \quad \forall s \in \sigma. \quad (8)$$

Here: $s = (\bar{p}, z), U(s,X) = \log c(s,X) - \psi h(s,X), s' = (P(s,X), Z(s,X)), U(s',\chi(s')) = \log c(s',\chi(s')) - \psi h(s',\chi(s'))$, and so on; $P(s,X)$ and $Z(s,X)$ are the PSE decision rules defined by equations (4) and (5); and $c(s,X)$ and $h(s,X)$ are the PSE allocation rules defined by equations (1) and (3). As a result, a MPE can be defined as a policy rule, $\chi : \sigma \rightarrow \mathbb{R}_+$, that solve the MA’s problem, (8), for all MA states, $s \in \sigma$.

I consider an alternative definition of MPE and derive the generalized Euler equation (GEE) from the MA’s maximization problem, both of which are due to Klein et. al. (2004).

**Definition 3** A Markov Perfect Equilibrium consists of a value function, $V$; decision rules, $P$ and $Z$; and a policy rule, $\chi$, such that for all $s = (\bar{p}, z) \in \sigma$:

- **given $P$, $Z$, and $\chi$:**

  $$V(s) = U(s,\chi(s),P(s,X),Z(s,X)) + \beta V(P(s,\chi(s)),Z(s,\chi(s))) ;$$

- **given $P$, $Z$, and $V$:**

  $$\chi(s) \in \arg\max_{\chi} \left[ U(s,X,P(s,X),Z(s,X)) + \beta V(P(s,X),Z(s,X)) \right] ;$$
Given $V$ and $\chi$:

$$\bar{p}^t = \hat{\lambda} \psi \left[ \frac{p(s, X, \bar{p}', z')^{\lambda - 1} + \beta p(s', \chi(s'), \bar{p}'', z'')^{\lambda - 1} \chi(s')^{\lambda} \chi(s')^{-1}}{p(s, X, \bar{p}, z')^{\lambda - 1} + \beta p(s', \chi(s'), \bar{p}'', z'')^{\lambda - 1} \chi(s')^{\lambda - 1}} \right],$$

and

$$\frac{p(s, X, \bar{p}', z')^{\lambda - 1}}{\bar{p}^{\lambda}} (\bar{p} - \psi) + \beta \left[ \frac{p(s', \chi(s'), \bar{p}'', z'')^{\lambda - 1}}{\bar{p}^{\lambda}} (\bar{p} - \psi) - \psi F^{-1}(z') \right] \geq \frac{p(s, X, \bar{p}', z')^{\lambda - 1}}{\bar{p}^{\lambda}} (\bar{p} - \psi) + \beta p(s', \chi(s'), \bar{p}'', z'')^{\lambda - 1} \left( \frac{\chi(s')}{\bar{p}'} \right) \left( \frac{\bar{p}'}{\chi(s') - \psi} \right),$$

with strict equality whenever $\varphi^* = F^{-1}(z') < \varphi_{\text{max}}$. Here, $\bar{p}' = P(s, X)$, $z' = Z(s, X)$, $s' = (P(s, X), Z(s, X))$, $\bar{p}'' = P(s', \chi(s'))$, $z'' = Z(s', \chi(s'))$, and $\bar{p} = \hat{\lambda} \psi$.

In this definition:

$$U(s, X, P(s, X), Z(s, X)) = \log c(s, X, P(s, X), Z(s, X)) - \psi h(s, X, P(s, X), Z(s, X)),$$

where $c(.)$ and $h(.)$ are the PSE allocation rules defined by equations (1) and (3), and $p(s, X, P(s, X), Z(s, X))$ is the PSE pricing rule defined by equation (2). Also, $P(s, X)$ and $Z(s, X)$ have been included as arguments of these functions to emphasize the fact that $U$, $c$, $h$, and $p$ depend on $\bar{p}'$ and $z'$, which in turn are given by the decision rules $P$ and $Z$.

Note that this definition neatly captures time consistency, as the policy rule is specified to coincide with the optimizing choice of money growth for all MA states.

### 5.1. The Generalized Euler Equation

The GEE is simply the FONC associated with the MPE policy rule, $\chi(.)$. The GEE characterization relies on differentiability of the policy and decision rules; with differentiability, $X$ must solve:

$$U_x C_X + U_h H_X + \beta \left( V_{p}^t P_X + V_{z}^t Z_X \right) = 0,$$

where

$$C_X = c_X + c_{\bar{p}} P_X + c_{z'} Z_X,$$

$$H_X = h_X + h_{\bar{p}} P_X + h_{z'} Z_X,$$
for all $s \in \sigma$. Here, $V'_{p}$ represents the derivative of the one-period ahead value function with respect to its first argument, and $V'_{z}$ is the derivative with respect to its second argument. Likewise, $c_{i}$ represents the derivative of the consumption allocation rule with respect to $i$, for $i = \{X, \bar{p}', z'\}$, and similarly for $h_{i}$.

To express the GEE in terms of primitives, note from the definition of the value function:

\[
V'_{p} = U'_{c}C_{p} + U'_{h}H_{p} + \beta \left( V''_{p}P'_{p} + V''_{z}Z'_{p} \right) .
\]

To simplify this expression, rearrange (9) to obtain:

\[
\beta \left( V''_{p}P'_{p} + V''_{z}Z'_{p} \right) = -\frac{\partial X'}{\partial \bar{p}} \left( U'_{c}C_{X} + U'_{h}H_{X} \right) ,
\]

where $X' = \chi (\bar{p}', z')$. After substitution:

\[
V'_{p} = U'_{c}C_{p} + U'_{h}H_{p} - \frac{P'_{p}}{P'_{X}} \left( U'_{c}C_{X} + U'_{h}H_{X} \right) .
\]

Manipulating $V'_{z}$ in the same way, and substituting it and $V'_{p}$ into (9) obtains the following expression for the GEE:

\[
U_{c}C_{X} + U_{h}H_{X} + \beta P_{X} \left[ U'_{c}C_{p} + U'_{h}H_{p} - \frac{P'_{p}}{P'_{X}} \left( U'_{c}C_{X} + U'_{h}H_{X} \right) \right] + \beta Z_{X} \left[ U'_{c}C_{z} + U'_{h}H_{z} - \frac{Z'_{z}}{Z_{X}} \left( U'_{c}C_{X} + U'_{h}H_{X} \right) \right] = 0.
\]

From a variational perspective, this states that the policy choice of the current MA affects both current utility and one-period ahead utility. These marginal effects are balanced to equal zero at an optimum. In terms of the current period, money growth affects the utility value of consumption via $U_{c}C_{X}$. This involves both direct and indirect effects on consumption. The direct effect, $c_{X}$, captures the erosion of the inherited normalized price, $\bar{p}$. The terms $c_{p'}P_{X}$ and $c_{z'}Z_{X}$ capture the indirect effects on current period pricing decisions, and hence, on the degree of relative price dispersion across firms. Similarly, $H_{X}$ captures the direct and indirect effects of current money growth on current period labor.

The remaining terms represent the effect of current policy on the future which arise through the effect on future inherited state variables, $s' = (\bar{p}', z')$. The first term involving
square brackets, $\beta P_X \{ \cdots \}$, can be split into two effects. The first is the indirect effect of the change in the inherited sticky price, $P_X$, on future utility values of consumption and labor, $U^c_0 \bar{C}_0$ and $U^h_0 \bar{H}_0$. The second is the induced effect of a change in current policy on future policy. Note that the term:

$$P_X \frac{P_p}{P_X} = \frac{\partial p'}{\partial X} \frac{\partial X'}{\partial p'} = \frac{\partial X'}{\partial X},$$

so that this reflects the change in future money growth affected by the current money growth influence on the sticky price, $p'$. Similarly, the second square bracket term, $\beta Z_X \{ \cdots \}$, summarizes the indirect and induced policy effects of $X$ on future utility via the fraction of flexible price firms, $z'$.

5.2. Optimizing Behavior at Full Price Flexibility

The GEE must be satisfied wherever policy and decision rules are differentiable. However, there is no reason to restrict attention to policy rules that are continuous when $z = 1$, i.e. when prices are fully flexible. When all prices are being reset, current money growth has no effect on the normalized price level, consumption, or labor. This can be readily seen in the equations (1) – (3) defining the rules $c(\cdot)$, $p(\cdot)$, and $h(\cdot)$. With no inherited price stickiness, outcomes can be very different from when $z < 1$, and allowing for a discontinuity in $\chi(\bar{p}, z)$ at $z = 1$ permits the illustration of this point in an obvious way.

6. ANALYZING MARKOV PERFECT EQUILIBRIUM

Here I study MPE in calibrated versions of the model of Section 2. Initially, attention is restricted to the case with differentiable policy rules. The first objective is to illustrate the multiplicity of equilibrium that arises due to monetary discretion. In the model there are two MPE: an optimistic equilibrium and a pessimistic equilibrium. The second objective is to characterize the degree of price rigidity in the steady state of the pessimistic equilibrium

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8These, in turn, involve direct and indirect effects via $C'_p$ and $H'_p$, analogous to those described in the previous paragraph.
and the model’s implications for real outcomes. Subsection 6.2 considers the case when the policy rule is discontinuous at \( z = 1 \), and illustrates the rich set of pessimistic equilibria that can be supported when the MA plays both pure and mixed strategies.

In the analysis, I restrict attention to economies with perfect foresight in the following sense: given that multiple equilibria exist, private sector expectations are coordinated so that there is no uncertainty about which equilibrium will prevail. Clearly, given the multiplicity, stochastic equilibria can be constructed in which a sunspot shock determines whether the optimistic or pessimistic outcome occurs; I do not analyze this possibility.\(^9\)

### 6.1. Quantitative Analysis of the Differentiable Case

Here I consider the play of differentiable policy rules by the MA. In particular, I consider policy rules in which:

\[
\chi(\bar{p}, 1) = \lim_{z \to 1} \chi(\bar{p}, z), \quad \forall \bar{p},
\]

despite the fact that the MA is indifferent between all values of money growth at full price flexibility. To make this operational, I solve for MPE by approximating the MA’s policy rule by a tensor product of Chebychev polynomials. Additional details on the solution method are contained in Appendix B.

The parameterization of the model is standard. The period length is taken to be six months, so that all firms make pricing decisions once a year. Accordingly, \( \beta = 0.98 \) to accord with a real, annual risk-free return of 4\%. The value of \( \lambda \) determines the demand elasticity of substitution across intermediate goods, and thus, the strength of the strategic complementarity across firms’ pricing decisions. As in much of the literature, I choose \( \lambda = 11 \) as a benchmark value so that the price-to-marginal-cost markup is 10\% in the zero-inflation steady state (see, for example, KW, and Devereux and Siu, 2004).\(^10\) In the numerical experiments, I also consider smaller values of \( \lambda \) (higher markups) to capture the full range of values used in monetary business cycle studies. I set the fraction of time spent

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\(^9\)For analysis, see KW.
\(^10\)Note that this is analogous to a positive inflation steady state with perfectly flexible prices.
in market activity in the zero-inflation steady state to $h_{ss} = 0.3$.

6.1.1. **Exogenous price rigidity** I first consider the case with the fixed cost distribution degenerate at infinity, or at least sufficiently large so that no firm chooses to reset its price more frequently than once a year. This corresponds to the KW model with exogenous price rigidity. Results from this model can be summarized as follows:

**Proposition 4** With exogenous price rigidity, the MPE policy rule, $\chi(\bar{p})$, is linear in $\bar{p}$. Hence, two locally isolated MPE exist: an optimistic equilibrium with low expected and realized inflation, and a pessimistic equilibrium with high inflation.

Obviously, this is not a new result. Proof of the linearity of the policy rule is contained in KW, in their discussion of the optimality of a *homogenous money stock rule*. The intuition behind this result is straightforward. From the PSE decision rules, (1) – (3), the effect of money growth on real outcomes is in direct proportion to the level of normalized preset prices, $\bar{p}$. Moreover, the gross money growth rate in no way affects the values of $p'$ or $z'$ chosen in the current period. As a result, optimal money growth, $\chi(\bar{p})$, is proportional to $\bar{p}$ as well. Note that this is also true with endogenous price rigidity. That is, given $z$, $\chi(.)$ is linear in $\bar{p}$; however, it is a non-linear function of the fraction of flexible price firms, $z$.

Given this linearity, Subsection 4.1 shows that the number of equilibria is exactly two. As a result, there are exactly two steady state MPE. In the optimistic steady state, the inflation rate is 1.9% per period (3.8% per year), while real output is 0.04% lower than in the zero inflation steady state. In the pessimistic steady state, inflation is much higher at 13.8% per period, and output is 1.91% lower than with zero inflation.

6.1.2. **Endogenous price rigidity** Here I consider a non-degenerate distribution of the fixed cost with finite support. In particular, I specify the distribution to be uniform on the interval $[0, \varphi_{max}]$. As in the case with exogenous price rigidity, the policy rule, $\chi(\bar{p}, z)$, is linear in $\bar{p}$.\textsuperscript{11} Hence, two locally isolated MPE exist, and two steady state MPE exist.

\textsuperscript{11}However, given $\bar{p}$, it is non-linear in $z$. 

24
With endogenous price rigidity, the fraction of firms that choose to set prices period-by-period depends on the inflation rate. The degree of price stickiness is greater in the optimistic equilibrium (with low inflation) than in the pessimistic one. The goal here is to determine – for quantitatively reasonable values of $\varphi_{\text{max}}$ – the degree of price stickiness in the pessimistic steady state.

First, I show that it cannot be the case that pessimistic equilibrium with full price flexibility always exists, for any finite value of $\varphi_{\text{max}}$. To see this, the relevant comparison is the difference in gross profits from choosing to be flexible relative to being sticky, and the highest fixed cost incurred by choosing flexibility. Flexible price profits are simply the discounted sum of two-period static monopoly profits. For any value of future money growth, there is a finite lower bound on sticky price profits: a sticky price firm can always set its price as an optimal markup over future marginal cost, and earn static monopoly profits in the second period of its price contract. The worst that can happen in the current period is that the firm’s relative price is so high that it generates zero demand and earns zero profit. Hence, the difference between flexible and sticky price profits is bounded. So as long as the maximal fixed cost is greater than this bounded difference, full price flexibility cannot be an equilibrium. Determining whether the model displays full price flexibility in pessimistic equilibrium is a quantitative issue.

To this end, I compute the pessimistic MPE for various values of $\varphi_{\text{max}}$ and find the minimal value such that the pessimistic steady state does not display full price flexibility. That is, I find the value – call it $\hat{\varphi}_{\text{max}}$ – such that for all fixed cost distributions with $\varphi_{\text{max}} > \hat{\varphi}_{\text{max}}$, $z < 1$ in the pessimistic steady state, and for all distributions with $\varphi_{\text{max}} < \hat{\varphi}_{\text{max}}$, $z = 1$ in the pessimistic steady state.

In Figure 5, I plot the value of $\hat{\varphi}_{\text{max}}$ for various calibrated values of $\lambda$. For the baseline calibration of $\lambda = 11$, $\hat{\varphi}_{\text{max}} = 8.9\%$ of semi-annual steady state firm revenue. That is, as long as the maximal fixed cost of a single price change is less than 8.9\% of firm revenue, all firms choose to incur the fixed cost, and the pessimistic steady state exhibits full price flexibility. As $\lambda$ decreases, so that the zero inflation steady state price-to-marginal-cost markup increases, the cutoff value, $\hat{\varphi}_{\text{max}}$, increases. For instance, when the steady state
markup is calibrated to 25% ($\lambda = 5$), the cutoff value is $\hat{\varphi}_{\text{max}} = 16.4\%$ of semi-annual revenue, and when the markup is 35% ($\lambda = 3.85$), $\hat{\varphi}_{\text{max}} = 18.9\%$.

To understand this, note that as $\lambda$ decreases, intermediate goods become less substitutable, so the optimal price of an individual firm is less sensitive to the prices set by other firms. Hence, as $\lambda$ decreases, so too does the strength of the strategic complementarity in firms’ pricing decisions. In order for a pessimistic equilibrium to exist, it must exist at higher levels of inflation and money growth. At higher inflation, the greater is the benefit to a firm in choosing flexibility over rigidity, and the greater is the degree of equilibrium price flexibility for a given fixed cost distribution. Hence, as $\lambda$ decreases, the range of $\varphi_{\text{max}}$ values for which pessimistic equilibrium displays fully flexible prices increases.

Note that the magnitude of $\hat{\varphi}_{\text{max}}$ over the range of calibrated $\lambda$ values is large. It is much larger than those used in calibrated monetary business cycle models with state-dependent pricing. For instance, Dotsey et al. (1999) consider a value of $\varphi_{\text{max}}$ which is equivalent 1.5% of semi-annual steady state firm revenue, while Devereux and Siu (2004) consider a value of $\varphi_{\text{max}} = 3.6\%$. More importantly, the magnitude of $\hat{\varphi}_{\text{max}}$ is much larger than that found from direct measurement of the fixed cost of price change. Zbaracki et al. (2000) is the leading study. They study the price-setting process of a US manufacturing firm and quantify all fixed costs associated with the issuance of the firm’s list prices – managerial (information-processing, decision-making), customer (communication, negotiation), and physical ‘menu’ costs. At a semi-annual frequency, their measured fixed cost of a one-time price change is equal to 2.5% of the firm’s revenue. Hence, for any reasonable magnitude of the maximal fixed cost, $\varphi_{\text{max}}$, prices are fully flexible in the pessimistic steady state.

Finally, real output and consumption are actually higher in the pessimistic, relative to the optimistic, steady state. In the optimistic steady state, real output is 0.04% lower than with zero inflation. But because prices are fully flexible in the pessimistic case, real output is identical to that of the zero inflation steady state. Hence, the predictions for real outcomes are opposite to those from the model with exogenous price rigidity.
6.2. Allowing for a Discontinuity in the Policy Rule

When \( z = 1 \), the MA inherits no sticky prices. Since all prices are being determined after current money growth, monetary policy has no real effect in these states of the world.\(^{12}\) As a result, the MA is indifferent between all values of money growth at full flexibility. This strict indifference opens up the possibility for a rich set of MPE policy rules, and a rich set of pessimistic steady states as well.

For instance, consider the following discontinuous policy rule:

\[
\hat{\chi}(\bar{p}, z) = \begin{cases} 
\chi(\bar{p}, z) & \text{for all } \bar{p} \text{ and } z < 1 \\
\hat{X} & \text{for all } \bar{p} \text{ and } z = 1 
\end{cases}
\]

(10)

where \( \chi(\bar{p}, z) \) is the differentiable MPE policy rule of Subsection 6.1 and \( \hat{X} \geq \beta \). It is easy to see that this rule is optimal on the part of the MA: for all \( z < 1 \), this rule coincides with the original MPE rule, \( \chi(\cdot) \); and at \( z = 1 \), any value of \( \hat{X} \) is optimal by indifference.

So to ensure that this discontinuous rule is a MPE policy rule, all that needs to be checked is that private sector best responses to (10) constitute equilibrium behavior. For all \( z < 1 \), (10) coincides with the original MPE rule, so the decision rules, \( P(s, X) \) and \( Z(s, X) \), given by (4) and (5) are optimal by definition. Moreover, at \( z = 1 \), current period pricing decisions are independent of current money growth. Hence, to ensure equilibrium, all that needs to be checked is that all firms that chose flexibility under the original policy rule (so that the MA inherits \( z = 1 \) today) continue to do so under (10). That is, what must be true is that at \( z = 1 \), no firm that chose flexibility finds it profitable to deviate to stickiness.

Since firms are differentiated only by their fixed cost, it suffices to check that firms with the highest fixed cost, \( \varphi_i = \varphi_{\text{max}} \), do not deviate. Clearly, the profitability of such a deviation depends on the value of \( \hat{X} \). For example, it cannot be the case that \( \hat{X} = 1 \). With zero money growth between periods, a firm could set a sticky price as an optimal markup over constant current and future marginal cost, and earn (discounted gross) two-period profits identical to those earned under flexible prices. Since this saves on incurring the fixed cost, all firms would find this deviation to stickiness profitable. Hence, for (10) to constitute

\(^{12}\)See the last paragraph of Section 3 for further discussion.
a MPE policy rule, it must be that \( \hat{X} \) is large enough to ensure no profitable deviation. In Appendix C, I characterize the smallest value of money growth – call this \( \hat{X}^{\text{min}} \) – so that no profitable deviation exists. From this, the following result holds:

**Proposition 5** Let \( \hat{X} \in [\hat{X}^{\text{min}}, \infty) \), where \( \hat{X}^{\text{min}} \) is defined in (14). Then the discontinuous policy rule (10) is a MPE policy rule.

Recall that the pessimistic steady state with a differentiable policy rule displays full price flexibility for any reasonable value of \( \varphi_{\text{max}} \). This result states that when discontinuous policy rules are considered, there are, in fact, a continuum of pessimistic steady states displaying arbitrarily high money growth and inflation. The only constraint on money growth is that it is greater than \( \hat{X}^{\text{min}} \).

In addition, the MA’s strict indifference at full flexibility allows me to consider MPE policy rules in which the MA plays a mixed strategy at \( z = 1 \). In particular, I consider policy rules of the following form:

\[
\tilde{\chi}(\bar{p}, z) = \begin{cases} 
\chi(\bar{p}, z) & \text{for all } \bar{p} \text{ and } z < 1 \\
\hat{X} & \text{for all } \bar{p} \text{ and } z = 1 \text{ with probability } \delta \\
1 & \text{for all } \bar{p} \text{ and } z = 1 \text{ with probability } 1 - \delta 
\end{cases}
\]  

where, \( \chi(\bar{p}, z) \) is the differentiable MPE policy rule of Subsection 6.1. When the MA inherits no sticky prices it generates positive money growth, \( \hat{X} > 1 \), with probability \( \delta \), and zero money growth otherwise. Again, showing that (11) is a MPE policy rule entails checking that at \( z = 1 \), no firm finds it profitable to deviate to stickiness.

Note, however, that MPE cannot exist for all values of \( \delta \). For instance, in the neighborhood of \( \delta = 0 \), the optimal two-period price implies a negative profit margin when \( \hat{X} \) is realized. The sticky price firm would choose – given the option – to shut down rather than meet demand and earn negative profits. Hence, for \( \delta \) sufficiently small, a sticky price firm will find it optimal to simply set a price in anticipation of zero money growth and shut down when the positive inflation state is realized. Allowing for the option of shut down puts a lower bound on the set of feasible mixing probabilities, \( \delta \). In Appendix C, I derive
the smallest value of $\delta$—call this $\delta_{\min}$—such that at $z = 1$, firms do not find it profitable to deviate to stickiness with shut down when $\tilde{X} > 1$ is realized. Hence, (11) is a MPE policy rule as long as $\delta \geq \delta_{\min}$. For each feasible $\delta$, there is a minimum value of money growth—call this $\tilde{X} (\delta)_{\min}$—analogous to $\hat{X}_{\min}$ for $\delta = 1$. Further discussion is contained in Appendix C; here I summarize as follows:

**Proposition 6** Let $\delta \geq \delta_{\min}$, where $\delta_{\min}$ is defined in (15). Then for $\tilde{X} \in \left[\tilde{X} (\delta)_{\min}, \infty\right)$, the mixed strategy policy rule (11) is a MPE policy rule.

As an illustration, Figure 6 plots $\tilde{X} (\delta)_{\min}$ for $\delta \in [\delta_{\min}, 1]$. The figure is plotted for the baseline calibration of the model as described in Subsection 6.1, with $\varphi_{\text{max}} = 3.6\%$ of steady state firm revenue (see Devereux and Siu, 2004). As $\delta$ increases, the smallest feasible value of positive money growth, $\tilde{X} (\delta)_{\min}$, at first falls and then increases. In Appendix C, I discuss the source of this non-monotonicity.

The existence of MPE with mixed strategy policy rules makes it clear that outcomes can differ drastically depending on whether prices are modeled as being exogenously or endogenously rigid. Hence, the characterization of pessimistic equilibrium differs drastically across the two models. With exogenous price rigidity, pessimism is reflected in a unique, high value of inflation in the steady state. But with endogenous price rigidity, a continuum of inflation rates can occur. Moreover, when the MA is playing mixed strategies, the pessimistic ‘steady state’ will display stochastic money growth and inflation. High inflation will prevail only occasionally (i.e. with probability $\delta$), and the rest of the time prices will be perfectly stable. In fact, for the calibration illustrated in Figure 6, it is possible that pessimism will be reflected in high inflation being realized less than half of the time.

Finally, it is worth noting that across all of these pessimistic equilibria with discontinuous and mixed strategy policy rules, the (real) outcomes for consumption and labor are identical to those from the case with a differentiable policy rule, discussed in Subsection 6.1.2. The only difference across equilibria are the predictions for (nominal) inflation rates. Moreover, the level of real output and consumption in the pessimistic equilibria are higher than those generated by the optimistic equilibrium.
7. A MODEL WITH UNIQUE EQUILIBRIUM

In the analysis presented above, the influence of monetary policy on real variables operates through the presence of sticky prices. Inflation erodes the real value of preset prices, mitigating the distortion due to monopolistic firms (generating a benefit of inflation), but simultaneously generates relative price distortions across otherwise identical production technologies (a cost of inflation). In Subsection 6.2, the rich set of MPE arises because with full price flexibility, money growth has no effect on either of these distortions, and thus on real outcomes or household welfare (again, see the discussion of Section 3). As a result, the MA is indifferent between all values of $X$. The only constraint on $X$ is that it ensures the choice of flexibility as being optimal for all firms making their pricing decisions. In this section, I consider an arbitrarily small perturbation to the model to break the MA’s indifference at full flexibility. This modification implies that equilibrium cannot exist with fully flexible prices. As a result, for quantitatively reasonable specifications of the fixed cost of price change, the model generates a unique MPE.

The modification I consider introduces a non-zero cost of inflation that is independent of the degree of price rigidity. In particular, suppose there is an arbitrarily small resource cost of money creation, $g = \varepsilon |X - 1|, \varepsilon > 0$. That is, printing money (or taking money out of circulation) is costly in terms of final goods. The MA finances money creation via lump-sum taxation, so that the MA’s budget constraint is:

$$
T_t = M_t - M_{t-1} - \theta_{t-1} \left( W_{t-1} h_{t-1} + \int_0^1 \Pi_{i,t-1} dt \right) - g_t, \quad \forall t.
$$

The modified model’s aggregate resource constraint is now:

$$
e + g = y.
$$

The rest of the model description is identical to Section 2. Apart from its effect on the monopoly distortion and relative price distortion, $X$ has a direct effect on utility by determining the fraction of produced output that is available for consumption. Hence, when prices are fully flexible, the maximizing MA sets $X = 1$ in order to minimize printing costs.
This strict preference for zero money growth at full flexibility introduces an obvious profitable deviation for flexible price firms. Suppose \( z = 1 \) so that \( X = 1 \). Given zero money growth, an individual firm considering a deviation to stickiness would set a sticky price as a markup over constant current and future marginal cost, \( \bar{p}' = \bar{p} = \lambda \psi \). The firm would earn identical gross profits by choosing stickiness relative to choosing flexibility, but without incurring the fixed cost. As a result, any individual firm would deviate to stickiness, meaning that \( z \neq 1 \). No equilibrium exists with fully flexible prices.

The quantitative analysis of Section 6 indicates that in the pessimistic steady state of the unmodified model, prices are fully flexible. Hence, allowing for an arbitrarily small cost of inflation independent of the degree of price rigidity implies that pessimistic equilibria do not exist. For reasonable quantitative specifications, the model predicts a unique optimistic MPE, so there is no multiplicity.

Finally, note that there are many ways to introduce a cost of inflation that is independent of price rigidity. For instance, within the context of the model presented in Section 2, a natural candidate would be explicit consideration of the interest rate distortion stemming from the cash-in-advance constraint. Without the income subsidy \( (\theta = 0) \) positive inflation drives a wedge between the real wage and the marginal rate of substitution in consumption-labor. By suitably modifying the specification of the subsidy, the wedge is minimized at \( X = 1 \), and when \( z = 1 \), this would be the MA’s optimal choice.\(^\text{13}\) Clearly, this is susceptible to the same deviation on the part of firms making their pricing decisions as before, so the characterization of the non-existence result would remain.

8. CONCLUSION

In this paper I have characterized time consistent equilibria in a model with monetary discretion and an endogenously determined degree of price rigidity. The endogeneity is introduced by allowing firms to determine their frequency of price change; more frequent price change involves incurring a fixed cost.

\(^{13}\)Further details are available from the author upon request.
When attention is restricted to differentiable policy rules, there exist two time consistent equilibria: an ‘optimistic’ equilibrium with low inflation, and a ‘pessimistic’ equilibrium with high inflation. This is in keeping with previous results from models with an exogenously determined degree of price rigidity. But for quantitatively reasonable specifications of the current model, the steady state of the pessimistic equilibrium displays full price flexibility. As a result, the implications for inflation and real output are very different. This implication is made clear by showing that a rich set of inflation outcomes can be supported in the pessimistic steady state. This is true when the monetary authority plays policy rules which are discontinuous, or which specify mixed strategies, at full flexibility. Moreover, real output is independent of inflation in the pessimistic steady state, and is in fact higher than in the optimistic low inflation steady state. Finally, I introduce an arbitrarily small cost of inflation into the model that is independent of price rigidity. I show that quantitative versions of this model display unique time consistent equilibrium. That is, for reasonably specified fixed costs of price change, this modification eliminates multiplicity of equilibrium.

**APPENDIX A**

Given a policy rule, $\chi(\cdot)$, I calculate the steady state corresponding to the PSE when $X \equiv \chi(\bar{p}, z)$.\(^{14}\) I use these values as the PS state $(\bar{p}, z, X) = (\bar{p}_{ss}, z_{ss}, \chi(\bar{p}_{ss}, z_{ss}))$. I then consider a range of prices for firms who choose to set a two-period price in the current period. For each of these sticky prices, $\bar{p}'_j$, the best response, $\bar{p}'_i$, is given by:

$$
\bar{p}'_i \equiv f(\bar{p}'_j, z') = \lambda \psi \left[ (1 - \gamma (\bar{p}'_j, z')) + \gamma (\bar{p}'_j, z') \chi (\bar{p}'_j, z') \right]. \quad (12)
$$

Here, dependence on $(\bar{p}, z, X)$ is suppressed for exposition. Note, that future money growth, $\chi \left( \bar{p}'_j, z' \right)$, and the relative weight on current versus future marginal cost must account for

---

\(^{14}\)If multiple steady states exist, I consider the one corresponding to the lowest value of normalized prices and inflation.
the fact that some firms choose price flexibility. That is:

\[ \gamma(p'_j, z') = \frac{p^{\lambda-1}}{p^{\lambda-1} + \beta p_j^{\lambda-1} \chi(p'_j, z')} \]

where

\[ p^{\lambda-1} = \left\{ \frac{1}{2} \left[ (1 - z) \left( \frac{\bar{p}}{\chi} \right)^{1-\lambda} + (1 - z') p_j^{1-\lambda} + (z + z') \bar{p}^{1-\lambda} \right] \right\}^{-1}, \]

\[ p_j^{\lambda-1} = \frac{1}{2} \left[ (1 - z') \left( \frac{\bar{p}_j}{\chi(p'_j, z')} \right)^{1-\lambda} + (1 - z'') p_j^{''1-\lambda} + (z' + z'') \bar{p}^{''1-\lambda} \right]^{-1}, \]

\[ \bar{p} = p' = \frac{\lambda \psi}{(\lambda - 1)}, \quad p'' = \text{P} \left( \bar{p}_j, z', \chi \left( \bar{p}_j, z' \right) \right), \text{ and } \quad z'' = \text{Z} \left( \bar{p}_j, z', \chi \left( \bar{p}_j, z' \right) \right). \]

Given \( \bar{p}_j' \), I find \( z' = z' \left( \bar{p}_j' \right) \) as the value which satisfies:

\[ \frac{p^{\lambda-1}}{p_j^{\lambda}} (\bar{p} - \psi) + \beta \left[ \frac{p_j^{\lambda-1}}{p_j} (\bar{p}_j' - \psi) - \psi F^{-1}(z') \right] = \frac{p^{\lambda-1}}{p_j^{\lambda}} (\bar{p}_j' - \psi) + \beta p_j^{\lambda-1} \left( \frac{\chi \left( \bar{p}_j, z' \right)}{\bar{p}_j'} \right) \left( \frac{\bar{p}_j'}{\chi(p'_j, z')} - \psi \right). \]

Hence, I view the weight, \( \gamma \left( \bar{p}_j', z' \right) = \gamma \left( \bar{p}_j', z' \left( \bar{p}_j' \right) \right) \) as a function of only \( \bar{p}_j' \). This allows me to map out the best response function:

\[ \bar{p}_i' = f \left( \bar{p}_j', z' \left( \bar{p}_j' \right) \right) = \bar{f} \left( \bar{p}_j' \right). \]

**APPENDIX B**

To be added.

**APPENDIX C**

C.1. Derivations for the Discontinuous Policy Rule

Here, I characterize \( \hat{X}^{\text{min}} \), the smallest admissible value of money growth at \( z = 1 \), such that (10) is a MPE policy rule. To this end, consider all values of \( s = (\bar{p}, z) \) such that
Let \( z' = Z(s, \chi(s)) = 1 \), where \( \chi(\cdot) \) is the original differentiable MPE policy rule of subsection 6.1. Denote these states as \( \hat{s} \subseteq \sigma \). Given that \( X' = \hat{X} \), flexible price profits for the \( \varphi_i = \varphi_{\max} \) firm at state \( \hat{s} \in \hat{\sigma} \) are given by:

\[
\tilde{\Upsilon}(\hat{s}) \equiv \frac{p^\lambda - 1}{\bar{p}^\lambda} (\tilde{p} - \psi) + \beta \left[ \frac{p^\lambda - 1}{\bar{p}^\lambda} (\tilde{p} - \psi) - \psi \varphi_{\max} \right],
\]

where \( \tilde{p} = \hat{\lambda} \psi \),

\[
p^\lambda - 1 = \left\{ \frac{1}{2} \left[ (1 - z) \left( \frac{\bar{p}}{\chi(\hat{s})} \right)^{1-\lambda} + (z + 1) \tilde{p}^{1-\lambda} \right] \right\}^{-1},
\]

\[
p'^\lambda - 1 = \left\{ \frac{1}{2} \left[ (1 - z') \bar{p}^{\lambda - 1} + (1 + z') \tilde{p}^{1-\lambda} \right] \right\}^{-1},
\]

and \( z'' = Z(., 1, .) \) and \( \bar{p}'' = P(., 1, .) \) are the optimal pricing decisions given \( z' = 1 \). Of course, given that \( z' = 1 \), the decision rules \( P(\cdot) \) and \( Z(\cdot) \) are independent of the values of \( \bar{p}' \) and \( X' \). I index \( \tilde{\Upsilon} \) by \( \hat{s} \) to emphasize that flexible price profits depend on \((\bar{p}, z)\) via \( p^\lambda - 1 \).

Conversely, if an individual firm chooses to deviate by charging a sticky price, it would earn profits:

\[
\tilde{\Upsilon}(\hat{s}, \hat{X}) \equiv \frac{p^\lambda - 1}{\bar{p}^\lambda} (\tilde{p}' - \psi) + \beta p'^\lambda - 1 \left( \frac{\hat{X}}{\bar{p}'} \right) \left( \tilde{p}' - \psi \right),
\]

where

\[
\tilde{p}' = \hat{\lambda} \psi \left( \frac{p^\lambda - 1 + \beta p'^\lambda - 1 \hat{X}^\lambda}{p^{\lambda - 1} + \beta p'^\lambda - 1 \hat{X}^{\lambda - 1}} \right).
\]

In order for the deviation to be unprofitable, it must be that:

\[
\tilde{\Upsilon}(\hat{s}) \geq \tilde{\Upsilon}(\hat{s}, \hat{X}). \tag{13}
\]

Let \( \hat{X}(\hat{s})^{\min} \) denote the smallest \( \hat{X} \) such that this holds at \( \hat{s} \). Moreover, condition (13) must hold for all \( \hat{s} \in \hat{\sigma} \). Hence, in order for \( \hat{\chi}(\tilde{p}, z) \) – with \( \hat{\chi}(\tilde{p}, 1) = \hat{X} \) – to constitute a MPE policy rule, it must be that \( \hat{X} \geq \hat{X}^{\min} \), where:

\[
\hat{X}^{\min} = \max_{\hat{s} \in \hat{\sigma}} \left\{ \hat{X}(\hat{s})^{\min} \right\}. \tag{14}
\]

I first characterize $\delta_{\text{min}}$, the smallest admissible mixing probability such that (11) is a MPE policy rule when firms have the option of shut down. Again, it suffices to check that firms with the highest fixed cost, $\varphi_i = \varphi_{\text{max}}$, do not deviate to sticky price-setting. Denote the states such that $z' = Z(s, \chi(s)) = 1$ as $\bar{\sigma} \subseteq \sigma$. Suppose the deviating firm adopts a strategy of shut down when $X$ is realized. Since it is pricing only for the zero money growth state in the future period, it charges a sticky price identical to the optimal flexible price, $\bar{p}' = \bar{p} = \hat{\lambda}\psi$. Profits from this deviation are:

$$\Theta(\tilde{s}) \equiv \frac{p^\lambda - 1}{\bar{p}^\lambda} (\bar{p} - \psi) + \beta (1 - \delta) \frac{p'^\lambda - 1}{\bar{p}^\lambda} (\bar{p} - \psi),$$

where $p^\lambda$ and $p'^\lambda$ are as given in subsection C.1 with $\hat{\chi}(\tilde{s})$ replaced by $\check{\chi}(\tilde{s})$, $\tilde{s} \in \tilde{\sigma}$. To ensure that this deviation is not profitable, it must be that $\check{\Upsilon}(\tilde{s}) \geq \bar{\Upsilon} \tilde{s}, X \check{\Upsilon}$; simplifying this condition indicates that this holds whenever $\delta \geq (\bar{p}^{\lambda}\varphi_{\text{max}}) / \left[p'^\lambda \left(\hat{\lambda} - 1\right)\right]$. Note that this condition is independent of $\tilde{s}$ and $\check{X}$. Hence, the smallest feasible mixing probability is:

$$\delta_{\text{min}} = \frac{\bar{p}^{\lambda}\varphi_{\text{max}}}{p'^\lambda \left(\hat{\lambda} - 1\right)}. \tag{15}$$

For all $\delta \geq \delta_{\text{min}}$, the value of $\tilde{X}(\delta)^{\text{min}}$ is defined in an identical fashion to $\check{X}^{\text{min}}$ in subsection C.1.

Finally, I briefly discuss the relationship between $\delta$ and $\tilde{X}(\delta)^{\text{min}}$. Again, $\tilde{X}(\delta)^{\text{min}}$ is defined as the value of money growth such that $\check{\Upsilon}(\check{s}) \geq \check{\Upsilon} \left(\tilde{s}, \tilde{X}\right)$ Since, flexible price profits are independent of $\delta$ and $\check{X}$, the slope $\partial \tilde{X}(\delta)^{\text{min}} / \partial \delta$ depends on the effect of $\delta$ on sticky price profits, $\partial \check{\Upsilon} / \partial \delta$, for a given value of $\tilde{X}$. Since $\partial \check{\Upsilon} / \partial \tilde{X} < 0$, $\text{sign} \left[\tilde{X}(\delta)^{\text{min}}\right] = \text{sign} \left[\partial \check{\Upsilon} / \partial \delta\right]$. The U-shaped pattern, then, is due to the fact that a change in $\delta$ affects sticky price profits via two offsetting channels – first, through the change in weight placed on profits across the positive and zero inflation states; and second, through the change in profits earned in each state, due to the effect of $\delta$ on the optimal sticky price, $\bar{p}'$. Whether the slope, $\partial \tilde{X}(\delta)^{\text{min}} / \partial \delta$, is positive or negative depends on the sign of each effect and the relative
strength of each effect at a particular value of $\delta$.$^{15}$

REFERENCES


$^{15}$Derivations are available from the author upon request.


Fig. 1: Best Response Function: Zero Money Growth Rate

The diagram illustrates the best response function, $f(p_{bar_j}')$, for a firm's price setting in a market with zero money growth rate. The vertical axis represents the best response price, $p_{bar_i}'$, while the horizontal axis represents the price set by all other firms, $p_{bar_j}'$. The $45^\circ$ line indicates the equality of the best response price and the price set by other firms. The function $f(p_{bar_j}')$ shows how the best response price changes in response to the price set by other firms.
Fig. 2: Best Response Function: Linear Policy Rule

The graph illustrates the best response function for a firm's price, $p_{bar_i}'$, to the price set by all other firms, $p_{bar_j}'$. The function $f(p_{bar_j}')$ represents the best response of firm $i$ to the price set by firm $j$. The 45° line indicates the equality of the best response and the price set by all other firms.
Fig. 3: Best Response Function: Non-Linear Policy Rule

The best response price, $p_{\text{bar}i}'$, is set by each firm in response to the price set by all other firms, $p_{\text{bar}j}'$. The function $f(p_{\text{bar}j}')$ describes how the best response price changes with the price set by other firms. The 45° line indicates the equality of the best response price and the price set by other firms.
Fig. 4A: Best Response Function: big max fixed cost

Fig. 4B: Degree of Price Flexibility: big max fixed cost

Fig. 4C: Best Response Function: small max fixed cost

Fig. 4D: Degree of Price Flexibility: small max fixed cost
Fig. 5: Markups, Fixed Costs, and Price Rigidity in the Pessimistic Steady State

- Some price rigidity in pessimistic steady state
- Full price flexibility in pessimistic steady state
Fig. 6: Minimum Money Growth Rates and Mixing Probabilities

feasible money growth rates, $\tilde{X}(\delta)$

not feasible