Monetary Policy and the Distribution of Money and Capital

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(Very Preliminary and Incomplete)

Abstract

Existing search-theoretical models of money have in general abstracted from the existence and accumulation of other assets, in particular, capital. In this paper we present a model where the optimal portfolio allocation decision of agents is explicitly modeled. Trade frictions in a decentralized consumption goods market give rise to an endogenous role for money. Capital goods are assumed to be type-specific and traded in a centralized market. Uninsurable idiosyncratic uncertainty in trading opportunities leads to a non-degenerate distribution of wealth. By focusing on stationary equilibria we characterize numerically the wealth distribution and its composition. We further analyze the effects of monetary policy on the equilibrium patterns of exchange, the distribution of wealth, capital accumulation, and welfare. In particular, we show that redistributive effects of monetary policy can lead to a positive optimal rate of inflation.

JEL Classification: E40, E50

Keywords: search; money; capital; monetary policy; redistribution; wealth distribution

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1 Introduction

The question of what is the optimal rate of inflation has interested and puzzled economist for a long time. While most traditional representative agent models used for monetary policy analysis prescribe the Friedman’s Rule as the optimal monetary policy rule, the discussion in most Central Banks seems to be centered on what (non-negative) rate of inflation to target. Several explanations have been proposed for this apparent paradox. Some authors have suggested that, given the need to finance some level of government expenditures, the inflation tax collected by following an expansionary monetary policy might be less distortionary than alternative forms of taxation. Other authors have stressed the difficulties in implementing a contractionary monetary policy. A recent branch in the literature has suggested an alternative explanation which representative agent models are unable to capture. A common feature to most of the literature that prescribes the optimality of the Friedman’s Rule is that, in those models, there is perfect insurance against any form of idiosyncratic risk (and thus the justification for a representative agent assumption). However, recent fundamental models of monetary economies based on microfoundations (Levine(1991), Molico(1997), Deviatov and Wallace(2001)) have stressed that the same frictions that give rise to an endogenous role for money (and some sort of market incompleteness) will also generate uninsurable idiosyncratic risk. This gives rise to endogenous heterogeneity and a potential insurance role for expansionary monetary policy through redistribution of wealth. In these models, although the same forces that arise in the models for which the Friedman Rule is optimal are at play, an additional redistributive effect of monetary policy can potentially lead to an optimal positive rate of monetary expansion.

A limitation of this recent work is that it assumes an extreme degree of market incompleteness - money is the only asset. In this paper, we extend this literature
by considering a search-theoretical model of money where agents can hold and accumulate both money and capital. By allowing agents to accumulate capital (and thus partially self-insure) we might reduce the insurance role provided by an expansionary monetary policy. On the other hand, by affecting the asset’s portfolio allocation decision of agents, monetary policy can potentially affect the productive capacity of the economy. In either case, a real balance effect of expansionary monetary policy is always present. The goal of this paper is to analyze and quantify the effects of expansionary monetary policy on the equilibrium patterns of exchange, the distribution of prices, the agent’s equilibrium portfolio allocation decision, the distributions of money and capital, and welfare.

We consider a search-theoretical model of money where both goods and money are perfectly divisible. Unlike most of the models in the literature we allow consumers to store their own (and only their own) consumption good, which can be used for consumption or as an input (capital) in the production of an agent’s own production good. By this assumption, we preclude capital (the stored consumption good) to compete with money as a medium of exchange, allowing it however to still serve as a store of value and to serve a productive role. We assume that there are two types of markets that open sequentially. A decentralized goods market where production and trade take place, and a centralize capital market where agents can re-optimize their asset’s portfolio but no production can take place. Trade frictions in the decentralized consumption goods market give rise to an endogenous

\[\text{In this paper we abstract from the issue of co-existence of different media-of-exchange and potential rate-of-return dominance. A recent paper that attempts to address those issues while abstracting from the redistributive effects of monetary policy is Lagos}(2004).\]

\[\text{The role of the centralized market is very different that the one in Lagos and Wright}(2004).\] There production of general goods takes place in the centralized market which together with the quasi-linear assumption on preferences the authors make, serves as an insurance against the idiosyncratic risk the agents face in the decentralized market. As a result the wealth distribution in that model is degenerate. Still, in both models the centralized market serves as a market for liquidity.
role for money. In this market agents meet randomly and bilaterally and bargain over the amounts of money and goods to be traded. In the centralized market agents are anonymous and trade money for capital (consumption good).\(^3\)

The presence of uninsurable idiosyncratic risk in trade opportunities leads to heterogeneity in wealth and in asset’s portfolio choices. Thus the state of the economy will be described by the joint distribution of money and capital among the agents in the economy, making it impossible to provide an analytical solution to the model. As such, we develop a numerical algorithm that allows us to solve for and characterize stationary equilibria. Furthermore, we use these numerical methods to provide quantitative answers to monetary policy questions.

Regarding the effects of monetary policy we show that money is always neutral in the long-run and also in the short-run if monetary injections are accomplished via proportional transfers. However, if the transfer mechanism generates a redistribution of wealth, increases in the level of the money supply will have short-run real effects. Furthermore, regarding the effects of changes in the rate of monetary growth, we show that proportional transfers are superneutral but that redistributive lump-sum transfer can have permanent real effects. Our preliminary numerical results suggest that a moderate rate of monetary expansion can lead to an increase in aggregate output, aggregate consumption, capital accumulation, and welfare. Also, the average fraction of time spent working might decrease.\(^4\)

As such, our example suggests that the model can potentially generate, for some parameter values, a positive optimal rate of inflation.

\(^3\)We assume that investment is reversible, that is, agents can eat their stored consumption good (capital) at the end of any period. So far, we have not analyzed the implications of the alternative assumption - irreversibility of investment.

\(^4\)Note that, due to computational restrictions, we only compare across steady states and thus ignore potential important effects along the transition path. As such, the usual care in interpretation of the results is required.
[Additional Literature Review - TO BE ADDED].

The remainder of the paper is organized as follows. In section 2, we describe the model economy. In section 3, we define a recursive competitive equilibrium for this economy. We then proceed to develop and present a numerical algorithm that allows us to compute stationary equilibria of the model. Finally, in section 5, we use this algorithm to characterize the stationary equilibria of the model and to illustrate the effects of a redistributive expansionary monetary policy.

2 The Model

Population, Goods and Specialization

Time is discrete. There is a $[0, 1]$ continuum of infinitely lived agents who specialize in the consumption, storage, and production of perfectly divisible goods. There are $N$ varieties of goods and $N$ types of agents, $i = 1, \ldots, N$ ($N \geq 3$), with an equal measure $\frac{1}{N}$ of agents of each type. Agent type $i$ consumes good $i$, has the ability to store solely good $i$, and produces good $i + 1 \pmod{N}$.

Technology

Production requires the use of capital (an agent’s stored consumption good) and labor inputs. Capital is production good specific. The production of good $i + 1$ requires the input of good $i$. Agent type $i$ combines good $i$ and his own labor.

\footnote{Note that, unlike what it is commonly assumed in the search-theoretical literature, e.g. Trejos and Wright (1995), we allow agents to store goods which can be used as an input in production. However, each type of agent can only store his/her own consumption good which prevents goods from being used as a medium-of-exchange, although they can serve as a store-of-value. The issues of coexistence of different media-of-exchange and potential rate of return dominance are interesting and important but we will not pursue them in this paper.}
effort to produce good $i+1$ according to a Cobb-Douglas production function:

$$y_{i+1} = f(k_i, l) = Ak_i^\alpha l^{1-\alpha}, \quad 0 < \alpha < 1, \ A > 0.$$ 

Capital depreciates at rate $0 < \delta < 1$.

Preferences

Agents derive utility $u(c)$ from consuming $c$ units of their type-specific consumption good and disutility $g(l)$ from providing $l$ units of labor effort. Assume $u$ is twice continuously differential with $u(0) = 0$, $u'(c) > 0$, and $u''(c) < 0$. Also, assume $g$ is twice continuously differentiable with $g(0) = 0$, $g'(l) > 0$, $g''(l) > 0$, $\lim_{l \to 0} g'(l) = 0$, and $\lim_{l \to 1} g'(l) = +\infty$. The per-period utility is given by

$$U(c, l) = u(c) - g(l).$$

Agents discount the future at discount factor $0 < \beta < 1$.

For notational convenience, let $v(y, k)$ denote the disutility of producing an amount $y$ of output when using an amount $k$ of capital.

$$v(y, k) \equiv g \left[ \left( \frac{y}{Ak^\alpha} \right)^{1-\alpha} \right]$$

Assets

In addition to capital, in this economy there is another, perfectly divisible, and costlessly storable, object which cannot be produced or consumed by any private individual, called fiat money. Agents can hold any nonnegative amount of money $\hat{m} \in \mathbb{R}_+$. Let $M_t$ denote the money supply at the beginning of period $t$.

Markets

\footnote{For simplicity, we assume capital depreciates at the same rate independently of whether it has been used in production or simply stored. More generally, one could consider different rates of depreciation.}
There are two markets which open sequentially during each period. A decentralized goods market and a centralized Walrasian capital market. In the decentralized goods market agents meet randomly and bilaterally, bargain, produce, and trade. In the centralized capital market agents are anonymous, and can optimize their assets portfolio by purchasing or selling capital, but cannot produce. Note that, the centralized market is a market for liquidity where stored goods can be traded for money but where no production is allowed. Allowing for production in this market would eliminate the need for a medium of exchange.\footnote{The role of the centralized market is very different than the one in Lagos and Wright(2000).[to be completed]}

**Monetary Policy**

Agents receive monetary transfers, $\hat{\tau}_t(\hat{m}, k)$ at the entrance of the centralized market, before trade occurs. In what follows we express all nominal variables as fraction of the beginning of the period money supply (before the current period’s monetary transfer).

$$m \equiv \frac{\hat{m}}{M}$$

$$\tau(m, k) \equiv \frac{\hat{\tau}(\hat{m}, k)}{M}$$

The aggregate money supply grows at a constant rate $\mu$,

$$M_{t+1} = (1 + \mu)M_t.$$  

**Decision Timing**

The timing of the model is as follows. At the beginning of each period agents are randomly matched with a potential trading partner in the decentralized goods market. If in the meeting there is single-coincidence-of-wants,\footnote{Note that, by assumption, we excluded the possibility of double-coincidence-of-wants meetings and thus barter. Allowing for barter does not change qualitatively the analysis of the paper.} agents bargain over
the quantity of money and goods to be exchanged, and production and exchange take place. For simplicity, we assume that capital depreciates after the decentralized market closes. After that, the centralized capital market opens. Monetary transfers take place, agents choose consumption and the asset’s portfolio allocation for next period, trade, and consume.\(^9\)

This concludes the description of the environment. In what follows, we will build gradually towards the definition of equilibrium.

### 3 Equilibrium

In this section we define a recursive equilibrium for this economy. We begin by describing the individual and aggregate state variables. An individual’s state variable consists of his current portfolio of money balances, as fraction of the current money supply, and capital. For notational convenience, we denote an individual’s state variable by \(s \equiv (m, k)\), where \(s \in S \equiv \mathbb{R}_{+} \times \mathbb{R}_{+}\). The aggregate state variable is, in turn, defined as the current probability measure over money holdings (measured as fraction of the current money supply) and capital, denoted \(\lambda\). Let \(\lambda : S \rightarrow [0, 1]\), where \(S \equiv B_{\mathbb{R}_{+}} \times B_{\mathbb{R}_{+}}\) denotes the Borel subsets of \(S\).

The agent takes as given the law of motion of the aggregate state variable, \(\lambda' = H(\lambda)\), where prime denotes the future period. Later, we will describe in detail the law of motion. Also, the agent takes as given the price of capital in the centralized market, \(p_k\), as a function of the current aggregate state, that is, \(p_k : \Lambda \rightarrow \mathbb{R}_{+} \setminus \{0\}\), where \(\Lambda\) denotes the space of probability measures over \(S\).

\(^9\)We assume that investment is reversible, i.e., agents can eat their stored consumption/capital good. Furthermore, they can potentially purchase consumption good from other agents of their type. The price of the good however, will never be cheaper that the price at which an agent can acquire the good in the decentralized market, as will be shown below.
In what follows we describe the value functions, taking as given the terms of trade in the decentralized market. Let $y(s_b, s_s; \lambda)$ and $d(s_b, s_s; \lambda)$ denote, respectively, the amount of goods and money (measured as a fraction of the current money supply) traded in the decentralized goods market between a buyer with individual state $s_b$, and a seller with individual state $s_s$, when the aggregate state is $\lambda$, taking as given the pricing function $p_k$. Later, we will specify the determination of the terms of trade.

Let $V(s; \lambda)$ be the value function for an agent holding portfolio $s$, when the aggregate state is $\lambda$, at the entrance of the decentralized market. Assume $V(\cdot; \lambda)$ is a continuous function. Also, let $W(\tilde{m}, \tilde{k}; \lambda)$ be the value of entering the centralized capital market holding the portfolio $(\tilde{m}, \tilde{k})$ when the aggregate state is $\lambda$.

We can then write the following functional equation expressing the relationship between the two value functions:\(^{10}\)

$$V(s; \lambda) = \frac{1}{N} \int_S W[m - d(s, s_s; \lambda), (1 - \delta)k + y(s, s_s; \lambda); \lambda] \lambda(ds_s)$$

$$+ \frac{1}{N} \int_S \left[ -v[y(s_b, s; \lambda), k] + W[m + d(s_b, s; \lambda), (1 - \delta)k; \lambda] \right] \lambda(ds_b)$$

$$+ \frac{N - 2}{N} W[m, (1 - \delta)k; \lambda]. \quad (1)$$

The first integral term on the r.h.s. is the expected value of being a buyer, the second integral term is the expected value of being a seller, and the last term is the expected value of a no single-coincidence-of-wants meeting, in which case no trade occurs.

The value of entering the centralized market holding portfolio $\tilde{s}$, given the

\(^{10}\)For now, assume $d(\cdot, \cdot; \lambda)$, $y(\cdot, \cdot; \lambda)$, and $W(\cdot; \lambda)$ are measurable functions. We will later show that, given $V(\cdot; \lambda)$ is continuous, $W(\cdot; \lambda)$ is continuous, and thus measurable, and that $d(\cdot, \cdot; \lambda)$ and $y(\cdot, \cdot; \lambda)$ can be defined as being measurable selections from a u.h.c. correspondence.
aggregate state $\lambda$, is defined as

$$
W(\bar{s}; \lambda) = \max_{c \geq 0, s' \geq 0} u(c) + \beta V(s'; \lambda')
$$

subject to:

$$
\frac{\bar{m}}{p_k(\lambda)} + \bar{k} + \frac{\tau(\bar{m}, \bar{k})}{p_k(\lambda)} = c + \frac{m'}{p_k(\lambda)}(1 + \mu) + k'
$$

$$
\lambda' = H(\lambda).
$$

By the Theorem of the Maximum, $W(\cdot; \lambda)$ is a continuous function and set of optimizers is a nonempty, compact-valued, and u.h.c correspondence. By the Measurable Selection Theorem, this correspondence permits a measurable selection. Define $\chi$, $\Psi$, and $\kappa$ as (measurable) sections of such correspondence, with

$$
c = \chi(\bar{s}; \lambda)
$$

$$
m' = \Psi(\bar{s}; \lambda)
$$

$$
k' = \kappa(\bar{s}; \lambda).
$$

We will now describe the determination, by bargaining, of the terms of trade in the decentralized market. We will adopt the generalized Nash bargaining solution where the buyer has bargaining power $\theta$, $0 < \theta < 1$, and with the threat point for an agent given by his continuation value $W(\bar{s}; \lambda)$.

For notational convenience, define $\Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda)$ and $\Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda)$ to be, respectively, the net surplus from trading an amount of good $\bar{y}$ for a fraction of the current money supply $\bar{d}$, for a buyer and a seller holding respectively, the portfolios $s_b$ and $s_s$, given the aggregate state $\lambda$.

$$
\Sigma_b(\bar{y}, \bar{d}, s_b, s_s; \lambda) \equiv W[m_b - \bar{d}, (1 - \delta)k_b + \bar{y}; \lambda] - W[m_b, (1 - \delta)k_b; \lambda]
$$

$$
\Sigma_s(\bar{y}, \bar{d}, s_b, s_s; \lambda) \equiv -v(\bar{y}, k_s) + W[m_s + \bar{d}, (1 - \delta)k_s; \lambda] - W[m_s, (1 - \delta)k_s; \lambda].
$$
Consider the following generalized Nash bargaining problem.

\[
\Sigma(s_b, s_s; \lambda) = \max_{(\tilde{y}, \tilde{d}) \in G(s_b, s_s; \lambda)} \Sigma_b(\tilde{y}, \tilde{d}, s_b, s_s; \lambda)\theta \Sigma_s(\tilde{y}, \tilde{d}, s_b, s_s; \lambda)^{1-\theta},
\]

where \(G : S \times S \to S\) is a correspondence defined by \(G(s_b, s_s; \lambda) = \{(\tilde{y}, \tilde{d}) \in S : 0 \leq \tilde{d} \leq m_b, 0 \leq \tilde{y} \leq Ak^s\}\). Once again, by the Theorem of the Maximum and the Measurable Selection Theorem the correspondence \(D(s_b, s_s; \lambda) = \{(\tilde{y}, \tilde{d}) \in G(s_b, s_s; \lambda) : \Sigma_b(\tilde{y}, \tilde{d}, s_b, s_s; \lambda)^{1-\theta} = \Sigma(s_b, s_s; \lambda)\}\) permits a measurable selection. Define \(y(s_b, s_s; \lambda) : S \times S \to \mathbb{R}_+\) and \(d(s_b, s_s; \lambda) : S \times S \to \mathbb{R}_+\) to be (measurable) sections of such correspondence.

Before defining a recursive equilibrium for this economy, we describe the law of motion \(\lambda' = H(\lambda)\). We begin by describing the evolution of the aggregate state from the beginning of the centralized market to the beginning of the next decentralized market. Define \(\Pi : S \times S \to [0, 1]\) to be

\[
\Pi(\tilde{s}, B; \lambda) = \begin{cases} 
1, & \text{if } [\Psi(\tilde{s}; \lambda), \kappa(\tilde{s}; \lambda)] \in B; \\
0, & \text{otherwise.}
\end{cases}
\]

Given that, for each \(\tilde{s}\), \(\Pi(\tilde{s}, \cdot; \lambda)\) is a probability measure on \((S, \mathcal{S})\), and, for each \(B \in \mathcal{S}\), \(\Pi(\cdot, B; \lambda)\) is a \(\mathcal{S}\)-measurable function\(^{13}\), \(\Pi\) is a well defined transition function. Let \(\Omega(\tilde{s})\) denote the distribution at the entrance of the centralized market, then

\[
\lambda'(B) = \int_S \Pi(\tilde{s}, B; \lambda) \; \Omega(\tilde{s}) \quad \forall B \in \mathcal{S}.
\]

We now describe the evolution of the aggregate state from the beginning of the decentralized market to the beginning of the centralized market. Let \(T =

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\(^{11}\)Note that, given \(W(\cdot, \cdot; \lambda)\) is continuous, the objective function is continuous. Furthermore, it is easy to show that the correspondence \(G\) is compact-valued and continuous.

\(^{12}\)Note that, since \((0, 0) \in G(s_b, s_s; \lambda)\) any solution to the Nash bargaining problem satisfies the participation constraints, that is, \(\Sigma_b[y(s_b, s_s; \lambda), d(s_b, s_s; \lambda), s_b, s_s; \lambda] \geq 0\) and \(\Sigma_s[y(s_b, s_s; \lambda), d(s_b, s_s; \lambda), s_b, s_s; \lambda] \geq 0\).

\(^{13}\)This follows immediately from the measurability of \(\Psi(\cdot; \lambda)\) and \(\kappa(\cdot; \lambda)\).
{buyer, seller, neither} and define the space \((T, T)\), where \(T\) is the \(\sigma\)-algebra. Define the probability measure \(\tau : T \rightarrow [0, 1]\), with \(\tau(buyer) = \tau(seller) = \frac{1}{N}\), and \(\tau(neither) = \frac{N-2}{N}\). Then, \((T, T, \tau)\) is a measure space. Define an event to be a pair \(e = (t, s)\), where \(t \in T\) and \(s \in S\). Intuitively, \(t\) denotes and agents trading status and \(s\) the portfolio of his current trading partner. Let \((E, E)\) be the space of such events, where \(E = T \times S\) and \(E = T \times S\). Furthermore, let \(\xi : E \rightarrow [0, 1]\) be the product probability measure. Define the mapping \(\phi(s, e) : S \times E \rightarrow S\), where

\[
\phi(s, e) = \begin{cases} 
[m - d(s, \cdot; \lambda), (1 - \delta)k + y(s, \cdot; \lambda)], & \text{if } e = (buyer, \cdot); \\
[m + d(\cdot, s; \lambda), (1 - \delta)k], & \text{if } e = (seller, \cdot); \\
[m, (1 - \delta)k], & \text{otherwise.}
\end{cases}
\]  

(11)

We can now define \(\Gamma : S \times S \rightarrow [0, 1]\) to be

\[
\Gamma(s, B; \lambda) \equiv \xi(\{e \in E|\phi(s, e) \in B\}).
\]  

(12)

Again, \(\Gamma\) is a well defined transition function.\(^{14}\) Then,

\[
\Omega(B) = \int_S \Gamma(s, B; \lambda) \lambda(ds) \quad \forall B \in S.
\]  

(13)

Finally, we can describe the law of motion of the aggregate state as

\[
\lambda'(B) = H(\lambda)(B) \equiv \int_S \int_S \Pi(s, \bar{s}; \lambda) \Gamma(s, ds; \lambda) \lambda(ds) \quad \forall B \in S.
\]  

(14)

We are finally ready to define a recursive equilibrium for this economy.

**Definition 1** (Recursive Equilibrium) A recursive equilibrium is a list of:

- **Pricing function:** \(p_k : \Lambda \rightarrow \mathbb{R}_+ \setminus \{0\}\);

- **Law of motion:** \(H : \Lambda \rightarrow \Lambda\);

\(^{14}\)By construction, for each \(s\), \(\Gamma(s, \cdot; \lambda)\) is a probability measure on \((S, S)\). Furthermore, given the measurability of \(d(\cdot, \cdot; \lambda)\) and \(y(\cdot, \cdot; \lambda)\), \(\Gamma(\cdot, B; \lambda)\) is a \(S\)-measurable function.
Value functions: $V : S \times \Lambda \rightarrow \mathbb{R}_+$ and $W : S \times \Lambda \rightarrow \mathbb{R}_+$;

Policy functions: $\chi : S \times \Lambda \rightarrow \mathbb{R}_+$, $\Psi : S \times \Lambda \rightarrow \mathbb{R}_+$, $\kappa : S \times \Lambda \rightarrow \mathbb{R}_+$; and

Terms of trade $y : S \times S \times \Lambda \rightarrow \mathbb{R}_+$, and $d : S \times S \times \Lambda \rightarrow \mathbb{R}_+$;

such that:

1. given the pricing function, the terms of trade, and the policy functions, the value functions satisfy the functional equations (1) and (2);

2. given the value functions and the law of motion of the aggregate state, the policy functions solve (2);

3. given the value functions, the terms of trade solve (8);

4. given the terms of trade the law of motion of the aggregate state is defined by (14);

5. the centralized market clears, $\int_S \int_S \psi(\bar{s}, \lambda) \Gamma(s, d\bar{s}; \lambda) \lambda(ds) = 1$.

In the remainder of the paper we will only focus on stationary recursive equilibria, where $\lambda = H(\lambda)$. 

13
4 Computation Algorithm [TO BE ADDED]

4.1 The Algorithm

4.2 Computational Considerations

5 Numerical Results [Very Preliminary and Incomplete]

In what follows we used the numerical algorithm presented in the last section to find and characterize stationary equilibria of the model. To numerically solve the model we adopt the following functional forms for the utility function and the disutility of supplying labor effort.

\[ U(c) = \frac{(c + b)^{1-\eta} - b^{1-\eta}}{1 - \eta} \]

\[ g(l) = \frac{l}{1 - l} \]

In addition, for now, we define a model period to be one year and pick parameter values reported in Table 1. [NOTE: We are currently calibrating the model to certain data observations, like the average level of velocity, capital-output ratio, fraction of time spent working, etc, for which the model delivers predictions. For now we will take the above parametrization as a numerical example to illustrate the equilibrium and the potential effects of monetary policy]
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</table>

Table 1: Parameter Values

Some of these parameters, given we take a model period to be one year, are standard. For example, $\beta$ and $\delta$ imply respectively, an annual interest rate of 4% and an annual depreciation rate of 7%. The others are taken from the original calibration in Lagos and Wright (2004). Ideally, we would like the model to match the average annual consumption velocity of money in the data. For the period 1959-2002, depending on whether one uses the monetary base or M1 as a measure of the money supply, these values are respectively, 11.2 and 4.0. As it will be clear below, to match the consumption velocity of money will require to calibrate the model to a shorter period than a year. In fact, by construction, the annual version cannot generate a velocity above $\frac{1}{N}$ given that only $\frac{1}{N}$ of the agents are buyers in each period of the model. Thus, even setting $N = 3$, the smallest possible number, will imply that a model period of one year cannot generate high enough velocity. We are in the process of addressing these issue.

5.1 Equilibrium Characterization

Given the functional forms and parameter values described above, we begin by analyzing the distribution of wealth and the portfolio allocation decision of the agents. Unless otherwise noted, we consider the case where there is no monetary
growth, \( \mu = 0 \). Figure 1 illustrates the typical distribution of wealth at the entrance of the decentralized market. Note that, not surprisingly, the distribution of wealth is non-degenerate and, for the parameter values we use, tends to be relatively jagged\(^{15}\). The jaggedness of the wealth distribution can be better understood once one looks at the distribution of prices in the decentralized market and the portfolio allocation decision of the agents.

Figure 2, shows the distribution of prices in the decentralized market, measured as percentage of the current money supply. The fact that the model generates price dispersion is not surprising if one notes that, in general, the terms of trade in the decentralized market will depend on both the wealth of the buyer and the seller as well as on their portfolio composition. For a given portfolio of a seller, the

\(^{15}\)The jaggedness of the wealth distribution is robust and is not a result of undersmoothing in the non-parametric density estimation process.
wealthier the buyer, holding constant his money holdings, the lower the price at which trade occurs. Also, for a given portfolio of the buyer and as long as the money constraint of the buyer is not binding, the wealthier the seller, keeping constant the amount of capital she holds, the higher the price at which trade occurs. (However, after a certain level of wealth of the seller, the money constraint of the buyer will be binding which will allow him to extract a higher fraction of the surplus.) This is a result of the fact that there is diminishing marginal value of wealth. On the other hand, the terms of trade will depend on the amount of money the buyer is holding and the amount of capital the seller has, and thus on their portfolio allocation decision. For a given seller’s portfolio, the more money the buyer brings to the bargaining table, holding fix his wealth, the worst terms of trade he will face (higher price). In particular, it is important to note that, if the

Figure 2: The Distribution of Prices in the Decentralized Market
money constraint of the buyer is binding, he is able to extract a higher fraction of the total surplus. Also, for a given buyer’s portfolio, the higher the amount of capital the seller brings into the bargaining, holding fixed his wealth, the more he will be willing to produce and thus the lower the price. Agents, are fully aware of all these considerations when making their optimal portfolio allocation decision. Thus the equilibrium price distribution depends on the equilibrium portfolio allocation decision and vice versa.

Figure 3 depicts the fraction of wealth held as real money balances at the entrance of the decentralized market. In what follows, I will try to provide some intuition to this optimal portfolio allocation decision. First consider the problem faced by a relatively poor agent. As a buyer, it is most likely that even if he holds a large fraction of his wealth as money, his money holdings are going to be binding
most of the time. As such he will be able to extract a larger fraction of the total surplus from trade. On the other hand, by holding a large fraction of his wealth as money he maximizes the expected value of the total surplus from trade. As a seller, even if she were to hold a large fraction of her wealth as capital, the cost of production would be high, and thus she would not be willing to produce much output. As a result the expected value of the total surplus from trade is small. Furthermore, since the agent is relatively poor, she would be able to extract a small fraction of the total surplus. Thus, a relatively poor agent will choose to hold a large fraction of their wealth as real money balances. As an agents wealth increases however, there is less on an incentive to leave a large fraction of that wealth as money. Note that, if the agent were to keep a constant fraction of his wealth as money, eventually, as his wealth increased, as a buyer, his money holdings constraint would be seldom binding. This would mean that the agent would be able to extract a smaller fraction of the total surplus. On the other hand as a seller, by holding more capital, the agent reduces the cost of production and increases the expected value of the total surplus from trade. As a result, in general, the wealthier the agent, the higher fraction of wealth he will hold as capital. However, given that there is diminishing marginal productivity of capital, eventually the expected gain of holding any additional capital is dominated by the depreciation cost of capital. After that point, as long as the rate of inflation is low enough, agents rather store any additional wealth as money.

Note that, in general, there are two types of situations that can arise in a bilateral meetings: either the money constraint of the buyer is binding or it is not.\footnote{Note that, on average other agents are going to be richer than him and thus have more (endogenous) bargaining power.} Note that, since capital depreciates, as long as the rate of inflation is low enough, money is a better store of value. For high rates of inflation though, capital is a better store of value. However, money also plays a role as medium of exchange and capital has a productive role. At each agent’s equilibrium portfolio allocation, the marginal value of each asset must be the same.\footnote{Note that, since capital depreciates, as long as the rate of inflation is low enough, money is a better store of value. For high rates of inflation though, capital is a better store of value. However, money also plays a role as medium of exchange and capital has a productive role. At each agent’s equilibrium portfolio allocation, the marginal value of each asset must be the same.}
not. If the constraint is binding, agents will trade on average at a lower price, if it is not the price on average will be higher. This fact, together with the optimal asset’s portfolio allocation and the distribution of wealth imply that the price distribution might not be unimodal. In turn, the fact that the distribution of prices is not unimodal leads to the jaggedness of the equilibrium distribution of wealth.

Finally, figure 4 shows consumption as a function of the wealth of the agent at the entrance of the centralized market.
5.2 The Effects of Monetary Policy

We will now address the effects of monetary policy. First, it is easy to show that changes in the level of the money supply are neutral in the long-run. To see this, note that although changes in the level of the money supply will have, in general, real effects in the short-term due to a one time redistribution of real wealth among agents in the centralized market, that change will not have permanent effects (the distribution of real wealth will eventually converge back to the same invariant distribution) and thus there will be no long-run real effects. Moreover, it is easy to see from equation (3) that, if the increase in the money supply is accomplished via proportional transfers, there will be no redistribution of wealth and thus money will be both short-run and long-run neutral. It follows that, if we consider the case of proportional transfers, money is also short-run and long-run supernuetral, i.e. changes in the rate of monetary growth will have no short-run or long-run effects on any real variable.

A more interesting case for our analysis is the one where money is injected via lump-sum transfers. These will have redistributive effects and, in general, lead to a permanent change in the distribution of wealth. In this case, changes in the rate of monetary growth will have both short-run and long-run real effects. At this moment, we are only able to solve the model for stationary equilibria of the model and thus we will not address any short-run effects of inflation. Furthermore, we will simply compare across steady-states with different rates of monetary growth/inflation ignoring the transition path. The following table summarizes our findings.

The above numerical example illustrates the potential of the model to generate
There are several different effects at work here and it is hard to disentangle them. In what follows we will try to provide some intuition for some of these effects. First, there is a redistribution effect. This has two components. On the one hand, for any given distribution of wealth that agents bring into the centralized market, independently of its composition between real money money balances and capital and keeping everything else constant, an expansionary monetary policy via lump-sum transfers will redistribute wealth from “rich” individuals to “poor” individuals decreasing the dispersion of wealth. Given that poor individuals have a higher marginal propensity to consume out of their wealth this will lead, in general, to an increase in aggregate consumption and decrease in aggregate savings. The magnitude of this effect depends on the degree of relative risk aversion among other things. On the other hand, the same monetary expansion will redistribute

### Table 2: Effects of Expansionary Monetary Policy - Lump-sum Transfers.

<table>
<thead>
<tr>
<th>Rate of Inflation ($\mu$)</th>
<th>0%</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output ($Y$)</td>
<td>0.162</td>
<td>0.169</td>
<td>0.179</td>
<td>0.182</td>
<td>0.176</td>
</tr>
<tr>
<td>Consumption ($C$)</td>
<td>0.138</td>
<td>0.142</td>
<td>0.146</td>
<td>0.147</td>
<td>0.142</td>
</tr>
<tr>
<td>Capital ($K$)</td>
<td>0.341</td>
<td>0.390</td>
<td>0.461</td>
<td>0.503</td>
<td>0.493</td>
</tr>
<tr>
<td>Average Hours</td>
<td>0.248</td>
<td>0.239</td>
<td>0.236</td>
<td>0.231</td>
<td>0.239</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>0.85</td>
<td>0.84</td>
<td>0.82</td>
<td>0.81</td>
<td>0.81</td>
</tr>
<tr>
<td>$K/Y$</td>
<td>2.10</td>
<td>2.31</td>
<td>2.58</td>
<td>2.78</td>
<td>2.80</td>
</tr>
<tr>
<td>$M/P_e$</td>
<td>0.978</td>
<td>0.655</td>
<td>0.542</td>
<td>0.508</td>
<td>0.462</td>
</tr>
<tr>
<td>Consumption Velocity</td>
<td>0.17</td>
<td>0.26</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\frac{M/P_e}{W}$</td>
<td>0.42</td>
<td>0.26</td>
<td>0.15</td>
<td>0.12</td>
<td>0.10</td>
</tr>
<tr>
<td>Ex-ante Welfare</td>
<td>138.78</td>
<td>139.32</td>
<td>139.87</td>
<td><strong>140.31</strong></td>
<td>140.27</td>
</tr>
</tbody>
</table>

a positive optimal rate of inflation. Note that moderate rates of inflation lead to a higher level of aggregate output, aggregate consumption, aggregate capital stock, and ex-ante welfare. It also leads to a reduction in the average fraction of time spent working.
wealth from agents that are holding large amounts of money at the entrance of the centralized market, typically agents that were sellers in the decentralized market, to agents that are holding little money, typically agents that were buyers. As such, in general, there will be redistribution of wealth from agents that were sellers to agents that were buyers. This diminishes the returns of holding capital and increases the returns of holding money. This effect tends to be stronger the higher the rate of monetary expansion since that will lead in equilibrium to a higher dispersion of real money balances. This effect is offset by a Tobin effect. Besides having a redistributive effect, the monetary expansion also affects the portfolio allocation decision of the agents. As the rate of inflation increases, the return of holding money decreases compared to holding capital. As such agents will want to hold a smaller fraction of their wealth as real money balances and a higher fraction of their wealth as capital. Note that, since the decentralized market is not competitive, the return of capital is not pinned down by the marginal productivity of capital. Finally, the marginal value of wealth will also decrease, leading eventually to less capital accumulation, less production, less consumption, and smaller welfare. [We are currently working on providing better intuition for these results]

Looking at the results for velocity, real money balances and the fraction of aggregate wealth held as real money balances it becomes clear that by considering a period to be an year we made search frictions too severe and magnified the role of money as a medium of exchange. This might explain the relatively high level of the optimal rate of inflation and the large effects on aggregate consumption, aggregate production, and other aggregate variables. We are currently addressing these issues.