Productivity spillovers, real exchange rates, and the ‘home market effect’: elements for a general equilibrium analysis

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Abstract

What are implications of growth and productivity gains for the international relative prices faced by a country? What is the sign of the international spillovers from countries that experience gains in production efficiency? This paper addresses these issues by introducing trade costs and endogenous entry and exit of firms or varieties in a standard two country model with monopolistic competition in production. When the number of goods supplied in equilibrium is endogenous, the macroeconomic and international effects of productivity differentials depend crucially on whether these differentials affect the manufacturing sector or the sector at the origin of new varieties. With standard assumption on elasticities, countries with higher productivity in manufacturing have higher GDP and consumption, but supply a smaller number of goods at a lower international price. Countries with lower entry costs also have higher GDP and consumption, but supply a larger array of goods at improved terms of trade. We also show that these two types of productivity gains (in production or in innovation) have potentially different international welfare spillovers.

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1 Introduction

Does the world economy benefit from a rise in production efficiency and output expansion in a specific country or region? A common view in trade and growth theory is that countries with higher growth and productivity gains experience a deterioration of their terms of trade: the increased supply of their products is absorbed by international markets at falling prices. Thanks to such movements in international relative prices, the benefits of output expansions are not confined exclusively to the country that experiences more rapid growth, but are transmitted positively to the rest of the global economy.

This pattern of international transmission is clearly consistent with the Harrod-Balassa-Samuelson hypothesis. As is well known, according to this hypothesis countries with higher productivity growth in the tradable sector should experience an appreciation of their real exchange rates — provided that the elasticity of substitution across tradables produced at home and abroad is sufficiently high. But in this case countries experiencing high productivity growth in tradables will simultaneously experience an appreciation of their real exchange rate, and a weakening of their terms of trade. To the extent that faster productivity growth translates into faster output expansion, an important implication of the association of higher growth with deteriorating terms of trade is that international trade contributes to a stable world income distribution, as emphasized by Acemoglu and Ventura (2002).

The workhorse models underlying this view typically assume that the set of commodities produced by different countries is given and time-invariant, and that no trade costs exist. Hence gains in efficiency translate into lower prices of the existing goods.\(^1\) When the assumption of a fixed number of product varieties is relaxed, as in a well-known contribution by Krugman (1989), the tenet that rapidly growing countries must experience a deterioration in their terms of trade becomes questionable. The argument is that countries can improve quality and attributes of their products. Quality improvements and product diversification can reduce or prevent altogether the fall in their terms of trade. The spillovers to the rest of the world are in this case less straightforward: the trade partners are hurt by higher import prices, but benefit from the availability of more varieties. An important issue in evaluating sign and size of the net gains from international transmission is the definition and use of appropriate welfare-based price indexes, whereas the product baskets should reflect variations in the number of goods available to consumers (a point stressed by Feenstra

\(^1\)A different view is discussed in Corsetti, Dedola and Leduc (2003), by means of a general equilibrium model calibrated for the US economy. In this model, US consumers have a home bias, and the price elasticity of US imports is calibrated as to match the standard deviation of the real exchange rate. As the resulting price elasticity is low, a higher supply of US goods can be absorbed in equilibrium only at a rising price.
In this paper, we reconsider these issues by analyzing the long-term determinants of the terms of trade and the world pattern of production within the framework of a stylized two-country macroeconomic model with imperfectly competitive product markets. Different from the standard specification, we let the set of varieties produced in each country to be endogenous and allow for transaction costs in international trade, thus encompassing the main elements of trade models that study the ‘home market effect’. Transaction costs induce a home bias in consumption, generating deviations from purchasing power parity (PPP) even though all goods are tradable. Different from the standard specification, also, we analyze international productivity differentials by drawing a distinction between productivity in manufacturing — the typical definition of productivity in standard macro models of international transmission — and innovation productivity related to the ability of creating new product varieties and operating new firms. This distinction plays a crucial role in our analysis, as productivity gains have very different implications for the equilibrium allocation depending on whether they reduce the costs of manufacturing goods, or the costs of operating new firms.

Our first result sheds light on the steady-state equilibrium implications of gains in manufacturing productivity. When the number of product varieties is endogenous, one may expect that countries with higher production efficiency in manufacturing tend to supply the widest array of goods. We show that this is not necessarily the case. Under standard assumptions about parameter values, firms’ competition in more productive countries drives down prices and profits, thus reducing the range of varieties supplied in the world markets. Domestic residents benefit from productivity gains mainly in terms of higher consumption of leisure. In steady state, they also experience a deterioration of the terms of trade and the real exchange rate.

Conversely, when productivity gains reduce the costs of introducing new varieties and operating new firms, we obtain the opposite result. The number of product varieties and the terms of trade rise. When the real exchange rate is conventionally measured without accounting for the greater availability of goods varieties, the country experiences a real appreciation.

2 When product markets are imperfectly competitive and internationally segmented, local demand conditions have a different impact on the profits of firms located in different countries. Because of trade costs, firms producing in the market with the stronger demand can take advantage of local market conditions better than firms producing elsewhere. Without entry, profits for the firms located in the country with the larger markets would increase relative to firms abroad. When entry is possible, the stronger market condition induce the creation of new firms producing new varieties. According to the ‘home market effect’, a change in demand for domestically produced goods raises the number of varieties more than proportionally, and/or raises domestic factor prices.
Overall, then, whether productivity improvement affects the production process or the innovation process is crucial to predict its effect on both the number of varieties, the terms of trade, and the real exchange rate. This is so even though both types of productivity improvements in the domestic economy lead to an increase in relative GDP vis-a-vis the rest of the world. Recent empirical evidence by Debaere and Lee (2003) appears to corroborate fully these results.

Notably, however, we also show that prices and quantities effects of productivity shocks may change sign when price elasticities of imports are sufficiently low — within the range of elasticities commonly adopted in calibration exercises.

Our model also predicts that, ceteris paribus, countries with a larger population supply a larger fraction of world goods, and have strong terms of trade. Their GDP and consumption is high relative to the rest of the world, but their real exchange rate (appropriately measured as to account for product varieties) tends to be weaker. Analogously, when government spending is biased towards domestic goods, countries with a larger fiscal sector have higher GDP, appreciated terms of trade, but weaker real exchange rate. High public consumption is associated with high private consumption.

All goods are tradable in our model, so that our framework differs from both the traditional Balassa-Samuelson view and its re-interpretation by Ghironi and Melitz (2004). As in the latter contribution, we trace the implications of using price indices that fail to account for product varieties, providing examples in which inappropriate indices of the real exchange rate would provide misleading information. It is worth stressing that the existence of frictions in international trade is crucial in our setup. Without trade frictions, the real exchange rate would be constant, no long-run deviation from PPP would appear and the nature of productivity shocks would not matter. Our model contributes to the recent but fast-growing literature building business cycle general-equilibrium model with endogenous tradability and product innovation, which shares many of the features of our model, including Bergin and Glick (2003) and Ghironi and Melitz (2004).

Section 2 presents the model setup. Section 3 discusses its equilibrium properties. Section 4 through 6 analyze productivity differentials, market size and government spending. Section 7 uses numerical examples to shed light on welfare results. Section 8 concludes. The appendix introduces an alternative specification of fixed cost to operate a new firm.

2 The model

The world economy consists of two countries, Home and Foreign — Foreign variables are denoted with a start. In each country there are households, firms, and a government.
Households consume a basket of differentiated tradable goods (varieties), both domestically produced and imported. They love variety of goods: they demand any new type of ‘brands’ of both domestic and imported goods supplied in the market. They supply labor to domestic firms only but own claims on firms’ profits worldwide. Labor is not mobile across borders. There are \( L_t \) households in the Home country and \( L_t^* \) households in the Foreign country.

Firms in both countries produce goods which are sold domestically or exported, using labor as the only input in production. The varieties produced by firms operating in the Home country are defined over a continuum of mass \( n_t \) and indexed by \( h \in [0, n_t] \). Similarly, Foreign varieties are indexed by \( f \in [0, n_t^*] \).

The number of varieties produced in each country is endogenously determined in the model. There is free entry in the goods sector, but firms face fixed entry costs to start production of a particular variety: entry costs consist of wages paid to the labor employed in operating a firm. Firms in both countries operate under conditions of monopolistic competition, so that each firm produces one variety only. Hence, an increase in \( n_t \) corresponds to both the introduction of new varieties in the Home country, and the creation of new Home firms.

Governments are assumed to purchase goods from national firms only. They finance their expenditures with lump-sum net taxes.

### 2.1 Firms

To produce final goods for the domestic and the export markets, firms have access to a linear technology in labor. So, the production function of the representative Home firm producing a specific variety \( h \) is:

\[
Y_t(h) = \alpha_t \ell_t(h)
\] (1)

where \( Y(h) \) is the output of variety \( h \), \( \ell(h) \) is labor used in its production, and \( \alpha_t \) is a country-specific labor productivity shock — common to all Home firms. Similarly, in the Foreign country we have:

\[
Y_t^*(f) = \alpha_t^* \ell_t^*(f)
\] (2)

To start the production of a variety \( h \) in the Home country, a firm needs to employ \( 1/\nu_t \) units of Home labor. Thus it faces a constant cost \( q(h) \):

\[
q_t(h) = w_t/\nu_t
\] (3)

Note that \( \nu_t \) is labor productivity in the sector at the origin of varieties, i.e. a parameter
affecting the cost of entry in the Home country’s goods sector. Similarly, the entry cost in the Foreign country is:

\[ q_t^*(f) = \frac{w_t^*}{\nu_t^*} \]  

(4)

Efficiency in setting up the production of new goods does not necessarily coincide with productivity in manufacturing. Thus in general \( \alpha \neq \nu \).

While there is free entry, it will never be profitable for a firm to produce a brand \( h \) or \( f \) already supplied by other firms, rather than introducing a new variety. Hence in equilibrium firms will be monopolistic suppliers of one good only. Assuming for simplicity that government spending falls entirely on local goods, variety \( h \) is sold to domestic agents (both private and public) or exported to foreign households. Shipping goods abroad entails transportation costs in the form of ‘iceberg’ costs. Accounting for transportation costs, then, the resource constraint for variety \( h \) will therefore be:

\[ Y_t(h) = L_t C_t(h) + (1 + \tau) L_t^* C_t^*(h) + G_t(h) \]  

(5)

where \( C_t(h) \) is consumption of good \( h \) by the representative Home resident, \( C_t^*(h) \) is consumption of good \( h \) by the representative Foreign resident, \( \tau \) is the transportation costs expressed in terms of unit of the export good, and \( G_t(h) \) is Home government purchases of good \( h \). Similarly, for the foreign representative firm we have:

\[ Y_t^*(f) = L_t (1 + \tau) C_t(f) + L_t^* C_t^*(f) + G_t^*(f) \]  

(6)

Let \( p_t(h) \) denote the Home-currency price of one unit of good \( h \) in the domestic market, and \( p_t(f) \) the Home-currency price of imports \( f \). Similarly \( p_t^*(h) \) is the Foreign-currency price of variety \( h \) imported by the Foreign country and \( p_t^*(f) \) is the Foreign-currency price of variety \( f \) in the Foreign country. Also, let \( \varepsilon_t \) denote the nominal exchange rate, defined as Home currency per unit of Foreign currency, while \( w_t \) is the nominal wage. Using the above notation, Home-currency operating profits are:

\[ \Pi_t(h) = p_t(h) L_t C_t(h) + \varepsilon_t p_t^*(h) L_t^* C_t^*(h) + p_t(h) G_t(h) - w_t \ell_t(h) \]  

(7)

Accounting for (5), the optimal prices \( p_t(h) \) and \( p_t^*(h) \) charged by the Home firm solves:

\[ 1 + \left( \frac{p_t(h) - w_t}{\alpha_t} \right) \frac{\partial (L_t C_t(h) + G_t(h)) / \partial p_t(h)}{L_t C_t(h) + G_t(h)} = 0 \]  

(8)

\[ \varepsilon_t + \left( \frac{\varepsilon_t p_t^*(h) - w_t}{\alpha_t} (1 + \tau) \right) \frac{\partial (L_t^* C_t^*(h)) / \partial p_t^*(h)}{L_t^* C_t^*(h)} = 0. \]  

(9)

Similar expressions hold for \( \Pi_t^*(f) \), \( p_t(f) \) and \( p_t^*(f) \).

\(^3\)In Appendix 2, we revisit our results when the fixed cost of entry is a convex cost function and increases with the number of existing varieties.
2.2 Households and government

The utility of the representative Home household is a positive function of consumption and a negative function of labor effort:

\[ U_t = \frac{C_t^{1 - \frac{1}{\psi}}}{1 - \frac{1}{\psi}} - \kappa \ell_t \] (10)

In the expression above, \( C_t \) is a composite good that includes all varieties:

\[ C_t = \left[ \int_0^{n_t} C_t(h)(h)^{1 - \frac{1}{\sigma}} dh + \int_0^{n_t^*} C_t^*(f)(f)^{1 - \frac{1}{\sigma}} df \right]^{\frac{\sigma}{\sigma - 1}} \] (11)

The parameters \( \psi \) and \( \sigma \) denote respectively the elasticities of intertemporal and intratemporal (i.e., across varieties) substitution, with \( \psi > 0 \) and \( \sigma \geq 1 \). In our model the number of variety is endogenous. We assume that household preferences are defined over a very large set of goods, so that their utility will be well defined (and increasing) in any new goods introduced in the market, given technology and labor endowment.

The budget constraint for the representative Home household is:

\[ \int_0^{n_t} p_t(h) C_t(h) dh + \int_0^{n_t^*} p_t(f) C_t^* f df + T_t + I_t = w_t \ell_t + \Pi_t \] (12)

where \( T_t \) are lump-sum net taxes denominated in Home currency, \( I_t \) is the ‘investment’ of households in firms (i.e. the financing of entry costs) and \( \Pi_t \) is revenue from dividends.

We assume incomplete market, but, in contrast with the standard assumption that households only own and finance domestic firms, we proceed by assuming that Household are endowed with a well-diversified international portfolio of claims on firms’ profits, so that they finance the same fraction of the cost of creating new varieties in each country. Resorting to the more standard assumption would not alter our result, but would complicate somewhat the analysis. So, Home households invest in a diversified portfolio of firms:

\[ I_t \equiv \frac{1}{L_t + L_t^*} \left( \int_0^{n_t} q_t(h) dh + \varepsilon_t \int_0^{n_t^*} q_t^* df \right) \] (13)

In return, each Home household receives an equal share of profits of all firms in the world:

\[ \Pi_t \equiv \frac{1}{L_t + L_t^*} \left( \int_0^{n_t} \Pi_t(h) dh + \varepsilon_t \int_0^{n_t^*} \Pi_t^* df \right) \] (14)

The representative Home household maximizes (10) with respect to \( C_t(h), C_t(f), \) and \( \ell_t \) subject to (12). The first order conditions yield:

\[ C_t(h) = \left( \frac{p_t(h)}{P_t} \right)^{-\sigma} C_t, \quad C_t(f) = \left( \frac{p_t(f)}{P_t} \right)^{-\sigma} C_t \] (15)
\[ w_t = \kappa P_t C_t^{1/\psi} \]  

(16)

where \( P_t \) is the utility-based consumer price index — specified below. In what follows, without loss of generality, we choose the appropriate nominal units of account such that \( w_t = \kappa \) or:

\[ C_t = P_t^{-\psi}. \]  

(17)

As domestic households provide labor in a competitive labor market both for production activities and for the creation of varieties, the resource constraint in the labor market is:

\[ Lt = \frac{1}{\alpha_t} \int_0^{n_t} Y_t(h) \, dh + \int_0^{n_t} \frac{q_t(h)}{w_t} \, dh. \]  

(18)

Assuming lump-sum taxes, we posit that the government balances its budget constraint every period. For the Home government we have:

\[ n_t G_t = L_t T_t \]  

(19)

where \( G_t \) is public consumption of a representative Home variety. For simplicity, we will assume that public demand for each specific variety is similar to private demand as derived in (15), so that:

\[ G_t(h) = \left( \frac{p_t(h)}{P_t} \right)^{-\sigma} G_t. \]  

(20)

### 2.3 Prices

Substituting expressions (15), (20) and their Foreign analogs in (8) and (9), we can easily derive the optimal prices charged by Home firms. These prices are equal to marginal costs — that in our setup coincide with labor costs per unit of product — multiplied by the equilibrium markup:

\[ p_t(h) = \frac{\sigma}{\sigma - 1} \frac{w_t}{\alpha_t} \equiv p_t \]  

(21)

\[ \varepsilon_t p_t^*(h) = \frac{\sigma}{\sigma - 1} \frac{w_t}{\alpha_t} (1 + \tau) = p_t(1 + \tau) \]  

(22)

Similar expressions hold in the Foreign country:

\[ p_t^*(f) = \frac{\sigma}{\sigma - 1} \frac{w_t^*}{\alpha_t} \equiv p_t^* \]  

(23)

\[ p_t^*(f) = \frac{\sigma}{\sigma - 1} \frac{w_t^*}{\alpha_t} (1 + \tau) = p_t^*(1 + \tau) \]  

(24)

Note that productivity gains that lower marginal costs reduce production price proportionally.
The equilibrium utility-based CPI are defined as the minimum expenditure required to purchase one unit of the basket $C (C^*)$, and are as follows:

$$P_t = \left[n_t p_t^{1-\sigma} + n_t^* \phi (\varepsilon_t p_t^*)^{1-\sigma}\right]^{-\frac{1}{\sigma}} \quad (25)$$

$$P_t^* = \left[n_t \phi (p_t/\varepsilon_t)^{1-\sigma} + n_t^* p_t^{1-\sigma}\right]^{-\frac{1}{\sigma}} \quad (26)$$

where — using a notation that is familiar to readers of the international trade literature — $\phi \equiv (1 + \tau)^{1-\sigma}$. Note that the parameter $\phi$ is positive and less than one. The case $\phi = 0$ corresponds to infinite trade costs and the case $\phi = 1$ to zero trade costs.

### 3 Profit, entry and exchange rates in general equilibrium

#### 3.1 Equilibrium conditions and steady state allocation

In this section we will characterize the model solution, and comment on the equilibrium link between profits and the creation of new varieties. To start with, using (17), (15) and (25), we express the private individual Home demand for the $h$ and $f$ goods in terms of the number of firms, relative prices and the real exchange rate:

$$C_t(h) = p_t^{-\sigma} p_t^{\sigma-\psi} = \frac{p_t^{-\psi}}{n_t + n_t^* \phi (\varepsilon_t p_t^*)^{1-\sigma}} \quad (27)$$

$$C_t(f) = (\varepsilon_t p_t^* (1 + \tau))^{-\sigma} p_t^{\sigma-\psi} = \frac{(1 + \tau)^{-\sigma} (\varepsilon_t p_t^*/p_t)^{-\sigma} p_t^{\sigma-\psi}}{n_t + n_t^* \phi (\varepsilon_t p_t^*/p_t)^{1-\sigma}} \quad (28)$$

Similarly, in the Foreign country:

$$C_t^*(f) = p_t^{*-\sigma} p_t^{*\sigma-\psi} = \frac{p_t^{*-\psi}}{n_t^* + n_t^* \phi (\varepsilon_t p_t^*)^{\sigma-1}} \quad (29)$$

$$C_t^*(h) = (p_t (1 + \tau)/\varepsilon_t)^{-\sigma} p_t^{*\sigma-\psi} = \frac{(1 + \tau)^{-\sigma} (\varepsilon_t p_t^*/p_t)^{\sigma} p_t^{*\sigma-\psi}}{n_t^* + n_t^* \phi (\varepsilon_t p_t^*/p_t)^{\sigma-1}} \quad (30)$$

In our specification, imperfect competitive firms earn positive operating profits. We can
write them as follows:

\[
\Pi_t(h) = \frac{p_t^{1-\psi}}{\sigma} \left[ n_t + n_t^* \phi (\varepsilon_t p_t^*/p_t)^{1-\sigma} \right] \frac{\sigma}{\sigma-1} + \frac{p_t}{\sigma} G_t \equiv \pi_t
\]

\[
\varepsilon_t \Pi_t^*(f) = \frac{p_t^{1-\psi}}{\sigma} \left[ n_t + n_t^* \phi (\varepsilon_t p_t^*/p_t)^{1-\sigma} \right] \frac{\sigma}{\sigma-1} + \frac{\varepsilon_t p_t^*}{\sigma} G_t^* \equiv \varepsilon_t \pi_t^* \tag{32}
\]

whereas \(\pi\) and \(\pi^*\) denote firm’s profit in own currency. Note that with constant markups, we can express the profits of the representative firm at Home and abroad as a constant fraction \(1/\sigma\) of global sales.

The above expressions stress an important feature of models that, similar to ours, account for the introduction of new varieties by imperfectly competitive firms. Namely, the sign of the relation between profits and the number of varieties depends on whether goods are substitutes or complements in the Edgeworth-Pareto sense, that is, on whether \(\sigma > \psi\) (substitutes) or \(\sigma < \psi\) (complements). This is because an increase in the number of varieties has two effects on profits via consumption demand. On the one hand, it leads to a fall of the price index \(P_t\) (25). This induces intertemporal substitution into current consumption, measured by \(\psi\). On the other hand, an increase in the number of goods implies intratemporal substitution away from existing goods, measured by \(\sigma\). The net effect is given by \(\sigma - \psi\).

An increase in the number of goods leads to lower consumption (and therefore lower sales and profits) if the intratemporal elasticity is larger than the intertemporal elasticity. In the literature, a benchmark parameterization for the intertemporal elasticity of substitution \(\psi\) is between 1 and 1/2. The appropriate choice of \(\sigma\) is less obvious, but in calibration exercises it is typically set above 1, with values between 5 and 10 not uncommon. In the rest of the paper, unless otherwise stated, we will carry out our analysis under the maintained assumption that: \(\psi \leq 1 < \sigma\).

Assuming that trade costs are strictly positive (i.e. \(\phi < 1\)), we can then use the expressions for profits to shed light on some crucial dimensions of the transmission mechanism further studied below. First, holding the number of varieties and the exchange rate constant, an increase in Home market size (an increase in \(L\)) raises operating profits at Home more than abroad. This is the ‘Home Market effect’ analyzed in trade models. By the same token, everything else constant, an increase in the number of Home firms lowers Home prof-
its more than Foreign profits. This is because trade costs partially shield Foreign firms from competition by Home firms. Furthermore, holding the number of Foreign and Home goods constant, a depreciation of the Home currency (an increase in $\epsilon$) raises profits at Home. This is a ‘competitive effect’ of real depreciation: Home produced goods become cheaper, and their demand increases. Finally, we know from optimal pricing that the ratio of $p^*$ to $p$ is proportional to the ratio of productivity $\alpha/\alpha^*$. Hence, for a given exchange rate and number of firms, a positive productivity shock in the manufacturing sector at Home (an increase in $\alpha$) corresponds to a drop in $p^*/p$, with two opposite effects on Home profits. By charging lower prices, Home firms become relatively more competitive and experience an increase in sales. However, because all Home firms experience the same rise in productivity, and charge the same lower price, when the intertemporal elasticity of substitution is less than one the value of sales decreases overall.

With free entry, optimal investment in new varieties implies that the value of a firm is equal to the cost of creating a variety and in equilibrium this must be equal to the value of operating profits. Thus competition in the goods market implies the following free entry conditions:

$$q_t = \frac{\kappa}{\nu_t} = \pi_t$$  \hspace{1cm} (33)$$
$$q^*_t = \frac{\kappa}{\nu^*_t} = \pi^*_t$$  \hspace{1cm} (34)$$

From our observation above, it follows that $\sigma > \psi$ is a necessary condition for the number of varieties to be a decreasing function of the cost of entry. It turns out that this inequality is also a condition for the symmetric steady state equilibrium with zero pure profit to be stable. Note that, because of free entry, the size of firms in manufacturing is pinned down by the entry cost.

Aggregating the households’ budget constraints in the Home country, and using the government budget constraint, we can write the balance of payments of the Home country as follows:

$$\frac{p_t^1}{\nu_t} - n_t^1 L_t^{\phi \epsilon_t^1} (p^*_t/p_t)^{\psi - \sigma} - \frac{p_t^1}{\nu_t} - n_t^1 L_t^{\phi \epsilon_t^1} (p^*_t/p_t)^{\psi - \sigma} - \frac{1 - \psi}{\nu_t} n_t^1 L_t^{\phi \epsilon_t^1} (p^*_t/p_t)^{\psi - \sigma} - \frac{1 - \psi}{\nu_t} n_t^1 L_t^{\phi \epsilon_t^1} (p^*_t/p_t)^{\psi - \sigma}$$  \hspace{1cm} (35)$$

All terms in equation (35) are expressed in Home nominal units. The first term on the left hand side represents Home exports, the second term Home imports, both inclusive of trade costs — their difference is therefore the trade balance. The third term corresponds to net profits paid by Home firms to Foreign households, the fourth term to net profits paid by
Foreign firms to Home households — their difference representing net factor payments. The balance of these four terms is the current account. The last two terms represent the capital account, i.e. the financing of Home firms by Foreign households and the financing of Foreign firms by Home agents.

Recalling that \( q_t = \kappa/\nu_t \), \( q_t^* = \kappa/\nu_t^* \), \( p_t = \sigma (\sigma - 1)^{-1} \kappa \alpha_t^{-1} \) and \( p_t^* = \sigma (\sigma - 1)^{-1} \kappa \alpha_t^*^{-1} \), the system of three equations (35), (33) and (34) determined the three endogenous variables \( \varepsilon_t \), \( n_t \) and \( n_t^* \) as a function of the exogenous variables \( \nu_t \), \( \nu_t^* \), \( \alpha_t \), \( \alpha_t^* \), \( L_t \), \( L_t^* \), \( G_t \) and \( G_t^* \) for given parameters \( \sigma, \psi, \kappa \) and \( \phi \). It is straightforward to verify that if \( \nu_t = \nu_t^* = \alpha_t = \alpha_t^* = L_t = L_t^* = 1 \) and \( G_t = G_t^* = 0 \), there is a symmetric equilibrium such that \( \varepsilon_t = 1 \) and \( n_t = n_t^* \). In what follows, we take this equilibrium as the baseline in our comparative statics exercises.\(^4\) This symmetric equilibrium is characterized as follows: \( p = p^* = \frac{\sigma}{\sigma - 1} \kappa \), \( w = w^* = \kappa \), \( \ell = \ell^* = \sigma n \). Note also that: \( P^{1 - \psi} = n \sigma \kappa \). The number of varieties produced by each country in steady state is:

\[
n = n^* = (\kappa \sigma)^{-\psi/(\sigma - 1)} (\sigma - 1)^{-(1 - \psi)/(\sigma - 1)} (1 + \phi)(\psi^{-1})/(\sigma - \psi)
\]

This characterization of varieties in a symmetric steady state suggests an interesting result on the effect of trade liberalization, defined as lower trade costs associated with a higher \( \phi \). Namely, when the intertemporal elasticity of substitution is less than one (a common assumption in the literature), trade liberalization actually reduces the number of varieties supplied at the world level. This contrasts with the prediction of the standard Krugman model with trade-induced variety expansion. Recently, Baldwin and Forslid (2004) have discussed a similar result in a model with firms heterogeneity after Melitz (2003). The above expression however makes it clear that the fall in good varieties after liberalization does not depend on firms’ heterogeneity.

In general, lowering trade costs have two opposite effects: on the one hand it raises demand for goods, increases profits and therefore induces entry. This effect of raising \( \phi \) is captured by the numerator of the expression (31) for Home profits. On the other hand, trade liberalization (if symmetric on both imports and exports) induces more competition by Foreign firms. This effect is captured by the presence of \( \phi \) in the denominator of equation (31). The number of varieties produced and traded by Home firms will fall with trade liberalization when (a) goods are substitutes in the Edgeworth-Pareto sense (i.e., \( \sigma > \psi \)), so that profits decrease when the number of varieties increases and (b) \( \psi < 1 \), so that the decrease in the Price index induced by trade liberalization does not lead to a large jump in

\(^4\)This symmetric equilibrium is characterized by the following: \( p = p^* = \frac{\sigma}{\sigma - 1} \kappa \), \( w = w^* = \kappa \), \( n = n^* = \left[ \kappa \sigma p^{\psi - 1} \right]^{-(\sigma - 1)/(\sigma - \psi)} (1 + \phi)^{\psi - 1}/(\sigma - \psi) \), \( \ell = \ell^* = \sigma n \). Note also that: \( P^{1 - \psi} = n \sigma \kappa \)
current consumption.

We conclude this section by observing that in each economy workers are employed either in manufacturing firms or in providing services to set up firms and create new varieties. The resource constraint can thus be written \( L_t = n_t Y_t / \alpha_t + n_t / \nu_t \). Now, using the fact that in manufacturing the size of firms is pinned down by the entry cost, we can write the size of each firm as a function of productivity levels \( \alpha \) and \( \nu \) as well as the elasticity \( \sigma \)

\[
Y_t = (\sigma - 1) \alpha_t / \nu_t.
\]

It follows that Home GDP can be written simply as \( u_t L_t = \sigma \kappa n_t / (L_t \nu_t) \). Then, the ratio of Home to Foreign GDP is: \( n_t / (\varepsilon_t L_t \nu_t n_t^*) \). These expressions will be useful in the analysis to follow.

### 3.2 Global patterns of production and the balance of payments

We now inspect again the mechanism of international transmission when product varieties are endogenous in greater detail, by differentiating the equilibrium conditions (33), (34) and (35) around a symmetric equilibrium.

We start by analyzing the free entry conditions equating profits to the operating costs \( q \). For the Home country, differentiating (33) around the symmetric equilibrium we obtain:

\[
\frac{\sigma - \psi}{\sigma - 1} \left( 1 + \phi^2 \right) \frac{dn_t}{n} = \left[ 2 \phi (\sigma - 1) - (1 + \phi^2) (1 - \psi) \right] d\alpha_t + (1 + \phi)^2 d\nu_t
\]

\[
+ \frac{(1 + \phi)^2}{\sigma - 1} dG_t n + (1 + \phi) dL_t + \phi [2 \sigma - \psi (1 - \phi)] d\varepsilon_t - 2 \phi \frac{\sigma - \psi}{\sigma - 1} d\varepsilon_t^* - \frac{1}{n^*}
\]

In itself, the free entry condition suggests the following observations. Under the assumption \( \psi \leq 1 < \sigma \), a higher productivity level in the manufacturing sector \( d\alpha_t > 0 \) leads to creation of new varieties only when trade is sufficiently open \( (\phi \text{ sufficiently close to one}) \). As already discussed above, higher Home productivity that translates into lower prices raises Home competitiveness, increasing demand by Foreign households and therefore Home profits, the more so the more open the country is. But when the intertemporal elasticity of substitution \( \psi \) is less than one, lower goods prices also decrease the value of domestic sales and profits. Conversely, we can now see that higher efficiency in the creation of new goods and firms \( dv_t > 0 \) decreases the cost of entry, therefore leading to the production of a larger number of varieties in the Home country. Higher government spending \( dG_t > 0 \) and larger market size \( dL_t > 0 \) also induce entry because they raise aggregate demand for domestic goods. A depreciation \( d\varepsilon_t > 0 \) has the same effect by making Home goods more competitive. An increase in the number of varieties produced abroad \( dn_t^* > 0 \), instead, imply more competition for Home firms, reducing their profits and therefore leading to exit.
Analogous considerations apply to the free entry condition for the Foreign country:

\[
\frac{\sigma - \psi}{\sigma - 1} (1 + \phi^2) \frac{dn_t^*}{n^*} = -2\phi (\sigma - \psi) d\alpha_t \\
+ \phi (1 + \phi) dL_t - 2\phi \frac{\sigma - \psi}{\sigma - 1} \frac{dn_t}{n} - \phi [2\sigma - \psi (1 - \phi)] d\epsilon_t 
\]

Gains in manufacturing productivity in the Home country reduce Foreign profits and lead to exit, so does depreciation of the Home currency. A larger Home market size raises the demand for Foreign varieties, increasing Foreign profits and causing entry. Note however that with strictly positive trade costs ($\phi < 1$), the magnitude of this effect is weaker than in the Home country. An increase in the number of varieties produced by Home firms brings about more competition, reducing Foreign profits and leading to exit.

The "Home market effect", a standard result in new trade theory that can be traced back to Krugman (1980), applies here as a special case of our model. It states that, in the presence of trade costs, an increase in market size (an increase in $L_t$) has a more than proportional effect on the number of varieties produced. Taking (37) and (38), and for a given exchange rate, it can be checked that:

\[
\frac{dn_t}{n} = \frac{\sigma - 1}{\sigma - \psi} \frac{dL_t}{1 - \phi}. 
\]

This expression is always more than one, so that the Home market effect applies, in the log utility case ($\psi = 1$). Note that the elasticity of the number of Home firms to market size increases as trade costs decrease ($\phi$ increases). The reason is that when trade costs are low (but not zero\(^5\)), firms can produce on the larger Home market so as to save on trade costs and still export to the smaller market easily. As we will see when we analyse general equilibrium effects of a change in market size, using a utility with less than unitary intertemporal elasticity of substitution and more importantly, taking into account the effect on the exchange rate qualifies this result in the sense that part of the effect of market size goes into the exchange rate.

Combining the two free entry conditions above allow us to study the implications of different economic factors on the overall number of varieties available to households worldwide:

\[
\left[ \frac{\sigma - \psi}{\sigma - 1} \right] \frac{dn_t + dn_t^*}{n} = dL_t + \frac{1}{\sigma - 1} \frac{dG_t}{n} - (1 - \psi) d\alpha_t + d\nu_t 
\]

The number of varieties produced at the global level unambiguously rises with changes in the size of the Home market, gains in efficiency in setting up firms and creating new goods, and higher government spending on Home goods. Conversely, the effects of gains in manufacturing productivity are ambiguous: the global number of varieties falls when the intertemporal elasticity of substitution is less than unity, raises otherwise. Intuitively, for a given cost of entry, productivity gains in the manufacturing sector reduce firms’ profits,

\(^5\)In the case in which $\phi$ is exactly equal to 1 (trade costs are zero) no Home Market effect arises and the location of production of varieties is indeterminate.
as competition leads firms to reduce their prices in line with the higher productivity. With a low $\psi$, a fall in the price index translates into lower sales revenue and profits. Recall that in our model the size of firms is pinned down by free entry and independent of $\alpha$. Thus, higher productivity that reduces the number of firms and varieties translates into lower labor effort. Interestingly, however, an economy-wide proportional productivity shock $d\alpha_t = d\nu_t$ unambiguously leads to entry. Note that exchange rate movements do not affect the global number of varieties: in our symmetric world changes in the exchange rate affect symmetrically — but with opposite sign — sales and profits in both countries, without global effects.

In our model we assume constant marginal disutility of labor, corresponding to an infinite Frisch elasticity of labor.6 One may think that this assumption is key to our results: as labor supply becomes more inelastic, domestic agents may take advantage of manufacturing productivity gains by increasing the array of goods supplied by the country, rather than leisure time. Interestingly, this is not the case: it can be shown that our results remain qualitatively identical when the Frisch elasticity of labor is less than infinite. Clearly, with a less elastic labor supply, the effect of higher manufacturing productivity on the global number of varieties is weaker, but remains negative as long as the intertemporal elasticity of substitution $\psi$ is less than 1.

Differentiating the balance of payment equation (35), we obtain:

$$
[2\sigma - 1 + \phi(2\psi - 1)]d\varepsilon_t = (1 + \phi)dL_t \\
- [(\sigma - \psi)(1 - \phi) + (1 + \phi)(\sigma - 1)] \left[ d\alpha_t + \frac{1}{\sigma - 1} \left( \frac{dn_t - dn_t^*}{n} \right) \right]
$$

(40)

Holding the number of varieties and productivity levels constant, an increase in market size $L_t$ depreciates the currency as it generates more imports. Higher productivity in the manufacturing sector $\alpha_t$ appreciates the exchange rate as falling Home prices make Home varieties more competitive and improve the trade balance. A nominal appreciation is required to correct the resulting surplus. Finally, an exogenous increase in the relative number of varieties produced in the Home country leads to an appreciation: a larger number of Home varieties raises Home exports. Note that a productivity improvement in the innovation sector only affects the exchange rate through the change in the number of varieties in both countries.

6 The Frisch elasticity is defined as the elasticity of labor supply relative to the wage keeping constant the marginal utility of consumption.
4 Productivity gains and the Home market effects

One may reasonably expect that countries with higher productivity be the world suppliers of most good varieties, sold abroad at a relatively low international price. In what follows, we will show that this is not necessarily the case. A crucial issue in this respect is the distinction between productivity in manufacturing, and efficiency gains reducing the costs of setting up firms and introduce new varieties.

In our specification, under standard assumptions on the value of the elasticity, countries with higher productivity in manufacturing will actually charge lower international prices (i.e., their terms of trade deteriorates), but supply a smaller (not a larger) set of goods to world markets. This is because higher competition in the goods markets will drive goods prices and profits down in the Home market, while Foreign firms will be shielded by trade costs. For a given cost of setting up a firm, lower goods prices following gains in Home manufacturing productivity will translate into exit.

Conversely, gains in efficiency related to innovation and firms creation improve a country’s terms of trade, and raise the number of varieties, both domestically and globally. In this case higher productivity does not spur more intensive competition in a given set of goods markets, but leads firms to expand the array of goods and services provided. Intuitively, introducing a new good when agents prefer varieties implies that demand rises in line with supply. More efficient countries will thus be able to expand their exports, without suffering a deterioration of their terms of trade. In the Foreign countries, more competition by Home firms can be partly offset by higher global demand. The net effect on entry and employment can be positive or negative.

4.1 Productivity gains in manufacturing

Consider first gains in manufacturing productivity by the Home country, i.e. an increase in \( \alpha \). We have already seen that, as long as \( \sigma > \psi \), an increase in Home productivity reduces the global number of varieties if the intertemporal elasticity of substitution \( \psi \) is below 1:

\[
\left[ \frac{\sigma - \psi}{\sigma - 1} \right] \frac{dn_t + dn_t^*}{n} = -(1 - \psi) d\alpha_t
\]

But what are its implications for the international allocation of production? The impact of gains in Home manufacturing productivity on the number of varieties is summarized below:

\[
\frac{1}{n} \frac{dn_t}{d\alpha_t} = \frac{-(\sigma - 1)(1 - \psi)}{\sigma - \psi} \left[ 1 - \frac{\psi (\sigma - 1) \phi}{(2\sigma - 1) [\sigma - \psi (1 - \phi)] - \sigma \phi} \right] < 0 \tag{41a}
\]

\[
\frac{1}{n} \frac{dn_t^*}{d\alpha_t} = \frac{-(\sigma - 1)(1 - \psi)}{\sigma - \psi} \left[ \frac{\psi (\sigma - 1) \phi}{(2\sigma - 1) [\sigma - \psi (1 - \phi)] - \sigma \phi} \right] < 0. \tag{41b}
\]
The number of varieties declines in both countries. It can be verified that the number of varieties falls more at Home than in the Foreign country. In the Home country, firms become larger.

To shed light on these results, it is useful to contrast their micro and macro dimensions. From the vantage point of an individual Home firm, productivity gains that reduce the marginal costs of production are an opportunity to expand the firm’s market and profits via a reduction in the product price. However, with an economy-wide shock, all Home firms simultaneously experience the same fall in marginal costs: they all compete with each other by cutting prices. As already discussed above, a fall in the price of consumption affects aggregate consumption demand with elasticity $\psi$. When this elasticity is below one, lower prices translates into lower profits for the Home firms. For given entry costs, lower profits reduce the number of varieties produced by Home firms. Conversely, when $\psi > 1$, a fall in prices raises demand more than proportionally, driving up profits and therefore the number of varieties supplied in equilibrium. There is not change in $n$ when $\psi = 1$.

In our model, all firms supply goods to both the domestic and the foreign markets, hence entry and exit at national level correspond one-to-one to entry and exit in the exports markets. Allowing for firms-specific productivity level (as in Ghironi and Melitz (2004)) introduces the possibility that some good be endogenously non traded in equilibrium, depending on prices and productivity levels. In this case, entry and exit in the export markets may differ from entry and exit of firms located in the Home country — they may even have different sign: a productivity shock at Home may reduce the number of varieties produced at Home, while simultaneously raising the number of varieties exported by the Home country. What our analysis highlights is the aggregate effect on total supply, that is sometimes blurred in analyses stressing heterogeneity.

What happens in the Foreign country is a direct implication of the change in production structure at Home. More Home firms and varieties means more competition for foreign firms, driving down their profits and hence causing exit. Conversely, a smaller set of varieties produced at Home means more room for Foreign production. These effects are consistent with important general equilibrium movements in international relative price.

While the nominal exchange rate appreciates (the more so the higher the trade costs) in response to a productivity shock:

$$\frac{dz_t}{d\alpha_t} = -\frac{[\sigma - \psi(1 - \phi) + (\sigma - 1)(1 + \phi)](\sigma - 1)}{(2\sigma - 1)[\sigma - \psi(1 - \phi)] - \sigma\phi} < 0.$$  \hspace{1cm} (42a)
the terms of trade deteriorate because of the fall in the price of domestic goods accompanying the gains in productivity:

$$\frac{dTOT_t}{d\alpha_t} = -1 - \frac{d\varepsilon_t}{d\alpha_t} < 0$$

(43)

Despite nominal appreciation, the movements in international relative prices are dominated by the effect of productivity on marginal costs of producing Home goods. Note that the terms of trade are customarily defined as the price of import over the price of exports, i.e.,

$$TOT_t = \frac{p_t}{p^*_t \varepsilon_t}$$. It can also be checked that with zero trade costs ($\phi$ approaches 1), then

$$\frac{dTOT_t}{d\alpha_t} = -1/\sigma$$.

Also the (welfare-based) real exchange rate depreciates,

$$\frac{dRER_t}{d\alpha_t} \frac{1}{RER} = \frac{-\psi (1-\phi) \frac{dn_t}{d\alpha_t} + \phi \frac{dn^*_t}{d\alpha_t} - \phi (\sigma - 1) \frac{dc_t}{d\alpha_t}}{(\sigma - \psi)(1 - \phi) + (\sigma - 1)(1 + \phi)} + 2 \frac{1 - \phi}{1 + \phi} > 0$$

(44)

moving in the opposite direction relative to the nominal exchange rate. Also the movement of the real exchange rate is therefore dominated by the relative movement of price indices. Consider first the Home CPI. The effect of higher Home manufacturing productivity on the welfare-based (correct) consumer Price Index is:

$$\frac{dP_t}{d\alpha_t} \frac{1}{P} = -\frac{1}{1 + \phi} - \frac{dn_t}{d\alpha_t} - \phi \frac{dn^*_t}{d\alpha_t} + \phi (\sigma - 1) \frac{d\varepsilon_t}{d\alpha_t}$$

(45)

Note that the first term is the direct negative effect of productivity changes. The price index decreases as a consequence of productivity gains. Analogously, for the Foreign price index we can write:

$$\frac{dP^*_t}{d\alpha_t} \frac{1}{P^*} = -\frac{1}{1 + \phi} - \phi \frac{dn_t}{d\alpha_t} + \phi (\sigma - 1) \frac{d\varepsilon_t}{d\alpha_t}$$

(46)

Here, the first channel is negative, but less strong than in the Home country because trade costs ($\phi < 1$) lower the impact of the direct effect of productivity innovations.

Recent literature has pointed out problems that may arise when consumption baskets are not properly constructed so to account for (endogenous) changes in the number of goods varieties. For instance, Ghironi and Melitz (2004) carries out an assessment of their model contrasting theoretically-consistent real exchange rates, with what they call ‘empirical exchange rates’ — based on price indexes that are constructed by averaging out prices over a fixed number of varieties. In the same spirit, we set $\frac{dn_t}{d\alpha_t} = \frac{dn^*_t}{d\alpha_t} = 0$ in the above expressions, and denote the resulting price index at Home and abroad with $\tilde{P}_t$ and $\tilde{P}^*_t$, respectively. If we use these price indices in computing the real exchange rate ($\tilde{RER}_t = \varepsilon_t \tilde{P}^*_t / \tilde{P}_t$), the effect of a productivity shock would still be a depreciation, but at a much lower rate:

$$\frac{d\tilde{RER}_t}{d\alpha_t} \frac{1}{\tilde{RER}} = \frac{1 - \phi}{1 + \phi} \left(1 + \frac{d\varepsilon_t}{d\alpha_t}\right) > 0$$

(47)
Note that without trade costs, $\phi = 1$, the real exchange rate (independently of the CPI definition) is always constant, i.e., PPP holds. But trade costs imply that productivity differences induce deviations from PPP even though all goods in our framework are traded.

4.2 Efficiency gains in creating new firms and new goods

Consider now productivity gains that reduces costs in the activities at the origin of new firms and varieties. An analysis of this type of productivity is obviously absent in standard models without entry: productivity is only confined to the manufacturing sector.

We have seen above that, under our maintained assumption $\sigma > \psi$, lower entry costs raise the number of varieties supplied at the global level. But the worldwide distribution of varieties clearly changes: more goods are produced by the Home country, less in the Foreign country. The country-specific effects of changes in the level of $\nu$ are given by:

$$\frac{1}{n} \frac{dn_t}{d\nu_t} = \frac{(\sigma - 1)}{(\sigma - \psi)} \left[ 1 + \frac{\psi \phi (1 - \psi)}{(2\sigma - 1)(\sigma - \psi(1 - \phi)) - \sigma \phi} \right]$$

(48a)

$$\frac{1}{n} \frac{dn_t^*}{d\nu_t} = -\frac{(\sigma - 1)}{(\sigma - \psi)} \frac{\phi \psi (1 - \psi)}{(2\sigma - 1)(\sigma - \psi(1 - \phi)) - \sigma \phi}$$

(48b)

Note that trade integration (a higher $\phi$) amplifies the effect of lower entry costs on Home supply of varieties. This is because new entry at Home raises competition worldwide, the more so the lower trade costs. Note also that, as the number of varieties increases at Home and decreases Abroad — employment correspondingly increases at Home and decreases Abroad.

To analyze the equilibrium response of relative prices, we first observe that production prices are not affected by $\nu_t$. The Home nominal exchange rate strengthens with efficiency gains in setting up new firms and varieties:

$$\frac{d\varepsilon_t}{d\nu_t} = \frac{-1}{2\sigma - 1 + \phi} \left[ 2 + \frac{\psi (\psi - 1)(1 - \phi)^2}{(2\sigma - 1)(\sigma - \psi(1 - \phi)) - \sigma \phi} \right] < 0$$

(49)

Interestingly, the effect on the exchange rate is stronger for higher trade costs — this is so because higher trade costs raise the rate of appreciation that brings about the required equilibrium change in relative demands for goods. Given product prices, the terms of trade move one-to-one with the nominal exchange rate: the country with lower costs of entry experiences an equilibrium appreciation of the terms of trade.

Most interestingly, however, despite stronger terms of trade and nominal exchange rate, the welfare-based real exchange rate actually depreciates:

$$\frac{dRER_t}{d\nu_t} \frac{1}{RER_t} = \frac{-\psi (1 - \phi)}{(\sigma - \psi)(1 - \phi) + (\sigma - 1)(1 + \phi)} > 0$$

(50)
The reason underlying this somewhat unexpected result is related to an important property of the welfare-based price index discussed above: a higher number of varieties reduces CPIs.

It follows that, when productivity affects the cost of creating new varieties, the sign of our comparative statics results will crucially depend on which real exchange rate is used, i.e. whether this is the welfare-based one or the ‘empirically relevant’ one. In fact, a measure of the real exchange rate including price indices that fail to account for changes in the number of varieties would move in the opposite direction relative to the welfare based real exchange rate:

\[
\frac{d \widetilde{RER}_t}{d \nu_t} \frac{1}{RER_t} = \frac{1 - \phi d\varepsilon_t}{1 + \phi d\nu_t} < 0
\]

(51)
i.e., it would point to a real exchange rate appreciation. The use of an inappropriate index would provide severely distorted information.

4.3 A comparison

The two shocks just analyzed have quite different equilibrium implications. Consider for instance the predicted GDP comovements and relative prices adjustment. Since the ratio of Home to Foreign GDP is: \(n_t / (\varepsilon_t L_t \nu_t n^*_t)\), it is easy to verify that relative GDP rises both with the improvement in productivity in the Home manufacturing sector, and asymmetric efficiency gains reducing Home entry costs relative to Foreign costs. However, depending on the sector experiencing productivity gains, a rise in Home GDP relative to Foreign GDP may be associated with either an appreciation or a depreciation of the (empirically relevant) real exchange rate — depending on whether the shock moves \(\nu\) or \(\alpha\). Predictions reminiscent of Balassa-Samuelson — richer countries have a stronger RER — would be generated by shocks to \(\nu\) only. Without proper adjustment for product varieties, the exchange rate would unambiguously appreciate with a rise in relative GDP driven by higher productivity in innovation.

Distinguishing between \(\alpha\) and \(\nu\) is a difficult and challenging empirical task. Their different macroeconomic implications, however, provide a strong motivation to collect evidence on productivity in manufacturing distinct from productivity in activities directly affecting entry costs. In this respect, it is worth mentioning that, with an economy-wide productivity change affecting all sectors, i.e. affecting equally production cost and the entry cost \((d\alpha_t = d\nu_t)\), the response of the (empirically relevant) real exchange rates is dominated by the latter: the country experiences a real depreciation.

Finally, the impact of the two productivity shocks on the terms of trade is opposite: whereas an increase in productivity in production activities leads to deterioration of the terms of trade, an increase in productivity in innovation produces an improvement. The
standard prediction of the positive association of growth and deterioration of the terms of trade only applies to gains in productivity in production activities.

Recent empirical studies have appealed to the intuition in Krugman (1989) in trying to measure the effect on the terms of trade of an increase in the supply of goods, as opposed to an increase in the varieties of goods supplied in the world markets. In a quite elaborated panel study, Debaere and Lee (2004) indeed find a positive association between spending in R&D and the terms of trade: more research is associated with stronger terms of trade. In this respect, the model in this paper provides a theoretical framework to empirical studies that, as in Debaere and Lee, try to trace the differential macroeconomic impact of efficiency and production in different sectors of the economy.

5 Market size: private expenditure and labor force

In this section, we analyze the implications of market size for the long-run equilibrium allocation with balanced trade, revisiting and generalizing some basic results of the trade literature on the ‘Home market effect.’ This literature has highlighted that in the presence of trade costs — holding the real exchange rate constant — market size raises the number of firms located in the country more than proportionally, or that market size raises factor prices. Our general equilibrium analysis contributes to this literature by reconsidering the home market effect in a two-country model, specifically endogenizing international prices and labor supply.

In other words, to a large extent our analysis of the effect of asymmetric market size (here measured by \( L_t \)) is closely related to the trade literature on the Home market effect. Nonetheless, we should note that our parameter \( L_t \) implies simultaneously higher domestic consumption demand and higher domestic labor supply. Hence, our results below will stem from the interaction of demand and supply considerations.

Consider first the general-equilibrium implications of a relatively larger Home market on the number of varieties produced worldwide and in each country. Equation (39) above shows that an increase in \( L_t \) raises the number of varieties supplied at the global level. We now show that the increase in the number of varieties produced at Home more than compensate for the fall of varieties produced in the Foreign country:

\[
\frac{1}{n} \frac{dn_t}{dL_t} = \frac{(\sigma - 1)}{(\sigma - \psi)} \left[ 1 + \frac{\sigma \phi (1 - \psi)}{(2\sigma - 1) [(\sigma - \psi)(1 - \phi)] - \sigma \phi} \right] \quad (52a)
\]

\[
\frac{1}{n} \frac{dn_t^*}{dL_t} = -\frac{(\sigma - 1)}{(\sigma - \psi)} \left[ \frac{\sigma \phi (1 - \psi)}{(2\sigma - 1) [(\sigma - \psi)(1 - \phi)] - \sigma \phi} \right] \quad (52b)
\]

Observe that when the utility from consumption is logarithmic (\( \psi = 1 \)), a larger Home market size \( L_t \) raises \( n_t \) one to one, but leaves \( n_t^* \) unaffected. In this case, the positive
implications of a larger Home market from the viewpoint of Foreign producers are exactly offset by the increasing competition from a larger number of Home firms: the number of Foreign firms remains unchanged. As shown in (32), in the more general case the one-to-one effect of a larger Home market size on Foreign sales revenue and profits needs to be compared to the effect of an increase in the number of world varieties (and competitors), measured by $(\sigma - \psi)/(\sigma - 1)$. When $\psi < 1$, this competition effect dominates the sales effect, profits decrease and Foreign firms exit.

Interestingly, the impact of the market size on the creation of Home varieties and destruction of Foreign varieties is higher in economies that are more closely integrated, i.e. they have lower trade costs (higher $\phi$). As long as trade costs are not exactly zero ($\phi = 1$), an increase in Home demand raises firms’ incentive to enter into the Home market. When trade costs are low enough, it is easy to invest in the high demand country and still export in the other market, so that the Home market effect becomes especially relevant. A small advantage (here in terms of market size) is amplified by small trade costs.

In our model, goods prices in domestic currency change only with productivity and the nominal wage rate, but not with market size. We can therefore analyze terms of trade by looking at movements in the nominal exchange rate only:

$$\frac{d\varepsilon_t}{dL_t} = \frac{- (1 - \phi)}{2\sigma - \psi (1 - \phi)} \left[ \frac{\sigma (1 - \phi)}{(\sigma - 1)} \frac{(1 - \psi)}{\sigma - \psi (1 - \phi)} - \sigma \phi \right] < 0$$

A large Home market appreciates the Home currency, and therefore improves the Home terms of trade. Note that this effect vanishes when trade costs approach zero ($\phi$ goes to 1). In this case terms of trade are constant since the increase in imports due to a larger market is exactly compensated by the increase in Home exports following the creation of new varieties.

The impact of higher market size on the welfare-based (correct) consumer Price Index is given by:

$$\frac{dP_t}{dL_t} = \frac{-1}{(1 + \phi) (\sigma - 1)} \left[ \frac{dn_t}{ndL_t} + \phi \frac{dn^*_t}{ndL_t} - \phi (\sigma - 1) \frac{d\varepsilon_t}{dL_t} \right]$$

The above expression highlights two channels through which market size affects the CPI: one is the number of goods varieties produced at Home and abroad (the sum of the first two terms in the second bracket), the other is the equilibrium movement of the exchange rate.

Now, we have seen above that the first channel is always positive ($\frac{dn_t}{ndL_t} + \phi \frac{dn^*_t}{ndL_t} > 0$), and that the nominal exchange rate is stronger in economies with larger market size. Thus, the overall impact is unambiguous: larger market size lowers the domestic welfare-based price

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7 In the case in which $\phi$ is exactly equal to 1 (trade costs are zero) no Home Market effect arises and the location of production of varieties is indeterminate.
As regards the Foreign price index, we can write:

\[
dP_t^* = \frac{1}{P} \frac{dP_t^*}{dL_t}  = \frac{-1}{(1 + \phi)(\sigma - 1)} \left[ \phi \frac{dn_t}{ndL_t} + \frac{dn_t^*}{ndL_t^*} + \phi (\sigma - 1) \frac{d\varepsilon_t}{dL_t} \right] \tag{54}
\]

Here the first channel is also negative, but less strong than in the Home country because trade costs \((\phi < 1)\) lower the impact of the higher number of varieties produced in the Home country. However, the second channel is positive: a stronger Home exchange rate raises the Foreign price level.

Using the above welfare-based price indices, it is easy to show that a larger market size unambiguously depreciates the Home currency in real terms:

\[
\frac{dRER_t}{dL_t} = \frac{1}{RER} \frac{1}{(1 - \phi)} \frac{(1 - \psi) \frac{d\varepsilon_t}{dL_t}}{(\sigma - \psi)(1 - \phi) + (\sigma - 1)(1 + \phi)} > 0 \tag{55}
\]

Note that the international price of consumption moves in the opposite direction relative to the international price of goods: a weaker real exchange rate corresponds to stronger terms of trade. Once again, this is an important implications of trade costs and endogenous entry.

Conversely, using the ‘empirically relevant’ price indices \(\tilde{P}_t\) and \(\tilde{P}_t^*\) to compute the real exchange rate \((\tilde{RER}_t = \varepsilon_t \tilde{P}_t^* / \tilde{P}_t)\), we would have a real appreciation:

\[
\frac{d\tilde{RER}_t}{dL_t} = \frac{1}{\tilde{RER}} \frac{1 - \phi \frac{d\varepsilon_t}{dL_t}}{1 + \phi \frac{d\varepsilon_t}{dL_t}} < 0 \tag{56}
\]

If the real exchange rate is not properly measured, changes in market size move the real exchange rate and the terms of trade in the same direction.

### 6 Market size: government spending

Do countries with higher government spending have a stronger real exchange rate? Does government spending strengthen the competitive position of domestic producers by creating a larger markets for their products? In this section we will use of general equilibrium model with the ‘Home market effect’ to address these issues.

Note that government spending \(G\) is different from market size \(L\) discussed in the previous section, in one important respect. Differences in \(L\) affect both goods demand and the labor force — the latter effect is obviously absent from an analysis of government spending. Moreover, we will assume that public consumption falls entirely on domestically produced goods — as case of complete home bias.

We start our analysis of the macroeconomic impact of government spending by showing
the effect of variations in $G$ on the number of varieties:

$$\frac{dn_t}{ndG_t} = \frac{[1 + \phi \psi (1 - \psi)]}{(\sigma - \psi)} \quad (57a)$$

$$\frac{dn^*_t}{ndG_t} = -\frac{\phi \psi (1 - \psi)}{(\sigma - \psi)} \quad (57b)$$

Under standard assumptions on elasticities, larger government spending is associated with a higher number of firms in the Home country. The number of firms in the Foreign country can fall or increase, depending on whether the intertemporal elasticity of substitution is below or above one, for the same reason discussed when analyzing variations in $L$. In particular, with $\psi < 1$ a higher number of Home firms implies stronger competition at the global level, which reduces the profitability of producing in the Foreign country. Note also that the effect of government spending on $n$ and $n^*$ is stronger, the deeper trade integration (higher $\phi$) is. When trade costs are low enough, it is easy to locate production in the high-demand country and still export in the other market: the competition by Home firms at global level is stronger.

Since under our normalization wages and prices do not respond to variations in government spending, the implications for the terms of trade $\varepsilon t p^*_t/p_t$ are once again captured by the nominal exchange rate:

$$\frac{d\varepsilon_t}{dG_t} = \frac{-\left(\frac{\sigma - \psi}{\sigma - 1}\right) (1 - \phi) + 1 + \phi}{[2\sigma - \psi (1 - \phi)] (\sigma - 1) + \sigma (1 - \phi) (1 - \psi)} < 0$$

The exchange rate appreciates, and therefore the terms of trade are stronger, due to the variations in the number of varieties produced in the two countries. Stronger terms of trade, coupled with higher Home employment, implies that Home GDP rises relative to the Foreign one.

The impact of higher government spending on the welfare-based (correct) consumer Price Index is given by:

$$\frac{dP_t}{dG_t} = \frac{-1}{(1 + \phi) (\sigma - 1)} \left[ \frac{dn_t}{ndG_t} + \phi \frac{dn^*_t}{ndG_t} - \phi (\sigma - 1) \frac{d\varepsilon_t}{dG_t} \right] \quad (58)$$

As discussed above, there are two channels to consider, via the number of varieties and the exchange rate. The first channel (the sum of the two first terms in the second bracket) is always positive ($\frac{dn_t}{ndG_t} + \phi \frac{dn^*_t}{ndG_t} > 0$). We have seen above the nominal exchange rate is stronger in economies with larger government spending (second channel). Thus, it is easy to verify that the overall impact is unambiguous: with (endogenously) constant nominal wages, higher government spending actually lowers the domestic welfare-based price index. As regards the Foreign price index, we can write:

$$\frac{dP^*_t}{dG_t} = \frac{-1}{(1 + \phi) (\sigma - 1)} \left[ \phi \frac{dn_t}{ndG_t} + \frac{dn^*_t}{ndG_t} + \phi (\sigma - 1) \frac{d\varepsilon_t}{dG_t} \right] \quad (59)$$
Here, the first channel is negative, but less strong than in the Home country because trade costs ($\phi < 1$) lower the impact of the higher number of varieties produced in the Home country. Conversely, the second channel is positive: a stronger Home exchange rate raises the Foreign price level. Note that without trade costs ($\phi = 1$), the real exchange rate would be constant and would not vary with $G$ (recall that PPP holds in this case).

As a result, while terms of trade are unambiguously stronger, the welfare-based real exchange rate ($RER_t = \varepsilon_t P_t^* / P_t$) tends to be weaker in countries where government spending is higher:

$$\frac{dRER_t}{dG_t} \frac{1}{RER} = \frac{-\psi(1 - \phi)}{(\sigma - \psi)(1 - \phi) + (\sigma - 1)(1 + \phi)} \frac{dz_t}{dG_t}$$

(60)

Even though the terms of trade appreciate with higher fiscal expenditure, its impact on the real exchange rate is more than offset by movements in the overall price index — movements that are driven by a changing number of varieties supplied at Home and worldwide. As discussed above, this result can be overturned if the CPI is computed using a consumption basket with a fixed number of varieties. Setting $\frac{dn_t}{dG_t} = \frac{dn_t^*}{dG_t} = 0$ in the above expressions, we obtain

$$\frac{d\widetilde{RER}_t}{dG_t} \frac{1}{\widetilde{RER}} = \frac{1 - \phi}{1 + \phi} \frac{dz_t}{dG_t}$$

(61)

Also in this case, the incorrectly measured real exchange rate would move in the same direction as the terms of trade and appreciate.

Overall, according to our model, countries with higher government spending will produce more varieties and have stronger terms of trade (competitive effect). However, consumers will tend to face a lower price of consumption (government spending weakens the real exchange rate). Employment is higher, but domestic households have higher real private consumption. The spillover on Foreign consumption is negative: higher domestic $G$ reduces private consumption spending abroad.

Our model thus predicts a positive association of public consumption with both private consumption and GDP — a result that is reminiscent of Mundell-Fleming, but occurs for reasons and over a time horizon that are entirely different from analysis in the Keynesian tradition. It does not predict stronger real exchange rates — as sometimes implied by models that do not make a distinction between terms of trade and the relative price of consumption. Indeed, in our analysis a higher domestic demand improves terms of trade — but is likely to depreciate the real exchange rate, when correctly computed. The mechanism underlying the appreciation of the terms of trade is more sophisticated than a simple aggregate demand effect on domestically produced goods. Specifically, the ‘home market effects’ highlights possible competitive advantages for domestic producers stemming from a stronger domestic demand. However, future work should extend the analysis to the case of distortionary
taxation, as to explore the trade-offs between market size and possible negative effects of a higher tax burden on investment and production.

7 Welfare and international spillovers

In this section we use numerical examples to explore the welfare properties of our model, and the sign of international spillovers from productivity gains and changes in market size and government spending.

An important issue in our setup is how we treat the welfare effects of variety. The Dixit-Stiglitz framework establishes by construction a direct relation between the elasticity of substitution among goods (determining the size of the markup) and individual preferences for goods’ varieties. However, there is no deep reason for such a strict and mechanical link. As discussed by the working paper version of the Dixit and Stiglitz (1977) paper, and more recently by Benassy (1996), love for varieties and elasticity of substitution need not be determined by the same parameter. Suppose that we replace (11) with the more general formulation:

\[ C_t = (n_t + n^*_t)^{\gamma - \frac{\sigma}{\sigma - 1}} \left[ \int_0^{n_t} C_t(h)^{1 - \frac{1}{\psi}} dh + \int_0^{n_t^*} C_t(f)^{1 - \frac{1}{\psi}} df \right]^{\frac{\psi - 1}{\psi}} \]  

(62)

As shown by Benassy (1996), \( \gamma \) measures the degree of love for variety independently of the elasticity of substitution. Notably, assuming the above more general specification of the consumption basket does not change our previous results about the equilibrium allocation. However, it has very important implications for welfare and the sign of international transmission. In the standard Dixit-Stiglitz consumption aggregate, \( \gamma = \frac{\sigma}{\sigma - 1} > 1 \): households derives utility from a larger number of varieties. Another natural benchmark is \( \gamma = 1 \), implying no love for variety even though varieties remain imperfect substitutes. In what follows, we analyze welfare and the sign of international transmission in these two polar cases: the Dixit-Stiglitz case where \( \gamma = \frac{\sigma}{\sigma - 1} > 1 \) and the case of \( \gamma = 1 \).

Parameters’ value are as follows. In our base case, the elasticity of substitution between goods \( \sigma \) is chosen to be 5. We also experiment with 2 and 10, which are two representative values in macro and trade studies. The intertemporal elasticity \( \psi \) is set equal to 0.7, but we also experiment with 0.5 and 1. Trade costs \( \tau \) are set as high as 50% — this is to be interpreted as including both transport and the costs of policy-induced trade barriers, but not retail and wholesale margins. We experiment with a low value of 20% and a high value of 70%. The latter value is close to the estimate by Anderson and van Wincoop (2004) in their survey of trade costs for the US. These authors estimate US trade costs as high
as 74%. The break down is 21% transportation costs, and 44% border-relate trade barriers (1.74 = 1.21 * 1.44). Without loss of generality, we normalize marginal disutility of labor units $\kappa = 1$.

Our results for $\gamma = \frac{\sigma}{\sigma - 1} > 1$ are shown in Table 1. We present welfare results in all cases. Results for the price index and labor effort in the base case only. In reading the table, recall that by (17), consumption and the price level are inversely related.

Table 1: the case with love for variety: $\gamma = \frac{\sigma}{\sigma - 1}$

<table>
<thead>
<tr>
<th></th>
<th>$L$</th>
<th>$G$</th>
<th>$\alpha$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base case: $\sigma = 5$</strong></td>
<td>$dP/P$</td>
<td>$dP^*/P$</td>
<td>$dP/P$</td>
<td>$dP^*/P$</td>
</tr>
<tr>
<td>$\psi = 0.5; \tau = 0.5$</td>
<td>-0.198</td>
<td>-0.024</td>
<td>-0.057</td>
<td>0.001</td>
</tr>
<tr>
<td>$dl/l$</td>
<td>-0.100</td>
<td>-0.011</td>
<td>0.233</td>
<td>-0.011</td>
</tr>
<tr>
<td>$dU$</td>
<td>0.258</td>
<td>0.030</td>
<td>-0.265</td>
<td>0.013</td>
</tr>
</tbody>
</table>

**Sensitivity analysis**

<table>
<thead>
<tr>
<th>$\sigma = 3$</th>
<th>$dU$</th>
<th>$dU^*$</th>
<th>$dU$</th>
<th>$dU^*$</th>
<th>$dU$</th>
<th>$dU^*$</th>
<th>$dU$</th>
<th>$dU^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.424</td>
<td>0.125</td>
<td>-0.466</td>
<td>0.060</td>
<td>1.059</td>
<td>0.040</td>
<td>1.550</td>
<td>0.121</td>
<td></td>
</tr>
<tr>
<td>$\sigma = 10$</td>
<td>0.123</td>
<td>0.002</td>
<td>-0.119</td>
<td>0.001</td>
<td>1.122</td>
<td>0.001</td>
<td>0.125</td>
<td>0.000</td>
</tr>
<tr>
<td>$\psi = 0.2$</td>
<td>0.278</td>
<td>0.029</td>
<td>-0.301</td>
<td>0.009</td>
<td>1.224</td>
<td>0.005</td>
<td>0.307</td>
<td>0.000</td>
</tr>
<tr>
<td>$\psi = 1$</td>
<td>0.221</td>
<td>0.029</td>
<td>-0.189</td>
<td>0.001</td>
<td>0.978</td>
<td>0.021</td>
<td>0.245</td>
<td>0.005</td>
</tr>
<tr>
<td>$\tau = 0.2$</td>
<td>0.211</td>
<td>0.070</td>
<td>-0.279</td>
<td>0.033</td>
<td>1.100</td>
<td>0.027</td>
<td>0.276</td>
<td>0.006</td>
</tr>
<tr>
<td>$\tau = 0.7$</td>
<td>0.272</td>
<td>0.018</td>
<td>-0.261</td>
<td>0.008</td>
<td>1.153</td>
<td>0.007</td>
<td>0.289</td>
<td>0.001</td>
</tr>
</tbody>
</table>

As apparent from the table, a larger Home market size, simultaneously increasing expenditure and labor supply, improves welfare at Home and Abroad. In both countries consumption rises and individual labor effort falls. Yet the welfare improvement is stronger in the Home country than abroad. Trade liberalization magnifies international spillovers: the benefits of a larger Home market size for Foreign agents increases (while the benefits for Home agents falls) when the trade cost is lowered from 70 to 20 percent.

Disregarding utility from public goods, higher government spending is beggar thyself: even though higher spending raises the number of varieties produced in the country and raises private consumption, the extra effort required to satisfy total demand dominates the welfare results. Interestingly, we can obtain an improvement in Home welfare for very low elasticities of intratemporal substitution ($\sigma$ below 1.5). In this case, agents love for variety is very large. Since the market provides far too few varieties, government spending is valued insofar as it can correct this market inefficiency.
International spillovers from a Home government spending are instead positive. Foreign welfare improves, despite the fall in Foreign consumption — the sign of the spillover is determined by the lower labor effort. Note that the highest gains are recorded when we lower the elasticity of substitution ($\sigma = 3$), and increase market integration ($\tau = .2$) relative to our base case.

Gains in manufacturing productivity at Home increases welfare at Home and Abroad. In both countries, private consumption increases, while labor effort decreases. At Home, the gain in consumption are due to the effect on prices of the exchange rate appreciation — which more than compensate for the fall in the number of varieties available to consumers. Abroad, the fall in the CPI is due to the combined effect of a weaker exchange rate, and lower supply price of imports. Once again, these effects more than compensate the reduction in the number of varieties in the market.

International spillovers are stronger when goods market are more integrated. Lowering trade costs raises the welfare gains in the Foreign country, while lowering the welfare gains at Home. The Home country gains instead with a higher elasticity $\sigma$ and a lower elasticity $\psi$. The welfare implications of gains in efficiency reducing entry costs are quite similar. Welfare improves in both countries, per effect of higher consumption and lower labor effort.

We now turn to the case without love for variety so that $\gamma = 1$. We only report the results for welfare.

Table 2: the case with love for variety; $\gamma = 1$

<table>
<thead>
<tr>
<th>Base Case</th>
<th>$L$</th>
<th>$G$</th>
<th>$\alpha$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dU$</td>
<td>$-0.172$</td>
<td>$-0.039$</td>
<td>$-0.261$</td>
<td>$-0.004$</td>
</tr>
<tr>
<td>$dU^*$</td>
<td>$1.085$</td>
<td>$0.043$</td>
<td>$0.264$</td>
<td>$-0.065$</td>
</tr>
</tbody>
</table>

Notably, the international implications of market size, government spending and productivity in innovations for the Foreign country are now negative: welfare in this countries falls with a higher $L$ and $\nu$. This is because now there is no welfare gain from an increased number of varieties — while stronger Home terms of trade hits foreign Households purchasing power. Effectively, Foreign households only face higher prices in the world market. Conversely, the international spillovers from higher productivity in Home manufacturing remain positive, and become stronger. Clearly, there is no negative welfare effect of a reduced number of varieties — while the fall in marginal costs dominates the price movements of Home products.

As discussed in the introduction, these results stress the importance of a reconsideration of the deep parameters underlying macro models. Not only the effects of productivity improvements on the equilibrium allocation depends on the nature of productivity gain
(whether it is in manufacturing or innovation), but also the sign of international spillovers depends on the relative strength of different effects from productivity — strengthening monopoly power vs. raising utility via additional goods varieties.

8 Conclusion (incomplete)

Understanding the determinants of international relative prices of a country’s output and consumption is a crucial challenge to economic theory and policy making. National wealth depends not only on the quantity of goods and services that a country can produce now and in the future, but also on the relative prices of such goods and services in the international markets. Unless asset markets provide full insurance, fluctuations in the terms of trade, i.e. the international relative price of exports in terms of imports, and the real exchange rate, i.e. the price of domestic consumption in terms of foreign consumption, induce important movements in relative national wealth, with potentially important implications for national welfare and equilibrium allocation.
References


[14] Hummels and Klenow


Appendix

In this appendix, we explore how some of our results change with a more general cost function for creating new varieties. We assume that the cost of creating a new variety, expressed in Home currency, is not constant as in the main text but takes the following form:

\[
q_t(h) = (n_t + \delta n_t^*)^\gamma w_t/\nu_t \tag{A.1a}
\]
\[
q_t^*(f) = (\delta n_t + n_t^*)^\gamma w_t^*/\nu_t^* \tag{A.1b}
\]

in the Home and the Foreign country respectively, with \( \gamma \geq 0 \) and \( 0 \leq \delta \leq 1 \). Interpreting the above expressions: it takes \((n_t + \delta n_t^*)^\gamma /\nu_t \) units of labor to create a new variety in the Home country. Note that \( \gamma = 0 \) corresponds to the case of a linear cost function as in the main text, \( \gamma > 0 \) to a convex cost function, whereas the cost of creating varieties depends positively on the number of existing varieties. The idea is that the cost of marketing and advertising that comes with the release of a new variety on the market rises when the number of existing varieties is already high. This may be due to the fact that it becomes increasingly difficult to differentiate products through marketing and advertising. More generally, \( \gamma \) can also be interpreted as a measure of how easily new varieties or new firms (if we interpret a variety as a firm) can be created. When they choose the optimal number of varieties to develop, firms take \( n_t \) and \( n_t^* \) in (A.1a) and (A.1b) as exogenous.

Because these goods are sold on both the Home and the Foreign markets, it is natural that both the Home and the Foreign produced goods enter the fixed cost function. However, there are various reasons why the number of locally produced and created goods may matter more, in terms of cost of creating new varieties, than the number of goods produced abroad. A first reason is the existence of some immobile factor of production in fixed supply in both countries — say, engineers and experts in marketing and advertising. For simplicity, we do not model explicitly this factor. It may also be that the effort of differentiating the product relative to existing ones is geared towards the Home market rather than the Foreign market. In our model, as the parameter \( \delta \) goes from 0 to 1, the cost function of creating new goods changes from ‘local’ to ‘global’: \( \delta = 0 \) corresponds to the case in which the cost of creating new varieties depends exclusively on the number of varieties produced (and created) at Home; with \( \delta = 1 \) the cost of creating varieties depends on the total number of varieties at the global level.

To simplify the analysis, we present the results of the general cost function (\( \delta \) and \( \gamma \) positive) in the log utility case (\( \psi = 1 \)).
The number of varieties produced in the Home country is unambiguously higher in the larger market, while the number of varieties produced abroad is unambiguously lower:

\[
\frac{1}{n} \frac{dn_t}{dL_t} = \frac{1 + \delta + \gamma}{(1 + \gamma)(1 + \delta + \gamma(1 - \delta))} \quad (A.2a)
\]

\[
\frac{1}{n} \frac{dn^*_t}{dL_t} = -\frac{\gamma \delta}{(1 + \gamma)(1 + \delta + \gamma(1 - \delta))} \quad (A.2b)
\]

Note that the effect of market size on the number of varieties produced at Home decreases with \(\gamma\) (which measures the convexity of the cost function), and increases with \(\delta\) (which measures the foreign content of the cost of creating new varieties). Both the general cost function version and the version with intertemporal elasticity of substitution less than one share the property that a larger Home market size leads to Foreign exit. In the main text version, the reason is the increased competition that reduces profits. In this version, the reason is higher cost of entry in the Foreign country.

The equilibrium effect on the nominal exchange rate is:

\[
\frac{d\varepsilon_t}{dL_t} = \frac{\gamma(1 + \phi)(1 - \delta) - (1 + \delta)(1 - \phi)}{[2\sigma - 1 + \phi][1 + \delta + \gamma(1 - \delta)]} \quad (A.3)
\]

The nominal exchange rate appreciates if:

\[
\gamma < \frac{(1 + \delta)(1 - \phi)}{(1 - \delta)(1 + \phi)} \quad (A.4)
\]

that is, for low values of \(\phi\) (high trade costs), low values of \(\gamma\) (weak convexity of the cost function) and high values of \(\delta\) (the local component of the cost of creating new goods is low).

The intuition is that when \(\gamma\) is low enough, firms can easily create new varieties in response to a larger demand, boosting Home exports. On the other hand, when \(\delta\) is high, the cost of creating new varieties abroad strongly depend on the number of varieties created at Home. By expanding the array of Home produced goods, a larger Home market size raises the cost of setting up firms abroad, hampering foreign exports.

The impact on the real exchange rate is:

\[
\frac{dRER_t}{dL_t} - \frac{1}{RER} = \frac{(1 - \phi)\left(1 - \frac{d\varepsilon_t}{dL_t}\right)}{2(\sigma - 1)} > 0 \quad (A.5)
\]

so that it is similar to the result in the main text.