Uncertainty and the Specificity of Human Capital.*

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Abstract

This paper studies an economy in which there is a trade-off between general and specific human capital. The trade-off arises because general human capital, while less productive, can *ex-post* be reallocated across firms. We show that the fraction of individuals with specific human capital is strictly decreasing in the extent of uncertainty in the economy, modelled as an informative signal that firms receive about their future productivity. While economies with more specific human capital are more productive, they are also more vulnerable to turbulence, modelled as a state of the world in which firms’ signals are completely uninformative.

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1 Introduction

Japan’s prolonged stagnation of the 1990’s has received a lot of attention in the recent past. A possible source of this stagnation, which has been pointed out in a number of studies, is the lack of reallocation of resources from less productive to more productive industries (see for example Kawai (1999) and Kawamoto (2004)). We argue that the observed lack of reallocation is due to the specificity of human capital rather than government policies which seem to have prevented reallocation. These protectionist policies implemented by the Japanese government may have simply been a way to fulfill commitments from optimal contracts, the same contracts that made possible the miraculous growth of the Japanese economy in previous decades.

Our argument is based on a trade-off between general and specific human capital, which arises in our model because general human capital, while less productive, can be reallocated across firms. Hence, the determining factor for the choice of human capital is the extent of uncertainty about future productivity faced by firms (and workers) when making investment decisions: economies with lower such uncertainty will have more workers with specific human capital.

We model this idiosyncratic uncertainty by introducing signals about future productivity that firms receive before making human capital investment decisions. We further introduce an aggregate state determining the ex-post accuracy of firms’ signals, which we interpret as turbulence. We parameterize this uncertainty in such a way that aggregate turbulence has no impact in economies where all individuals acquire general human capital. This turbulence, however, has a lasting negative impact in economies where some individuals acquire specific human capital. Furthermore, the extent of the negative effect is increasing in the fraction of workers with specific human capital. Our theory thus predicts that while economies with more firm-specific human capital are in general more productive, they are also more vulnerable to turbulence.

The environment we consider consists of an overlapping generations model where individuals accumulate human capital when young, produce when middle-aged, and retire when old. Cohorts of firms (projects) are clearly identified with generations of
workers. Upon paying a fixed cost of entry, firms receive a signal about their future productivity and hire young workers accordingly. Firms are only productive during the second period of their existence, as are their workers. An aggregate state (turbulence) determines the ex-post precision of the ex-ante signals that firms received.

In this model, *ex ante* idiosyncratic uncertainty determines the allocation of human capital investment across firms. More uncertainty leads to less specific human capital. Ex-post turbulence endogenously generates aggregate uncertainty, as the negative impact of turbulence is monotonically related to the fraction of individuals with specific human capital in the economy.

The issues addressed in this paper are quite different from those in the traditional literature on investment under uncertainty (see for example Dixit and Pindyck (1994)). This literature focuses on the need/desire for insurance against idiosyncratic risk faced by investors. By contrast, we consider an environment where investors are able to completely pool idiosyncratic uncertainty, and focus instead on the output (profit) maximizing allocation of the investment. The trade-off we capture is between efficiency and flexibility, which arises as firms and workers choose between general and specific human capital, as in the standard theory of human capital developed by Becker (1964).

There is a number of recent papers that investigate the effects of increased turbulence on labor markets and aggregate outcomes, focusing mainly on the recent European experience. Ljungqvist and Sargent (1998, 2002) and den Haan et al. (2001) investigate whether increased turbulence—defined as an increase in the probability that an individual looses his/her skills following a layoff or a job separation—can account for the dramatic rise in unemployment rates in Europe. While we do not directly address unemployment in this paper, our model is consistent with the finding that an increase in turbulence leads to an increase in demand for reallocation of workers. In a similar context, Wasmer (2003) shows that an economy with more general human capital (US) is able to adapt better to an increase in turbulence than an economy with more firm-specific human capital (Europe). While some of our results are similar, we derive our results without appealing to frictions to the labor market, nor do we rely on exogenous government policies. Krueger and Kumar (2003) focus
on the US-Europe growth difference since the 1980s. They build a model of education and technology adoption to argue that the European focus on specialized, vocational education might have worked well during the 1960s and 1970s, but not as well during the subsequent information age when new technologies emerged at a more rapid pace. While the underlying economic mechanism in that paper is quite different from ours, the increased frequency of switching technologies they consider could be interpreted as one of the sources of productivity uncertainty in our model.

The rest of the paper is organized as follows. The next section present the economic environment and defines equilibrium. Section 3 presents some basic results regarding efficient allocations in this economy. Depending on parameter values, four types of equilibria may emerge, as indexed by the type(s) of human capital firms use. Section 4 presents a general way to find the solution to any type of equilibrium, and shows that any economy converges at the same pace to its unique steady state. The impact of turbulence is discussed in section 5, and section 6 concludes.

2 The Environment

We consider a closed economy populated by overlapping generations of individuals who live for 3 periods. Individuals invest in human capital when young, work when middle-aged, and retire when old. Human capital investment can be of two distinct types - general and firm-specific. While investment in specific human capital is more productive, such capital cannot be used in any firm other than the one it was acquired for. Each period, a single perishable consumption good is produced by a continuum of firms using human capital as the only input. For convenience, we assume that firms also live for three periods. Upon paying a fixed cost of entry, firms draw a signal about their second-period productivity level. Using that information, they decide how many young workers to hire and what type of human capital to employ. At the beginning of the second period, the productivity of the firm is realized, the firm adjusts its labor force if desired, and production takes place.

We assume that firms have access to perfect insurance markets while individuals have no such access. Thus, as long as there is no aggregate uncertainty, firms will offer
workers a deterministic wage. We further assume that households cannot borrow and that firms pay young workers they hire an advance on their future wages. In their second period of life, firms pay middle-aged workers their promised wages (less the advance), a fraction of which is saved for retirement. Note that this arrangement is equivalent to one where firms provide workers with retirement benefits, as the cheapest way for firms to deliver a certain reservation utility corresponds to the allocation workers would choose by maximizing their utility on the corresponding budget. The savings of the middle-aged are deposited into a mutual fund which finances new firms who need resources to finance their entry cost and wage advances to young individuals.

2.1 Individuals

There is a continuum (of measure one) of individuals born every period. They live and consume for three periods. During their first period of life, individuals accumulate human capital which becomes productive when middle-aged and depreciates fully when old. All individuals have the same preferences represented by the utility function

\[ U(c^1, c^2, c^3) - v(h + g) = \ln c^1 + \beta \ln c^2 + \beta^2 \ln c^3 - \eta(h + g), \]  

where \( c^j \) represents consumption at age \( j \), \( h \) and \( g \) respectively represent specific and general human capital acquired when young, \( 0 < \beta < 1 \) is a discount factors and \( \eta \) is the utility cost of accumulating 1 unit of human capital.

2.2 Firms

Each period a measure of ex-ante identical potential entrants (firms or projects) are born. Should they choose to enter, they must pay a fixed cost \( \phi \). We denote \( \mu_t \) the measure of firms entering in period \( t - 1 \), as these firms will only produce in period \( t \).

Upon entering, each firm draws an individual signal \( s \in \{g, b\} \) about their future productivity, where the good signal \( g \) occurs with probability \( \rho \) and the bad signal occurs with complementary probability. The actual productivity levels, drawn at the beginning of the following period (\( t \)), can also take on two values: \( A \in \{A_H, A_L\} \),
where $A_H > A_L$. The probability that a firm with signal $s$ draws high productivity is denoted $\pi_s$, with $\pi_g > \pi_b$.

In Section 5 we introduce an aggregate state which determines the ex-post accuracy of the signal and takes one of two values: $z \in \{P, N\}$. The environment described here corresponds to the precise state state, $P$, while in non-precise state $N$ the signals are be \textit{ex-post} completely uninformative: the probability of a firm drawing high productivity ($\pi^N_t$) will not depend on the signal. To keep the resulting distribution of productivities unchanged, we set $\pi^N = \rho \pi_g + (1 - \rho) \pi_b$. Accordingly, one can think of state $N$ as a state of the world in which productivities are re-shuffled across firms while maintaining the same \textit{ex-post} measure of firms with low and high productivity as in state $P$.

Given the signal received in period $t-1$, each firm decides how many young workers to hire, and sign binding contracts with these workers specifying the type and amount of human capital to be acquired by the worker as well as (contingent on aggregate state) payments to workers in the current and the following periods. The labor market of young workers is competitive. Once their productivity level $A_t$ is realized, firms can hire additional workers with general human capital from other firms or “lend out” its own workers. The market for middle-aged workers with general human capital is also competitive. The production function of an individual firm is:

$$ F(H, G) = A(H + \gamma G)^\theta, $$

where $H$ and $G$ respectively denote total stocks of specific and general human capital employed by the firm, $\gamma \in [0, 1]$ measures the relative efficiency of general human capital, and $\theta \in (0, 1)$.

Notice that firm-specific human capital is more productive than general human capital, the more so the lower $\gamma$ is. The advantage of general human capital is that it offers firms flexibility, as workers with general human capital can be reallocated from unexpectedly unproductive firms to unexpectedly productive firms.

One can think of this environment as follows: many projects can be undertaken in any given period. In order to get an idea as to how productive a project is, a fixed cost needs to be paid (research cost, market analysis, etc.). At this stage labor is allo-
cated among the projects and human capital accumulation choices are made. Firms expecting to be more productive will be larger. In the following period, firms draw their actual productivity conditional on their signal from one of two distributions. In tranquil times, their signals turn out to be more informative than in “turbulent” times. Upon realization of the productivity levels reallocation of workers with general human capital takes place. Clearly, there will be more demand for such reallocation during “turbulent” times.

2.3 Financial Intermediaries

There are several allocation-equivalent ways to model the financial (and insurance) side of our economy. A transparent one is to think of a competitive mutual fund financed or created every period by middle-aged workers. This mutual fund both pools future idiosyncratic risk and advances credit to newly created firms.

Entering firms borrow from the intermediary to pay the entry cost. Upon realization of the signal about future productivity, they borrow additional funds to pay young workers they hire. Effectively, this borrowing cannot be disentangled from the insurance against future idiosyncratic productivity shocks that these young firms are purchasing. The “repayment” of these loans is contingent on the realized productivity level next period. In fact, firms could even receive further funds from the intermediary next period if their productivity level were very low relative to expectations derived from the signal. But these are financed from extraordinarily high “repayments” from the “lucky” firms and do not involve intergenerational transfers.

The ownership of firms is irrelevant, since competitive entry and full insurance against idiosyncratic risk guarantees that their value is zero. One could thus imagine that the mutual fund effectively owns all the firms it financed and gets all the revenue less wages paid. This “ownership” features unlimited liability.
2.4 Timing of Events

In each period $t$, events occur in the following order:

1. The aggregate state $z_t$ is realized.
2. Middle-aged firms learn their productivity levels $A_t$.
3. Middle-aged workers with general human capital are reallocated.
4. Middle-aged firms produce.
5. Inter-firm payments for reallocated workers take place.
6. Middle-aged firms repay their loans and insurance arrangements are fulfilled.
7. Middle-aged firms pay their workers.
8. Middle-aged firms pay whatever is left (nothing) as dividends to the mutual fund.
9. Old mutual fund pays old individuals.
10. Middle-aged workers make their savings decisions. They create and finance a new mutual fund.
11. New entrants borrow from the new mutual fund and pay entry costs.
12. Entrants receive signals about future productivity.
13. New firms make hiring and contractual decisions, borrow from the mutual fund, and pay their newly hired young workers.
14. Young workers acquire human capital, as contracted with firms.
15. Consumption takes place.

2.5 Market Arrangements

In the environment described above, firms and workers sign long-term contracts. This market arrangement is important when there is potential exposure of risk-averse workers to idiosyncratic risk. Specifically, if the efficient allocation has some workers accumulating firm-specific human capital, the uncertainty about future productivity of these firms is non-trivial, and firms but not workers have access to insurance against idiosyncratic productivity shocks, then long term contracts between firms and workers are essential to decentralize the efficient allocation.

Note also that long-term contracts would be meaningless without commitment, not only from the firm, but also from the financial intermediary (insurer) who ends up bailing out firms (and workers) with low realized productivity.
2.6 Competitive Equilibrium

Let us abstract from aggregate uncertainty for now. That is \( z_t = P \) for all \( t \). In our market arrangement, new firms offer young workers contracts that specify payments in the first and second period. These firms are maximizing profits subject to keeping the utility of young workers above some reservation value. For simplicity of exposition, we will use below an equivalent representation where firms offer young workers a second period wage and young workers choose how to allocate it across periods given market interest rate.

A Competitive Equilibrium in this environment consists of a sequence of prices \( \{w_t^H, w_t^G, r_t\}_{t=0}^\infty \), allocations for individuals \( \{c_{t-1}^1, c_t^2, c_{t+1}^3, h_t, g_t\} \) and for firms \( \{H_t(s), \Pi_t(s), G_t(A, H), R_t(A, H)\} \), and aggregates \( \{\mu_t, M_t\} \) such that

1. given prices, individuals’ allocations maximize utility subject to their (maximized) present-value budget constraint:

\[
\max_{c,h,g} \left\{ \ln c_{t-1}^1 + \beta \ln c_t^2 + \beta^2 \ln c_{t+1}^3 - \eta(h_t + g_t) \right\}
\text{s.t. } c_{t-1}^1(1 + r_t) + c_t^2 + \frac{c_{t+1}^3}{1 + r_{t+1}} \leq \max \{w_t^H h_t, w_t^G g_t\} \tag{HHP}
\]

2. given prices and stock of specific human capital, \( G_t(A, H) \) maximizes profits in the second period:

\[
R_t(A, H) \equiv \max_{G \geq 0} \left[ A(H + \gamma G)^\theta - w_t^G G \right] \tag{FP2}
\]

3. given prices, signal and \( R_t(A, H), H_t(s) \) maximizes expected present value of profits:

\[
\Pi_t(s) \equiv \max_H \left\{ E[R_t(A, H)|s] - w_t^H H \right\} \tag{FP1}
\]

4. expected profits of entrants are 0:

\[
\frac{E[\Pi_t(s)]}{1 + r_t} = \phi \tag{3}
\]
5. markets clear:

\[ M_t g_t = \mu_t \left[ \rho (\pi_g G_t(A_H, H_t(g)) + (1 - \pi_g)G_t(A_L, H_t(g))) + (1 - \rho)(\pi_b G_t(A_H, H_t(b)) + (1 - \pi_b)G_t(A_L, H_t(b))) \right] \tag{4} \]

\[ (1 - M_t) h_t = \mu_t \left[ \rho H_t(g) + (1 - \rho)H_t(b) \right] \tag{5} \]

\[ \frac{c_{t+1}^2}{1 + r_{t+1}} = \mu_t \phi + c_t^1 \tag{6} \]

where \( M_t \) is the fraction of individuals born in \( t - 1 \) who choose to accumulate general human capital.

3 Some Basic Results

Before proceeding to the analysis and implications of the model, we establish some basic results that prove useful in the analysis.

First, because of the linearity of the utility function in human capital, all individuals will accumulate the same amount of human capital, given by \((1 + \beta + \beta^2)/\eta\). We thus normalize \( \eta = (1 + \beta + \beta^2) \), so that individuals will accumulate either \( h \) or \( g \) equal to 1. Second, we state without a formal proof that whenever both general and firm-specific human capital is accumulated in equilibrium, young workers have to be indifferent between the two types of human capital. It follows that they will receive the same wage regardless of the type of labor services they supply, i.e. \( w^H_t = w^G_t = w_t \).

More importantly, the first welfare theorem holds in this environment since the interest rate has to be positive in the long run, as shown in the following proposition:

**Proposition 1** *Any competitive equilibrium is efficient.*

**Proof.** Since the economy converges to a steady state (see proposition 4 in section 4), we only have to show that the interest rate is strictly positive in that steady state. To
see this, note that consumers with logarithmic utility allocate income across periods in fixed proportions. If we let \( \hat{W} \) denote the steady state lifetime wealth of individuals in terms of middle-age consumption goods, then

\[
\frac{c^3}{1 + \hat{r}} = \hat{W} \frac{\beta^2}{1 + \beta + \beta^2},
\]

\[
c^1(1 + \hat{r}) = \hat{W} \frac{1}{1 + \beta + \beta^2}.
\]

These expressions imply that equation (6) cannot hold in the steady state unless the interest rate is strictly positive.

We use this equivalence to show two basic results below. The first result establishes that in any efficient allocation, all individuals fully specialize either in specific or general human capital.

**Proposition 2** Efficient allocations cannot have a positive measure of individuals acquiring both firm-specific and general human capital.

**Proof.** Suppose, by contradiction, that in an efficient allocation a firm \( i \) hires a positive measure \( \lambda \) of individuals who make fraction \( \kappa \) of their human capital investment general and fraction \( (1 - \kappa) \) firm-specific. Consider an alternative allocation where \( \kappa \lambda \) individuals only acquired general human capital and the remaining \( (1 - \kappa) \lambda \) only acquired firm-specific human capital. The alternative allocation results in weakly greater output for all productivity levels, while keeping the cost of acquiring human capital constant. The output of the two allocations is the same if there is no ex-post reallocation, and the output of the alternative allocation is greater if some of the workers under consideration are reallocated to other firms (as long as \( A_L > 0 \)). When workers with general human capital are reallocated (\( \kappa \lambda \) of them), there is now \( (1 - \kappa) \lambda \) workers with specific human capital who remain productively employed in firm \( i \). That portion of the human capital stock was lost during reallocation in the original allocation. Note further that the event in which the reallocation occurs has strictly positive probability, since otherwise firm \( i \) would make all its employees obtain firm-specific human capital only. So, the alternative allocation always delivers at least as much output as the original, and delivers strictly more output (revenue)
with strictly positive probability, while keeping the costs constant. This implies that the original allocation could not have been efficient. ■

The second result establishes sufficient conditions for an economy to feature a positive measure of individuals with firm-specific human capital.

**Proposition 3** If the low productivity level is bounded away from zero \((A_L > 0)\), then the measure of individuals with firm-specific human capital is strictly positive in any efficient allocation.

**Proof.** This follows directly from the fact that once all firms have received their idiosyncratic productivity shock, even the firm with the smallest productivity shock will be operating and so would hire a positive amount of firm-specific human capital. In other words, even if a firm knew for sure that its productivity level tomorrow will be low, it would still want to hire workers with specific human capital. ■

4 Equilibria

In this section we study the different types of equilibria that this economy can generate, assuming that there is no aggregate uncertainty in the sense that the signal is always informative. Four types of equilibrium may emerge, depending on parameter values. The first type is one where all individuals acquire general human capital, whereas the second is one where all individuals acquire specific human capital. In the last two cases, a fraction of individuals acquire specific human capital and the rest acquire general human capital. What differentiates these cases is whether all ex-post productive firms hire generalists or only those who received a bad signal hire generalists.

4.1 Solving for an Equilibrium

In the appendix, we show in details how to construct the full solution to the model for each case. As it turns out, the general way to find a solution is common to all four cases. The algorithm proceeds as follows.
At the beginning any period $t$, the state of the economy is given by the number of firms that entered in the previous period as well as the amount of consumption that is currently promised to the old, $(\mu_t, c^3_t)$. We now briefly demonstrate how to obtain $(\mu_{t+1}, c^3_{t+1})$ from the current state.

We show in the appendix that in each case the labor market clearing condition implies that the wage rate in period $t$ is completely determined by the current measure of producing firms:

$$ w_t = B\mu_t^{1-\theta}, \quad (7) $$

where $B$ is a constant, the value of which depends on the type of equilibrium. Since the labor share of output is given by $\theta$, i.e. $w_t = \theta Y_t$, we can write aggregate output as a function of $\mu$:

$$ Y_t = \frac{B\mu_t^{1-\theta}}{\theta}. \quad (8) $$

The aggregate income of middle-aged individuals is given by $Y_t - c^3_t \equiv I_t$. With logarithmic preferences, middle-aged individuals will save a fraction of their income, $X_t = \left(\frac{\beta}{1+\beta}\right) I_t$, and consume the remainder, $c^2_t = \left(\frac{1}{1+\beta}\right) I_t$. The market clearing condition for savings and investment then implies that

$$ X_t = \left(\frac{\beta}{1+\beta}\right) (Y_t - c^3_t) = \mu_{t+1}\phi + \frac{w_{t+1}}{(1+r_{t+1})(1+\beta + \beta^2)}. \quad (9) $$

In other words, the resources saved by the current middle-aged will be used to pay the consumption of young individuals in period $t$, which they optimally chose to be a fraction of the wage they will receive tomorrow, as well as the entry cost of firms that will produce tomorrow.

Next we use the free entry condition to establish a relationship between the interest rate and the measure of entering firms. First, notice that expected profits for entering firms at date $t$ can be written as

$$ E[\Pi_{t+1}(s)] = \rho E[F_{t+1}(H, G)|g] + (1-\rho) E[F_{t+1}(H, G)|b] $$

$$ -w_{t+1}\left[\rho \pi_y G_{t+1}(A_H, H_{t+1}(g)) + \rho H_{t+1}(g) + (1-\rho) \pi_y G_{t+1}(A_H, H_{t+1}(b)) + (1-\rho) H_{t+1}(b) \right]. \quad (10) $$
The first term in (10) corresponds to output per firm in period \( t + 1 \) \( (Y_{t+1}/\mu_{t+1}) \), and the term multiplying \( w_{t+1} \) is equal to the total number of individuals working in period \( t + 1 \) \( (1/\mu_{t+1}) \) (see equations (4) and (5)).\(^1\) The free entry condition (3) can thus be written as

\[
\frac{Y_{t+1}/\mu_{t+1} - w_{t+1}/\mu_{t+1}}{1 + r_{t+1}} = \phi,
\]

or, using (7) and (8),

\[
1 + r_{t+1} = \frac{(1 - \theta)B}{\theta \phi} \mu_{t+1}^{-\theta}.
\]

We can thus use equation (11) together with the equation for output (8) in equation (9) to solve for the measure of firms that will be producing in period \( t + 1 \):

\[
\mu_{t+1} = \Lambda_1 \mu_t^{1-\theta} - \Lambda_2 c^3_t,
\]

where

\[
\Lambda_1 = \left( \frac{\beta B}{\theta (1+\beta)} \right) \left[ \phi + \frac{\theta \phi}{(1-\theta)(1+\beta+\beta^2)} \right]^{-1},
\]

\[
\Lambda_2 = \left( \frac{\beta}{1+\beta} \right) \left[ \phi + \frac{\theta \phi}{(1-\theta)(1+\beta+\beta^2)} \right]^{-1}.
\]

Finally, consumption of the old in period \( t+1 \) is given by the return on the period \( t \) savings of the middle-aged, that is,

\[
c^3_{t+1} = (1 + r_{t+1}) \left( Y_t - c^2_t \right),
\]

where \( r_{t+1} \) is given by (11).

While this algorithm is independent of the type of equilibrium, the constant \( B \) does depend on the the type of equilibrium under study, which itself depends on parameter values. We now partition the parameter space into four regions corresponding to each type of equilibrium.

### 4.2 Types of Equilibria

Parameter values completely determine the type of equilibrium we obtain. As Proposition 3 shows, the only way to have an economy in which all individuals acquire

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\(^1\)Equation (10) omits generalists working for firms receiving low productivity, as these firms never hire generalists.
general human capital is when the low productivity level is zero \((A_L = 0)\). Otherwise a positive fraction of individuals will be specialists. We show in the Appendix that when the ratio of relative productivities is such that

\[
\frac{A_H}{A_L} > \frac{1 - \pi_g}{\gamma - \pi_g},
\]

then the equilibrium is one where all firms with high realized productivity level, regardless of the signal they received, hire generalists. Similarly, when the relative productivities is such that

\[
\frac{1 - \pi_g}{\gamma - \pi_g} \geq \frac{A_H}{A_L} > \frac{1 - \pi_b}{\gamma - \pi_b},
\]

then the equilibrium is one where only firms that received a bad signal but a high realized productivity level hire generalists. Finally, if

\[
\frac{A_H}{A_L} \leq \frac{1 - \pi_b}{\gamma - \pi_b},
\]

then all individuals will be specialists.

It should be noted that these conditions completely characterize the path of the economy, in the sense that if we start in one of these cases, the economy will remain in that case.\(^2\)

### 4.3 Convergence

For any of these cases, the transitional dynamics are very tractable. The following lemma establishes that we can take initial conditions to have the property that promised consumption to the old are a constant fraction of income.

**Lemma 1** For any \((c^3_t, \mu_t)\), promised consumption to the old in period \(t + 1\) is a constant fraction of output in period \(t + 1\), i.e. \(c^3_{t+1} = \alpha Y_{t+1}\).

**Proof.** First note that using equation (11), equation (9) implies that

\[
X_t = \mu_{t+1} \left[ \phi + \frac{\theta \phi}{(1 - \theta)(1 + \beta + \beta^2)} \right].
\]

\(^2\)This may not be the case with aggregate uncertainty.
Using (11) and the first equality in (9), we can also rewrite equation (13) as

\[ c_{t+1}^3 = \left( \frac{(1 - \theta)B}{\theta \phi} \right) \mu_{t+1}^{-\theta} \left( 1 + \frac{\beta}{\beta} \right) X_t \]

\[ = \left( \frac{(1 - \theta)B}{\theta \phi} \right) \left[ \phi + \frac{\theta \phi}{(1 - \theta)(1 + \beta + \beta^2)} \right] \left( 1 + \frac{\beta}{\beta} \right) \mu_{t+1}^{1-\theta}. \]

Finally, using (8), it follows that \( c_{t+1}^3 = \alpha Y_{t+1} \), where

\[ \alpha = \left( \frac{(1 - \theta)B}{\theta \phi} \right) \left[ \phi + \frac{\theta \phi}{(1 - \theta)(1 + \beta + \beta^2)} \right] \left( 1 + \frac{\beta}{\beta} \right) \left( \frac{\theta}{B} \right) \]

\[ = \left[ \frac{(1 - \theta)(1 + \beta + \beta^2) + \theta}{1 + \beta + \beta^2} \right] \left( 1 + \frac{\beta}{\beta} \right). \]

This lemma establishes that we can take initial conditions to imply that consumption promised to the old is a fraction \( \alpha \) of output, which is useful for the following proposition.

**Proposition 4** In any of the four possible equilibria, the economy converges to its unique steady state at rate \( 1 - \theta \).

**Proof.** Following Lemma 1, we can take \( c_t^3 = \alpha Y_t \). Then equation (12) implies that

\[ \mu_{t+1} = \left[ \frac{\theta \Lambda_1}{B} - \alpha \Lambda_2 \right] Y_t. \]

It follows that \( \mu_{t+1}/\hat{\mu} = Y_t/\hat{\dot{Y}} \), where \( \hat{\dot{x}} \) indicates the steady state value of variable \( x \). Finally, equation (8) implies that

\[ \frac{Y_{t+1}}{\hat{\dot{Y}}} = \left( \frac{\mu_{t+1}}{\hat{\mu}} \right)^{1-\theta} \]

\[ = \left( \frac{Y_t}{\hat{\dot{Y}}} \right)^{1-\theta}. \]
5 The Impact of Turbulence

Although we cannot solve analytically for the solution to the model with aggregate uncertainty, we nevertheless illustrate below the impact of a one time shock to the precision of firms’ signals. We do so by modelling an unexpected turbulent state, and making sure that the equilibrium allocation is the limit of equilibrium allocations of a general model with aggregate uncertainty as the probability of the turbulent state goes to zero.

To do so, we first establish that in the model with aggregate uncertainty and complete markets, resources in both states are allocated in the same proportions between the young, middle-aged and old individuals. We use this result to allocate resources following an unexpected one-time shock in an economy without aggregate uncertainty.

Let $\delta$ denote the probability of the aggregate state $z = P$ and let $q_t(z)$ denote the price in period $t$ of an Arrow security that pays one unit of consumption good in period $t + 1$ contingent on the aggregate state being $z$. Finally, let $DI_t = Y_t - c_t^3$ denote the disposable income of middle-aged workers in period $t$.

Proposition 5 The fractions of total output allocated to old and middle-aged workers are the same in all states:

\[
\frac{c_t^3(P)}{DI_t(P)} = \frac{c_t^3(N)}{DI_t(N)}.
\]

Proof. Utility maximization by middle-aged workers implies the following state-contingent savings:

\[
q_{t-1}(P)c_t^3(P) = \frac{\delta \beta}{1 + \beta} DI_{t-1},
\]

\[
q_{t-1}(N)c_t^3(N) = \frac{(1-\delta) \beta}{1 + \beta} DI_{t-1},
\]

so that

\[
\frac{c_t^3(P)}{c_t^3(N)} = \frac{\delta}{1-\delta} \frac{q_{t-1}(N)}{q_{t-1}(P)}. \tag{17}
\]
Similarly, utility maximization of young workers in period \(t-1\) implies that

\[
q_{t-1}(P)DI_t(P) = \frac{(\beta + \beta^2)\delta}{1 + \beta + \beta^2} PVI_{t-1},
\]

\[
q_{t-1}(N)DI_t(N) = \frac{(\beta + \beta^2)(1 - \delta)}{1 + \beta + \beta^2} PVI_{t-1},
\]

where \(PVI_t\) denotes the present value of lifetime income of young individuals born in period \(t\). It follows that

\[
\frac{DI_t(P)}{DI_t(N)} = \frac{\delta}{1 - \delta} \frac{q_{t-1}(N)}{q_{t-1}(P)}. \tag{18}
\]

The statement of the Proposition follows from equations (17) and (18).

To illustrate the effects of turbulence, we study an economy where the probability \(\delta\) of the turbulent state \(N\) is 0. For illustrative purposes we take an economy where all workers acquire specific human capital (condition (16) holds). In this case,

\[
Y_t(P) = \Gamma^{1-\theta} \mu_t^{1-\theta},
\]

where constant \(\Gamma = \rho E_{\theta}^{1-\theta} + (1 - \rho) E_{b}^{1-\theta}\), and where \(E_{\theta} = \pi_{\theta} A_H + (1 - \pi_{\theta}) A_L\) and \(E_{b} = \pi_{b} A_H + (1 - \pi_{b}) A_L\) are expected productivities given respective signals.

If, unexpectedly, signals are completely uninformative ex-post (all firms have equal chance \(\pi^N = \rho\pi_{\theta} + (1 - \rho)\pi_{b}\) of high productivity), then total output produced is

\[
Y_t(N) = \frac{\mu_t^{1-\theta}}{\Gamma^{1-\theta}} (\pi A_H + (1 - \pi) A_L) \left( \rho E_{\theta}^{1-\theta} + (1 - \rho) E_{b}^{1-\theta} \right).
\]

The output following a shock, i.e. in state \(N\), is lower than expected output, i.e. in state \(P\), as can be seen in equation (19). The proportional decrease in output in the period of an unexpected “noise shock” is

\[
\frac{Y_t(P) - Y_t(N)}{Y_t(P)} = \rho(1 - \rho)(A_H - A_L)(\pi_{\theta} - \pi_{b}) \frac{E_{\theta}^{1-\theta} - E_{b}^{1-\theta}}{\rho E_{\theta}^{1-\theta} + (1 - \rho) E_{b}^{1-\theta}}. \tag{19}
\]

Observe that the proportional decrease in output relative to the “tranquil” state is the same in and out of steady state, as equation (19) is independent of \(\mu\). In other words, (19) shows the extent of the damage of turbulence relative to where the
economy would have been without turbulence, not relative to where the economy was last period.

Also note that the extent of damage (decrease in output) is greater in economies with higher expected precision of signals (higher $\pi_g$ and lower $\pi_b$). Intuitively, a higher expected precision of signals increases output in the precise state ($Y_t(P)$) and decreases output in the non-precise state ($Y_t(N)$), thus making the extent of damage greater in economies with higher expected precision. This suggests that economies in which a high fraction of the labor force are specialists are more vulnerable to turbulence.¹

To summarize, equation (19) establishes the extent of the initial downturn in the economy. Proposition 5 establishes how the allocation of resources is distributed among individuals of different ages in the period of the shock, thereby guarantying that the subsequent convergence is as established in proposition 4.

6 Conclusion

This paper studies an economy in which there is a trade-off between general and specific human capital. The trade-off arises because general human capital is on the one hand less productive, but one the other hand more flexible than specific human capital. We show that the fraction of individuals with specific human capital is strictly decreasing in the extent of uncertainty in the economy. In an economy in which there are two productivity levels, this uncertainty is modelled as an informative signal that firms receive about their future productivity.

We further introduce a state of the world in which firms’ signals are completely un-informative, which we interpret as turbulence. We show that the impact of turbulence is larger in economies that have a larger share of their labor force trained in specific human capital. In other words, economies with low uncertainty are more susceptible to turbulence. We conjecture that the common practice of life-time employment in Japan, which is conducive to the accumulation of specific human capital, may be

¹A formal proof of this conjecture is forthcoming.
responsible not only for the rapid growth of the economy in the 1960’s and 1970’s, but also for the prolonged stagnation of the 1990’s. Whether Japan should abandon the practice of life-time employment depends on whether turbulence experienced in the 1990’s was a one-time shock or a new regime.
References


A  Equilibria: Details

Since all types of equilibrium are very similar, we only present in details how to construct an equilibrium for one of these cases. For the other cases, we only present the value of the constant $B$, which is sufficient to construct an equilibrium as shown in section 4.

A.1  All High Productivity Firms Hire Generalists

A.1.1  Middle-Aged Firms’ Problems

There are four (4) types of middle-aged firms: firms who received a good signal when young can either have a high or low productivity level, and similarly for firms who received a bad signal.

**Good Signal ($s = g$), High Productivity ($A = A_H$)**  Firms who received a good signal when young decided to hire and train $H(g)$ individuals in their firm-specific human capital. Their problem when middle-aged is as follows:

$$R(A_H, H(g)) \equiv \max_{G \geq 0} \left\{ A_H(H(g) + \gamma G) - w^G G \right\},$$

where $w^G$ is the wage rate of generalists and $G$ is the number of generalists to be hired. Optimality implies that

$$w^G = \theta A_H \gamma (H(g) + \gamma G)^{\theta - 1}, \quad (20)$$

and the number of generalists to hire is given by

$$G(g) = \frac{1}{\gamma} \left[ \left( \frac{\gamma \theta A_H}{w^G} \right)^{1/\theta} - H(g) \right]. \quad (21)$$

$^4$Since firms with low productivity do not hire generalists, it should be clear that $G(g)$ denotes the number of generalists hired by firms who received a good signal (and a high productivity level).
The revenue function for these firms is therefore given by

\[ R(A_H, H(g)) = A_H \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{\theta}{1-\theta}} - \frac{w^G}{\gamma} \left[ \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{1}{1-\theta}} - H(g) \right] \]

\[ = (1 - \theta) A_H \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{\theta}{1-\theta}} + \frac{w^G}{\gamma} H(g). \]

**Good Signal (s = g), Low Productivity (A = A_L)** Although these firms have a low realized productivity, they still received a good signal when young and thus also decided to hire and train \( H(g) \) individuals. Since these firms will definitely not hire generalists (they would want to get rid of some of their specialists, so \( G = 0 \) for these firms), their revenue function when middle-aged is given by

\[ R(A_L, H(g)) \equiv A_L H(g)^{\theta}. \]

**Bad Signal (s = b), High Productivity (A = A_H)** Firms who received a bad signal when young decided to hire and train \( H(b) \) individuals in their firm-specific human capital. Their problem when middle-aged is as follows:

\[ R(A_H, H(b)) \equiv \max_{G \geq 0} \left\{ A_H (H(b) + \gamma G)^{\theta} - w^G G \right\}. \]

Optimality implies that

\[ w^G = \theta A_H \gamma (H(b) + \gamma G)^{\theta - 1}, \]

and the number of generalists to hire in this case is given by\(^5\)

\[ G(b) = \frac{1}{\gamma} \left[ \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{1}{1-\theta}} - H(b) \right], \]

so their revenue function is given by

\[ R(A_H, H(b)) = (1 - \theta) A_H \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{\theta}{1-\theta}} + \frac{w^G}{\gamma} H(b). \]

\(^5\)Since firms with low productivity do not hire generalists, it should be clear that \( G(b) \) denotes the number of generalists hired by firms who received a bad signal (and a high productivity level).
Bad Signal \((s = b)\), Low Productivity \((A = A_L)\) These firms realized a low productivity after getting a bad signal, so they hired \(H(b)\) specialists and will not hire any generalists \((G = 0)\), so their revenue function when middle-aged is given by

\[
R(A_L, H(b)) \equiv A_L H(b)^\theta.
\]

From equations (20) and (22), we get the following relationship between high productivity firms’ hiring decisions:

\[
H(g) - H(b) = \gamma[G(b) - G(g)].
\]

A.1.2 Young Firms’ Problems

There are two types of young firms: those with a good signal and those with a bad signal. Both types of firm need to make a hiring decision when young based on their expected profits when middle-aged.

Good signal \((s = g)\) The problem for firms who receive a good signal is as follows:

\[
\max_H \left\{ \pi_g R(A_H, H) + (1 - \pi_g) R(A_L, H) - w^H H \right\},
\]

where \(w^H\) is the wage rate per unit of firm specific human capital. Optimality thus requires that

\[
w^H = \pi_g \frac{w^G}{\gamma} + (1 - \pi_g) \theta A_L H^{\theta - 1},
\]

which means that

\[
H(g) = \left( \frac{\gamma (1 - \pi_g) \theta A_L}{\gamma w^H - \pi_g w^G} \right)^{\frac{1}{1 - \theta}}. \tag{24}
\]

Bad signal \((s = b)\) The problem for firms who receive a bad signal when young is as follows:

\[
\max_H \left\{ \pi_b R(A_H, H) + (1 - \pi_b) R(A_L, H) - w^H H \right\}.
\]

Optimality thus requires that

\[
w^H = \pi_b \frac{w^G}{\gamma} + (1 - \pi_b) \theta A_L H^{\theta - 1},
\]
which means that

\[ H(b) = \left( \frac{\gamma (1 - \pi_b) \theta A_L}{\gamma w^H - \pi_b w^G} \right)^{\frac{1}{1-\theta}}. \]

(25)

Notice that by replacing the expression for \( H(g) \) (equation (24)) into the expression for \( G(g) \) (equation (21)), we have

\[ G(g) = \left( \frac{\gamma}{\gamma-w^G} \right)^{\frac{1}{1-\theta}} \left[ \left( \frac{A_H}{w^G} \right)^{\frac{1}{1-\theta}} - \left( \frac{(1 - \pi_g) A_L}{\gamma w^G - \pi_g w^G} \right)^{\frac{1}{1-\theta}} \right], \]

which is strictly positive if the following condition holds:6

\[ \frac{A_H}{A_L} > \frac{1 - \pi_g}{\gamma - \pi_g}, \]

that is, this type of equilibrium occurs if (i) high productivity firms are sufficiently more productive than low productivity firms, (ii) the probability of getting high productivity conditional on having received a good signal is sufficiently high, and (iii) if firm-specific human capital is sufficiently more productive than general human capital. Another way to write the previous expression is as follows:

\[ \gamma A_H > E[A|s = g], \]

which simply says that the good productivity level is sufficiently high and sufficiently productive relative to expected productivity once a firm knows her signal.

Similarly, by replacing the expression for \( H(b) \) (equation (25)) into the expression for \( G(b) \) (equation (23)), we can see that \( G(b) \) will be strictly positive if the following condition holds:7

\[ \frac{A_H}{A_L} > \frac{1 - \pi_b}{\gamma - \pi_b}, \]

which means that when this condition is not satisfied, there will be no generalists in equilibrium.

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6This condition corresponds to equation (14) in the main text.
7This condition corresponds to equation (15) in the main text.
A.1.3 Labor Market Clearing Condition

The labor market clearing condition is that the total number of workers hired adds up to unity. Since a fraction $\rho$ of firms, of which there are a total of $\mu$, receive a good signal and hire $H(g)$ individuals, and similarly for firms who receive a bad signal, we have

$$1 - M = \mu \rho H(g) + \mu (1 - \rho) H(b).$$

Similarly, middle-aged firms with good signals who’s productivity turns out to be high ($\rho \pi g$) hire $G(g)$ generalists, and similarly for firms with bad signals, so that

$$M = \mu \rho \pi g G(g) + \mu (1 - \rho) \pi b G(b).$$

Our market clearing condition thus reads

$$\mu \rho (\frac{\gamma (1 - \pi_g) \theta A_L}{\gamma w^H - \pi_g w^G})^{\frac{\theta}{1-\theta}} + \mu (1 - \rho) (\frac{\gamma (1 - \pi_b) \theta A_L}{\gamma w^H - \pi_b w^G})^{\frac{\theta}{1-\theta}}$$

$$+ \mu \rho \pi_g \frac{1}{\gamma} \left[ (\frac{\gamma \theta A_H}{w^G})^{\frac{\theta}{1-\theta}} - \left( \frac{\gamma (1 - \pi_g) \theta A_L}{\gamma w^H - \pi_g w^G} \right)^{\frac{\theta}{1-\theta}} \right]$$

$$+ \mu (1 - \rho) \pi_b \frac{1}{\gamma} \left[ \left( \frac{\gamma \theta A_H}{w^G} \right)^{\frac{\theta}{1-\theta}} - \left( \frac{\gamma (1 - \pi_b) \theta A_L}{\gamma w^H - \pi_b w^G} \right)^{\frac{\theta}{1-\theta}} \right] = 1,$$

which, since $w^S = w^G = w_t$, simplifies to

$$w_t = B \mu_t^{1-\theta},$$

where $B$ is defined by

$$B^{\frac{1}{1-\theta}} = \theta \rho \left[ \pi_g A_H (\gamma \theta A_H)^{\frac{\theta}{1-\theta}} + (1 - \pi_g) A_L \left( \frac{\gamma (1 - \pi_g) \theta A_L}{\gamma - \pi_g} \right)^{\frac{\theta}{1-\theta}} \right]$$

$$+ \theta (1 - \rho) \left[ \pi_b A_H (\gamma \theta A_H)^{\frac{\theta}{1-\theta}} + (1 - \pi_b) A_L \left( \frac{\gamma (1 - \pi_b) \theta A_L}{\gamma - \pi_b} \right)^{\frac{\theta}{1-\theta}} \right].$$
A.2 Only High Productivity Firms with Bad Signals Hire Generalists

The only difference with the previous section is that $G(g) = 0$. It follows that the constant $B$ from the labor market clearing condition is now defined as

$$B^{\frac{1}{1-\sigma}} = \rho \left( \theta E_g \right)^{\frac{1}{1-\sigma}} + \theta(1-\rho) \left[ \pi_b A_H \left( \gamma \theta A_H \right)^{\frac{1}{1-\sigma}} + (1-\pi_b) A_L \left( \frac{\gamma(1-\pi_b)\theta A_L}{\gamma - \pi_b} \right)^{\frac{1}{1-\sigma}} \right],$$

where $E_g = \pi_g A_H + (1-\pi_g) A_L$.

A.3 The Economy with only Firm-Specific Human Capital

The only difference with the previous case is that $G(g) = G(b) = 0$. It follows that the constant $B$ from the labor market clearing condition is now defined as

$$B^{\frac{1}{1-\sigma}} = \rho \left( \theta E_g \right)^{\frac{1}{1-\sigma}} + (1-\rho) \left( \theta E_b \right)^{\frac{1}{1-\sigma}},$$

where $E_b = \pi_b A_H + (1-\pi_b) A_L$.

A.4 The Economy with only General Human Capital

Finally, we have

$$B^{\frac{1}{1-\sigma}} = \pi \left( \gamma^\theta \theta A_H \right)^{\frac{1}{1-\sigma}} + (1-\pi) \left( \gamma^\theta \theta A_L \right)^{\frac{1}{1-\sigma}},$$

where $\pi$ is the fraction of firms who receive a high productivity shock. Note that this expression simplifies further as $A_L = 0$ is a necessary condition for this type of equilibrium to occur.